Exact Bit Error Probability of Orthogonal Space-Time Block Codes with Quadrature Amplitude Modulation [1]

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Abstract

In this paper, for the linear orthogonal space-time block codes including the orthogonal space-time codes introduced by Alamouti [1], Tarokh [7], and Xia [5], the exact expression for the pairwise error probability in the slow-varying Rayleigh fading channel is derived in terms of the message symbol distance between two message vectors rather than the codeword symbol distance between two transmitted orthogonal space-time codewords. Using one-dimensional component symbol error probability, the exact closed form expressions for the bit error probability of linear orthogonal space-time codes in the slow-varying Rayleigh fading channel are derived for QPSK, 16-QAM, 64-QAM, and 256-QAM.

1. Introduction

Alamouti proposed a simple transmit diversity scheme with two transmit antennas which employs the 2×2 complex orthogonal design [1] and the space-time block codes which generalize Alamouti's scheme were introduced in [7].

Recently, Simon [4] and Taricco and Biglieri [6] independently worked on the exact expression of the pairwise error probability for space-time codes and an approximation of the bit error probability for some space-time trellis codes.

Lu, Wang, Kumar, and Chugg [3] also derived the exact pairwise error probability and the exact bit error probability of BPSK and QPSK for some orthogonal space-time block codes. Independently, we derived the exact closed form expression of the bit error probability for any orthogonal space-time block codes with square QPSK, 16-QAM, 64-QAM, and 256-QAM. It is clear that QPSK can be considered as 4-QAM.

2. System Model

Let L_t and L_r be the numbers of transmit antennas and receive antennas in the wireless communication systems, respectively. The codeword $\mathbf{X}$ is a matrix with N rows and L_t columns, which consists of $N \cdot L_t$ complex valued symbols. Let x_i^n be the n-th row and the i-th column element of the codeword matrix $\mathbf{X}$. At time n, the L_t symbols in each row of $\mathbf{X}$ are simultaneously transmitted via the L_t transmit antennas, $1 \le n \le N$. We assume slow Rayleigh fading channel, where fading is assumed to be constant over the duration of a codeword matrix. Let $\alpha_{i,j}$ be the channel coefficient from the i-th transmit antenna to the j-th receive antenna, the set of which are independent complex Gaussian random variables with zero mean and unit variance. Let $\mathbf{A} = [\alpha_{i,j}]$ be an $L_t \times L_r$ channel matrix and known to the receiver. At time n, the received signal at j-th receive antenna is denoted by y_j^n. Let w_j^n be the independently and identically distributed (i.i.d.) complex Gaussian noise with zero mean and unit variance. Let $\mathbf{Y}$ and $\mathbf{W}$ be $N \times L_r$ matrices which consist of y_j^n's and w_j^n's, respectively. Thus we have

$$\mathbf{Y} = \sqrt{\frac{\overline{\rho}}{E_m}}\mathbf{X} \cdot \mathbf{A} + \mathbf{W}$$

where $\overline{\rho}$ is the average signal to noise ratio (SNR) and E_m the average energy transmitted from all L_t transmit antennas combined during a symbol period.

In this paper, square and rectangular orthogonal space-time block codes are considered, where the codeword matrices have the property of columnwise orthogonality. The message vector of length L_s is denoted by $\mathbf{s}$, which is given by

$$\mathbf{s} = (s_1, s_2, \cdots, s_{L_s}). \tag{1}$$

Let b_s be the number of bits per message symbol s_k, which is determined by employed modulation scheme. Since QAM is defined in the two-dimensional signal space, it is necessary to split the L_s-dimensional complex message vector in (1) into the $2L_s$-dimensional real vector given by

$$\mathbf{s}' = (s_{1,x}, s_{1,y}, s_{2,x}, s_{2,y}, \cdots, s_{L_s,x}, s_{L_s,y}) \tag{2}$$

where $s_k = s_{k,x} + j s_{k,y}, 1 \le k \le L_s$.

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Let $\mathcal{C}(\cdot)$ be a mapping from an L_s-tuple complex message vector to the columnwise orthogonal $N \times L_t$ space-time block codeword matrix given by $\mathbf{X} = \mathcal{C}(\mathbf{s})$.

It is said to be a *linear space-time block code* if each element of a codeword matrix $\mathcal{C}(\mathbf{s})$ is a linear combination of the message symbols s_k and their complex conjugates. Various linear complex orthogonal space-time block codes are introduced in [1, 5, 7] , including

$$\mathcal{C}_1 = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}, \mathcal{C}_2 = \begin{pmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(-s_1-s_1^*+s_2-s_2^*)}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{(s_2+s_2^*+s_1-s_1^*)}{2} \end{pmatrix},$$

$$L_s = 2, N = L_t = 2, \quad L_s = 3, N = 4, L_t = 3, \text{code rate} = 3/4,$$

$$\mathcal{C}_3 \quad = \quad \begin{pmatrix} s_1 & s_2 & s_3 & 0 & s_4 \\ -s_2^* & s_1^* & 0 & s_3 & s_5 \\ s_3^* & 0 & -s_1^* & s_2 & s_6 \\ 0 & s_3^* & -s_2^* & -s_1 & s_7 \\ s_4^* & 0 & 0 & -s_7^* & -s_1^* \\ 0 & s_4^* & 0 & s_6^* & -s_2^* \\ 0 & 0 & s_4^* & s_5^* & -s_3^* \\ 0 & -s_5^* & -s_6^* & 0 & s_1 \\ s_5^* & 0 & s_7^* & 0 & s_2 \\ -s_6^* & -s_7^* & 0 & 0 & s_3 \\ s_7 & -s_6 & -s_5 & s_4 & 0 \end{pmatrix},$$

$$L_s = 7, N = 11, L_t = 5, \text{code rate} = 7/11.$$

By using the columnwise orthogonality of the linear orthogonal space-time codes, the $L_t \times L_t$ complex matrix $\mathcal{C}(\mathbf{s})^H\mathcal{C}(\mathbf{s})$ can be derived as follows

$$\mathcal{C}(\mathbf{s})^H\mathcal{C}(\mathbf{s}) = \text{diag}\{\sum_{k=1}^{L_s} g_{k,1} \cdot |s_k|^2, \cdots, \sum_{k=1}^{L_s} g_{k,L_t} \cdot |s_k|^2\} \quad (3)$$

where $(\cdot)^H$ denotes the Hermitian operator and diag{} denotes a diagonal matrix with elements as indicated between the braces.

Linear orthogonal space-time codes can be classified according to the values of $g_{k,i}$. Let a linear orthogonal space-time code be called *homogeneous* if $g_{k,i}$ is constant, i.e., g for all k and i and otherwise, *nonhomogeneous*. It is clear that $\mathcal{C}_1$ and $\mathcal{C}_2$ are homogeneous and $\mathcal{C}_3$ is nonhomogeneous.

For a homogeneous code, (3) can be simplified as $\mathcal{C}(\mathbf{s})^H\mathcal{C}(\mathbf{s}) = g\sum_{k=1}^{L_s} |s_k|^2 \cdot I$, where I is the $L_t \times L_t$ identity matrix.

Let E_s be the average symbol energy of s_k. Thus the average energy transmitted from all antennas combined during a symbol period, E_m can be expressed as

$$E_m = \frac{1}{N}\sum_{i=1}^{L_t}\sum_{k=1}^{L_s} g_{k,i} \cdot E_s$$

which for a homogeneous linear space-time code, can be simplified to $E_m = g\frac{L_t L_s}{N}E_s$.

Recently, Simon [4], Taricco and Biglieri [6] independently worked on the closed expression of the exact pairwise error probability of space-time codes. Let $\mathbf{X}$ and $\hat{\mathbf{X}}$ be two distinct $N \times L_t$ codeword matrices of space-time transmission. According to the result in [3, 4], the pairwise error probability that $\mathbf{X}$ is decoded to $\hat{\mathbf{X}}$, is given as

$$P(\mathbf{X} \to \hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L_t}\left[1 + \frac{\overline{\gamma}}{4\sin^2\theta}\sum_{n=1}^{N}|x_i^n - \hat{x}_i^n|^2\right]^{-L_r} d\theta \quad (4)$$

where $\overline{\gamma} = \frac{\overline{\rho}}{E_m}$.

Let $\mathcal{C}$ be a linear orthogonal space-time code. Let $\mathbf{p} = \{p_1, p_2, \cdots, p_{L_s}\}$ and $\mathbf{q} = \{q_1, q_2, \cdots, q_{L_s}\}$ be distinct message vectors and $\mathcal{C}(\mathbf{p})$ and $\mathcal{C}(\mathbf{q})$ be the corresponding codewords, respectively. Let $\mathbf{X} = \mathcal{C}(\mathbf{p})$ and $\hat{\mathbf{X}} = \mathcal{C}(\mathbf{q})$. From the columnwise orthogonality of $\mathcal{C}(\cdot)$, the difference matrix $\mathbf{X} - \hat{\mathbf{X}}$ also has a columnwise orthogonal property. Using (3), we can modify (4) into

$$P(\mathcal{C}(\mathbf{p}) \to \mathcal{C}(\mathbf{q})) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L_t}\left[1 + \frac{\overline{\gamma}}{4\sin^2\theta}\sum_{k=1}^{L_s} g_{k,i}|p_k - q_k|^2\right]^{-L_r} d\theta. \quad (5)$$

It should be noted that the exact expression for the pairwise error probability in (5) is derived in terms of the message symbol distance between two message vectors rather than the codeword symbol distance between two transmitted codeword matrices.

For homogeneous codes, the equation can be further simplified as

$$P(\mathcal{C}(\mathbf{p}) \to \mathcal{C}(\mathbf{q})) = \frac{1}{\pi}\int_0^{\pi/2}\left[1 + \frac{\overline{\gamma} \cdot g}{4\sin^2\theta}\sum_{k=1}^{L_s}|p_k - q_k|^2\right]^{-L_t \cdot L_r} d\theta. \quad (6)$$

Using the result in [?], (6) can be integrated into the closed form expression as

$$P(\mathcal{C}(\mathbf{p}) \to \mathcal{C}(\mathbf{q})) = \frac{1}{2}\left\{1 - \sqrt{\frac{\delta}{1+\delta}}\sum_{m=0}^{L_t \cdot L_r - 1}\binom{2m}{m}\left(\frac{1}{4(1+\delta)}\right)^m\right\}, \quad (7)$$

where $\delta = \frac{\overline{\gamma}g}{4}\sum_{k=1}^{L_s}|p_k - q_k|^2$. This can be used to derive the exact bit error probability of homogeneous linear orthogonal space-time codes in the following section.

3. One-dimensional Component Symbol Error Function

In this section, we will discuss the pairwise one-dimensional component symbol error function, referred to as *component error function*. Let $d_h(\mathbf{a}, \mathbf{b})$ be the Hamming distance between two vectors $\mathbf{a}$ and $\mathbf{b}$. Let $\mathbf{s}$ and $\hat{\mathbf{s}}$ be vectors such that $s_{k,x} \neq \hat{s}_{k,x}$ and $d_h(\mathbf{s}', \hat{\mathbf{s}}') = 1$ where $\mathbf{s}'$ and $\hat{\mathbf{s}}'$ are defined in (2). That is, we consider two message vectors which differ by one component $s_{k,x}$. Using (5), the pairwise error probability between $\mathcal{C}(\mathbf{s})$ and $\mathcal{C}(\hat{\mathbf{s}})$ is given as

$$P(\mathcal{C}(\mathbf{s}) \to \mathcal{C}(\hat{\mathbf{s}})) = P(s_{k,x} \to \hat{s}_{k,x}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L_t}\left[1 + \frac{\overline{\gamma}}{4\sin^2\theta}g_{k,i} \cdot |s_{k,x} - \hat{s}_{k,x}|^2\right]^{-L_r} d\theta.$$

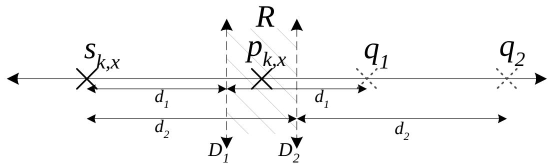

Figure 1: An example of a one-dimensional component symbol error from $s_{k,x}$ to $\hat{s}_{k,x}$.

Thus we can define the component error function as a function of the distance $l = |a_{k,x} - b_{k,x}|$ between two points $(a_{k,x}, c_{k,y}), (b_{k,x}, c_{k,y})$ in the two-dimensional signal space as follows

$$\mathcal{Q}_{k,x}(l) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L_t}\left[1+\frac{\overline{\gamma}}{4\sin^2\theta}g_{k,i}\cdot l^2\right]^{-L_r} d\theta. \quad (8)$$

In a similar manner, the component error function $\mathcal{Q}_{k,y}(l)$ is defined as a function of the distance $l = |a_{k,y} - b_{k,y}|$ between two points $(c_{k,x}, a_{k,y}), (c_{k,x}, b_{k,y})$ in the two-dimensional signal space. It is clear that $\mathcal{Q}_{k,x}(l)$ is equal to $\mathcal{Q}_{k,y}(l)$. It should be noted that $\mathcal{Q}_{k,x}(\cdot)$ is a function of l and $\overline{\gamma} = \frac{\overline{\rho}}{E_m}$.

For the case of homogeneous linear orthogonal space-time codes, the component error functions $\mathcal{Q}_{k,x}(l)$ and $\mathcal{Q}_{k,y}(l)$ have the same form for all k denoted by $\mathcal{Q}_c(l)$ and reduce to the closed expression, i.e.

$$\mathcal{Q}_c(l) = \frac{1}{2}\{1-\sqrt{\frac{g\overline{\gamma}\cdot l^2}{4+g\overline{\gamma}\cdot l^2}}\sum_{m=0}^{L_t\cdot L_r-1}\binom{2m}{m}\times\left(\frac{1}{4+g\overline{\gamma}\cdot l^2}\right)^m\} \quad (9)$$

from (7) because all $g_{k,i}$ are equal. Note that for the nonhomogeneous linear orthogonal space-time codes, the component error function $\mathcal{Q}_{k,x}(l)$ in (8) should be numerically integrated.

In order to find the symbol error probability for the multilevel modulation scheme, the one-dimensional component symbol error probability that $s_{k,x}$ is decoded into $\hat{s}_{k,x}$, where R is the decision region of $\hat{s}_{k,x}$, must be found as in Fig. 1. Using the component error function defined in (8) and two fictitious points q_1 and q_2 in Fig. 1, the probability that $b'_{k,x}$ crosses the decision boundary D_1 and D_2 can be expressed as $P(s_{k,x} \to q_1)$ and $P(s_{k,x} \to q_2)$, respectively. The component error function that $s_{k,x}$ is decoded into $\hat{s}_{k,x}$ is then derived as

$$\begin{aligned} &P(b'_{k,x} \in R | s_{k,x} \text{ transmitted}) \\ &= P(s_{k,x} \to q_1) - P(s_{k,x} \to q_2) \\ &= \mathcal{Q}_{k,x}(2d_1) - \mathcal{Q}_{k,x}(2d_2), \end{aligned} \quad (10)$$

which will be used to derive the exact bit error probability of the space-time codes with various modulation schemes in the following section.

4. Exact Bit Error Probability for QPSK

The alphabet size in QPSK modulation is $L = 2^{b_s} = 4$. Thus each message symbol is encoded from two bits and the average energy per symbol is computed as $E_s = 2d^2$. Clearly, each dimension of QPSK modulation corresponds to BPSK modulation. The bit error probability for the one-dimensional component symbol $s_{k,x}$ is easily derived as $P_e(s_{k,x}) = \mathcal{Q}_{k,x}(d-(-d)) = \mathcal{Q}_{k,x}(2d)$, as is that for $s_{k,y}$. Thus the bit error probability for QPSK is given as

$$\begin{aligned} P_{QPSK} &= \frac{1}{2L_s}\sum_{k=1}^{L_s}\{P_e(s_{k,x}) + P_e(s_{k,y})\} \\ &= \frac{1}{L_s}\sum_{k=1}^{L_s}\mathcal{Q}_{k,x}(2d) \end{aligned}$$

where $\mathcal{Q}_{k,x}(\cdot)$ is given in (8). Using (9), the exact expression for the bit error probability of homogeneous linear orthogonal space-time block codes with QPSK modulation is given as

$$\begin{aligned} P_{QPSK} &= P_e(s_{k,x}) = \mathcal{Q}_c(2d) \\ &= \frac{1}{2}\{1-\sqrt{\frac{g\overline{\gamma}d^2}{1+g\overline{\gamma}d^2}}\sum_{m=0}^{L_t\cdot L_r-1}\binom{2m}{m} \\ &\quad\times\left(\frac{1}{4(1+g\overline{\gamma}d^2)}\right)^m\}, \end{aligned} \quad (11)$$

where $\overline{\gamma} = \overline{\rho}/E_m = \frac{N\overline{\rho}}{gL_tL_sE_s} = \frac{N\overline{\rho}}{gL_tL_s\cdot 2d^2}$.

5. Exact Bit Error Probability for 16-QAM

Each message symbol in 16-QAM is expressed in 4 bits. The average symbol energy of 16-QAM can be computed as $E_s = 10d^2$. The first and the third bits in the binary expression of the message symbol s_k determine the real part $s_{k,x}$ of s_k and the other two bits determine the imaginary part $s_{k,y}$. It is clear that in order to find the exact bit error probability expression for the linear orthogonal space-time codes with 16-QAM, it sufficient to find $P_e(s_{k,x})$. First, 4-ary amplitude shift keying(ASK), which corresponds to the real or the imaginary part of 16-QAM should be considered. $R_i, 1 \le i \le 4$ denotes the decision regions for the constellation points c_i, which is expressed in two bits. Assume that c_i is transmitted. The number of bit errors for each decoded constellation point is then defined as

$$B_{c_i} = \{d_h(c_1, c_i), d_h(c_2, c_i), d_h(c_3, c_i), d_h(c_4, c_i)\}.$$

Thus it can be classified into two different cases according to the constellation points.

Case 1: c_1 or c_4 is transmitted:

Suppose that c_1 is transmitted and thus $s_{k,x} = -3d$. The number of bit errors for each decoded constellation point can then be counted as $B_{c_1} = \{0, 1, 2, 1\}$. Similar to (10), let $P_{c_j}(R_i)$ be the probability that the transmitted message component symbol c_j is decoded into c_i, that is, $P(b'_{k,x} \in R_i | c_j \text{ transmitted})$. The bit error probability for the transmitted symbol c_1, $P_{k,1}$ can then be calculated as

$$P_{k,1} = \frac{1}{b_s^{(x)}} \sum_{i=1}^{4} d_h(c_1, c_i) \cdot P_{c_1}(R_i) \qquad (12)$$

where $b_s^{(x)} = b_s/2$ is the number of bits in the binary expression of $s_{k,x}$. Similar to QPSK, we can calculate

$$\begin{aligned} P_{c_1}(R_2) &= \mathcal{Q}_{k,x}(2d) - \mathcal{Q}_{k,x}(6d) \\ P_{c_1}(R_3) &= \mathcal{Q}_{k,x}(6d) - \mathcal{Q}_{k,x}(10d) \\ P_{c_1}(R_4) &= \mathcal{Q}_{k,x}(10d) \end{aligned}$$

and (12) can be rewritten as $P_{k,1} = \frac{1}{2}\{\mathcal{Q}_{k,x}(2d) + \mathcal{Q}_{k,x}(6d) - \mathcal{Q}_{k,x}(10d)\}$. The bit error probability for the transmitted symbol c_4, $P_{k,4}$ is exactly the same as $P_{k,1}$.

Case 2: c_2 or c_3 is transmitted:

Suppose that c_2 is transmitted and thus $s_{k,x} = -d$. Similar to the previous case, we can obtain $B_{c_2} = \{1, 0, 1, 2\}$ and thus the exact bit error probability for this case can be easily obtained as $P_{k,2} = P_{k,3} = \frac{1}{2}\{2 \cdot \mathcal{Q}_{k,x}(2d) + \mathcal{Q}_{k,x}(6d)\}$.

Combining the two cases, the bit error probability for the message component symbol $s_{k,x}$ is given as

$$\begin{aligned} P_e(s_{k,x}) &= \frac{1}{4}(P_{k,1} + P_{k,2} + P_{k,3} + P_{k,4}) \\ &= \frac{3}{4}\mathcal{Q}_{k,x}(2d) + \frac{1}{2}\mathcal{Q}_{k,x}(6d) - \frac{1}{4}\mathcal{Q}_{k,x}(10d) \end{aligned}$$

and thus the exact expression of bit error probability for 16-QAM is given as

$$\begin{aligned} P_{16QAM} = \frac{1}{L_s} \sum_{k=1}^{L_S} \{ &\frac{3}{4}\mathcal{Q}_{k,x}(2d) \\ &+ \frac{1}{2}\mathcal{Q}_{k,x}(6d) - \frac{1}{4}\mathcal{Q}_{k,x}(10d)\} \qquad (13) \end{aligned}$$

where $\mathcal{Q}_{k,x}(\cdot)$ is defined in (8). For the homogeneous linear orthogonal space-time block codes with 16-QAM, (13) can be simplified as

$$P_{16QAM} = \frac{3}{4}\mathcal{Q}_c(2d) + \frac{1}{2}\mathcal{Q}_c(6d) - \frac{1}{4}\mathcal{Q}_c(10d),$$

the close form expression.

6. Exact Bit Error Probability for 64-QAM and 256-QAM

Let us consider square 64-QAM and 256-QAM such that the minimum distance between constellation points is $2d$ like 16-QAM. In the same way, the exact bit error rate for 64-QAM and 256-QAM can be derived as

$$\begin{aligned} P_{64QAM} = \frac{1}{L_s} \sum_{k=1}^{L_s} \frac{1}{12} \{ &7\mathcal{Q}_{k,x}(2d) + 6\mathcal{Q}_{k,x}(6d) \\ &- \mathcal{Q}_{k,x}(10d) + \mathcal{Q}_{k,x}(18d) - \mathcal{Q}_{k,x}(26d)\} \\ P_{256QAM} = \frac{1}{L_s} \sum_{k=1}^{L_s} \frac{1}{32} \{ &15\mathcal{Q}_{k,x}(2d) + 14\mathcal{Q}_{k,x}(6d) \\ &- \mathcal{Q}_{k,x}(10d) + 5\mathcal{Q}_{k,x}(18d) + 4\mathcal{Q}_{k,x}(22d) \\ &- 5\mathcal{Q}_{k,x}(26d) - 4\mathcal{Q}_{k,x}(30d) + 5\mathcal{Q}_{k,x}(34d) \\ &+ 4\mathcal{Q}_{k,x}(38d) - 3\mathcal{Q}_{k,x}(42d) - 2\mathcal{Q}_{k,x}(46d) \\ &+ \mathcal{Q}_{k,x}(50d) - \mathcal{Q}_{k,x}(58d)\} \end{aligned}$$

and for homogeneous codes they can be easily given in closed form expressions. Note that $E_s = 42d^2$ for 64-QAM and $E_s = 170d^2$ for 256-QAM.

7. Conclusions

In this paper, the exact bit error probability of linear orthogonal space-time codes is derived as a function of SNR and the number of transmit and receiver antennas for various modulation schemes such as QPSK, 16-QAM, 64-QAM, and 256-QAM.

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