# Pairwise Error Probability of Quasi-Orthogonal Space-Time Block Codes [1]

*Jae-Dong Yang$^{O}$, *Xianglan Jin, *Jong-Seon No, and **Dong-Joon Shin
*School of Electrical Engineering and Computer Science, Seoul National University
**Division of Electrical and Computer Engineering , Hanyang University
Email: yjdong@ccl.snu.ac.kr

Abstract

In this paper, we derive the exact pairwise error probabilities (PEPs) of various quasi-orthogonal space-time block codes (QO-STBCs) using the moment generating function. By classifying PEP of each QO-STBC into several types, we derive the closed-form expression for each type of PEP. Based on these closed-form expressions, we obtain the union bound on the symbol error probability (SEP). Through simulation, it is shown that this union bound is quite tight.

## 1. Introduction

In a multiple-input multiple-output (MIMO) communication system, transmit diversity is a very attractive technique to achieve high reliability and spectral efficiency. Space-time code is a good solution to get transmit diversity. Tarokh, Seshadri, and Calderbank proposed space-time codes and their performance criterion [12]. Alamouti introduced a simple transmit diversity scheme with two transmit antennas [1]. Space-time block codes (STBCs) from orthogonal designs were proposed [11], which have low decoding complexity and achieve full diversity and full rate. However, it is known that the complex orthogonal design with full rate exists only for the case which is Alamouti's scheme [1].

Since it is not possible to construct the orthogonal STBC with full diversity and full rate for more than two transmit antennas, Jafarkhani [3] introduced a quasi-orthogonal space-time block code (QO-STBC), which sacrifices full diversity but keeps full rate. Another QO-STBC was proposed by Tirkkonen, Boariu, and Hottinen [13], which has the same performance and similar feature as that in [3]. Recently, Sharma and Papadias showed that QO-STBCs can achieve full diversity by rotating signal constellation [7].

Taricco and Biglieri derived the exact pairwise error probability (PEP) of space-time codes [10]. Lu, Wang, Kumar, and Chugg analyzed the performance of space-time codes [6]. Kim, Kang, and No derived the exact bit and symbol error probability for the orthogonal STBC with quadrature amplitude modulation [4] [5]. Simon analyzed the performance of space-time codes by deriving the exact PEP of space-time trellis codes (STTCs) based on the moment generating function [8].

In this paper, using the moment generating function as in [8], we derive the exact PEPs and union bound on the symbol error probability (SEP) of various QO-STBCs. We also verify that our union bound is quite tight through simulation.

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## 2. Channel Model and Quasi-Orthogonal Space-Time Block Codes

Consider a wireless communication system with $L_t$ transmit antennas and $L_r$ receive antennas. Let $\mathbf{X}$ be an $L_t \times L_t$ codeword matrix. From the first row to the last row, the symbols in a row of the codeword matrix $\mathbf{X}$ is transmitted via transmit antennas simultaneously. We assume that the Rayleigh fading channel is quasi-static such that the channel remains constant over the duration of a codeword matrix. Then the received signal can be expressed as the following matrix form

$$\mathbf{Y} = \mathbf{XH} + \mathbf{N} \tag{1}$$

where $\mathbf{Y}$ is the $L_t \times L_r$ matrix of the received signal, $\mathbf{N}$ is the $L_t \times L_r$ matrix of the additive white Gaussian noise (AWGN), and $\mathbf{H}$ is the $L_t \times L_r$ channel coefficient matrix. The elements of $\mathbf{H}$ are independent complex Gaussian random variables with mean zero and variance 0.5 per dimension. The elements of $\mathbf{N}$ are also independent complex Gaussian random variables with mean zero and variance $\sigma^2$ per dimension.

A codeword matrix $\mathbf{X}$ of QO-STBC is an $L_t \times L_t$ matrix which is not an orthogonal matrix, but some columns are orthogonal to each other. Since the difference matrix between any two distinct codewords does not always have full rank, QO-STBCs cannot achieve full diversity. QO-STBCs have the decoding complexity higher than orthogonal STBCs but lower than STTCs. When maximum likelihood (ML) decoding is performed, unlike orthogonal STBCs, a pair of two symbols of $4 \times 4$ QO-STBCs are jointly decoded.

Jafarkhani proposed the $4 \times 4$ QO-STBC [3], which has the following codeword matrix.

$$\mathbf{X}=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}. \quad (2)$$

Note that the first and the fourth columns are not orthogonal but they are orthogonal to the second and the third columns, whereas the second and the third columns are not orthogonal. Since the difference matrix between two distinct codeword matrices does not guarantee full rank, this scheme cannot achieve full diversity.

Tirkkonen, Boariu, and Hottinen also introduced the $4\times 4$ QO-STBC [13], which has the following codeword matrix.

$$\mathbf{X}=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}. \quad (3)$$

This scheme has the same performance as Jafarkhani scheme and does not guarantee full diversity.

Since Jafarkhani and TBH schemes do not have full diversity, Sharma and Papadias proposed the constellation rotation technique to achieve full diversity in QO-STBCs [7]. The codeword matrix of Sharma and Papadias scheme is given as

$$\mathbf{X}=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2^* & -x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & -x_1 & x_2 \\ x_4^* & x_3^* & -x_2^* & -x_1^* \end{bmatrix}. \quad (4)$$

Without constellation rotation, (4) cannot have full diversity. However, if we rotate the constellation corresponding to symbols $x_3$ and $x_4$, we can achieve full diversity.

## 3. Exact Pairwise Error Probability

In [8], the exact pairwise error probability of STTCs was derived using the moment generating function. We also use the moment generating function to evaluate the exact pairwise error probability of QO-STBCs.

Let $vec(\mathbf{A})$ denote the vectorization operator which stacks the columns of $\mathbf{A}$ and $\otimes$ denote the matrix Kronecker product. Then, (1) can be rewritten as

$$\mathbf{y}=(\mathbf{I}_{L_r}\otimes\mathbf{X})\mathbf{h}+\mathbf{n}$$

where $\mathbf{y}=vec(\mathbf{Y})$, $\mathbf{h}=vec(\mathbf{H})$, $\mathbf{n}=vec(\mathbf{N})$, and $\mathbf{I}_n$ is the $n\times n$ identity matrix.

Assume that the perfect channel state information (CSI) is available at the receiver. Then, the ML decoding metric becomes

$$m(\mathbf{Y},\mathbf{X})=\|\mathbf{Y}-\mathbf{X}\mathbf{H}\|^2=\|\mathbf{y}-(\mathbf{I}_{L_r}\otimes\mathbf{X})\mathbf{h}\|^2. \quad (5)$$

The PEP conditioned on the channel, that ML decoder decodes correct codeword matrix $\mathbf{X}$ into incorrect codeword matrix $\hat{\mathbf{X}}$, is given as

$$\begin{aligned} P(\mathbf{X}\to\hat{\mathbf{X}}|\mathbf{H}) &= Pr\{m(\mathbf{Y},\mathbf{X})\geq m(\mathbf{Y},\hat{\mathbf{X}})|\mathbf{H}\} \\ &= Pr\{m(\mathbf{Y},\mathbf{X})-m(\mathbf{Y},\hat{\mathbf{X}})\geq 0|\mathbf{H}\}. \end{aligned} \quad (6)$$

After plugging (5) into (6) and doing some manipulation, we can obtain

$$P(\mathbf{X}\to\hat{\mathbf{X}}|\mathbf{H})=Pr\{z\geq\mu|\mathbf{H}\}=Q_{z|\mathbf{H}}\left(\sqrt{\frac{\mu}{4\sigma^2}}\right)$$

where $\mu=\|\,[\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]\,\mathbf{h}\,\|^2$, $Q_{z|\mathbf{H}}(\cdot)$ is the Q-function, and $z=2Re\{\mathbf{n}^H[\mathbf{I}_{L_r}\otimes(\hat{\mathbf{X}}-\mathbf{X})]\mathbf{h}\}$ is a conditionally zero mean real Gaussian random variable with variance $\sigma_z^2=4\sigma^2\mu$. If we normalize the average transmitted symbol energy from each antenna, i.e. $E\{|x_i|^2\}=1$, then the noise variance $\sigma^2$ would be $L_t/2\gamma$, where $\gamma$ is the average signal-to-noise ratio. Using the Craig's result [2], the conditional exact PEP is given as

$$P(\mathbf{X}\to\hat{\mathbf{X}}|\mathbf{H})=\frac{1}{\pi}\int_0^{\pi/2}\exp\left[-\frac{\mu}{8\sigma^2\sin^2\theta}\right]d\theta. \quad (7)$$

Plugging $\mu=\|\,[\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]\,\mathbf{h}\,\|^2$ and $\sigma^2=L_t/2\gamma$ into (7), we can obtain

$$\begin{aligned} P(\mathbf{X}\to\hat{\mathbf{X}}|\mathbf{H}) =& \frac{1}{\pi}\int_0^{\pi/2}\exp\Bigg[-\frac{\gamma}{4L_t\sin^2\theta} \\ & \times\|[\,\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]\mathbf{h}\|^2\Bigg]d\theta. \end{aligned} \quad (8)$$

To evaluate the exact PEP, we need to average the conditional PEP in (8) over the channel matrix. Since taking expectation over the channel matrix $\mathbf{H}$ is equivalent to taking expectation over the channel vector $\mathbf{h}$, we get

$$\begin{aligned} P(\mathbf{X}\to\hat{\mathbf{X}}) &= E_{\mathbf{H}}\{P(\mathbf{X}\to\hat{\mathbf{X}}|\mathbf{H})\} \\ &= \frac{1}{\pi}\int_0^{\pi/2}E_{\mathbf{h}}\Big\{\exp\Big[-\frac{\gamma}{4L_t} \\ &\quad \times\|[\,\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]\mathbf{h}\|^2\frac{1}{\sin^2\theta}\Big]\Big\}d\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2}M_p\left(-\frac{1}{\sin^2\theta}\right)d\theta \end{aligned}$$

where

$$p=(\gamma/4L_t)\mathbf{h}^H[\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]^H[\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]\mathbf{h}$$

and $M_p(s)=E_p\{\exp(sp)\}$. Considering that $p$ is the quadratic form of complex Gaussian random variables, we can use the moment generating function of the quadratic complex Gaussian random variables. In [14], Turin derived the characteristic function of quadratic complex Gaussian random variables and using this result, we can obtain the exact PEP as follows.

$$P(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\Big[\det\Big(\mathbf{I}_{L_rL_t}+\frac{\gamma_s}{4\sin^2\theta} \times[\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]^H[\mathbf{I}_{L_r}\otimes(\mathbf{X}-\hat{\mathbf{X}})]\Big)\Big]^{-1}d\theta = \frac{1}{\pi}\int_0^{\pi/2}\Big[\det\Big(\mathbf{I}_{L_t}+\frac{\gamma_s}{4\sin^2\theta} \times(\mathbf{X}-\hat{\mathbf{X}})^H(\mathbf{X}-\hat{\mathbf{X}})\Big)\Big]^{-L_r}d\theta \quad (9)$$

where $\gamma_s=\gamma/L_t$. Using (9), we will find the closed-form expressions for the exact PEP of various QO-STBCs in the followings.

To derive the exact PEP of Jafarkhani's QO-STBC in (2), we have to calculate the determinant in (9) as follows.

$$\det\left[\mathbf{I}_4+\frac{\gamma_s}{4\sin^2\theta}(\mathbf{X}-\hat{\mathbf{X}})^H(\mathbf{X}-\hat{\mathbf{X}})\right] = [(1+a)^2-b^2]^2 \quad (10)$$

where $a=(\gamma_s/4sin^2\theta)\sum_{i=1}^4|x_i-\hat{x}_i|^2$ and $b=(\gamma_s/4sin^2\theta)2Re\{(x_1-\hat{x}_1)(x_4-\hat{x}_4)^*-(x_2-\hat{x}_2)(x_3-\hat{x}_3)^*\}$. It is already known that the ML decoding of QO-STBC in (2) is done pair by pair, i.e. $x_1$ and $x_4$ are jointly decoded and so are $x_2$ and $x_3$, independently [3]. Therefore, we will only consider $x_1$ and $x_4$. Plugging (10) into (9) and after some algebraic manipulations, we can obtain the following exact PEP.

$$P(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\left[1+\frac{\gamma_s}{4\sin^2\theta}|(x_1-\hat{x}_1)+(x_4-\hat{x}_4)|^2\right]^{-2L_r} \times\left[1+\frac{\gamma_s}{4\sin^2\theta}|(x_1-\hat{x}_1)-(x_4-\hat{x}_4)|^2\right]^{-2L_r}d\theta. \quad (11)$$

Similarly to the Jafarkhani scheme, symbols $x_1$ and $x_3$ in TBH scheme in (3) are jointly decoded and so are $x_2$ and $x_4$, independently. To evaluate the exact PEP, we have to calculate the determinant in (9), which is the same as (10) except for $b=(\gamma_s/4sin^2\theta)2Re\{(x_1-\hat{x}_1)(x_3-\hat{x}_3)^*-(x_2-\hat{x}_2)(x_4-\hat{x}_4)^*\}$. Using the same method as the Jafarkhani scheme, we can derive the PEP of TBH scheme as

$$P(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\left[1+\frac{\gamma_s}{4\sin^2\theta}|(x_1-\hat{x}_1)+(x_3-\hat{x_3})|^2\right]^{-2L_r} \times\left[1+\frac{\gamma_s}{4\sin^2\theta}|(x_1-\hat{x}_1)-(x_3-\hat{x_3})|^2\right]^{-2L_r}d\theta. \quad (12)$$

In order to find the exact PEP of Sharma and Papadias scheme in (4), we also have to calculate the determinant in (9) as follows.

$$\det\left[\mathbf{I}_4+\frac{\gamma_s}{4\sin^2\theta}(\mathbf{X}-\hat{\mathbf{X}})^H(\mathbf{X}-\hat{\mathbf{X}})\right] = [(1+a)^2+b^2]^2 \quad (13)$$

where $a=(\gamma_s/4sin^2\theta)\sum_{i=1}^4|x_i-\hat{x}_i|^2$ and $b=(\gamma_s/4sin^2\theta)2jIm\{(x_1-\hat{x}_1)(x_3-\hat{x}_3)^*+(x_2-\hat{x}_2)(x_4-\hat{x}_4)^*\}$ with $j$ equal to $\sqrt{-1}$. Plugging (13) into (9), we can obtain the following exact PEP only using $x_1$ and $x_3$.

$$P(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\left[1+\frac{\gamma_s}{4\sin^2\theta}|(x_1-\hat{x}_1)+j(x_3-\hat{x}_3)|^2\right]^{-2L_r} \times\left[1+\frac{\gamma_s}{4\sin^2\theta}|(x_1-\hat{x}_1)-j(x_3-\hat{x}_3)|^2\right]^{-2L_r}d\theta. \quad (14)$$

Note that (14) has the similar expression as (11) and (12).

Now, we obtained the exact PEPs of three QO-STBCs. From (11), (12), and (14), the exact PEP of three QO-STBCs is given as

$$P(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+(\gamma_s/4)u}\right)^{2L_r} \times\left(\frac{\sin^2\theta}{\sin^2\theta+(\gamma_s/4)v}\right)^{2L_r}d\theta \quad (15)$$

where $u$ and $v$ can be easily derived from the first and the second factors of the integrands in (11), (12), and (14), respectively. Depending on $u$ and $v$, the exact PEP can be classified into three types.

Type I is when either $u$ or $v$ is equal to zero. In this case, there are two symbol errors between $\mathbf{X}$ and $\hat{\mathbf{X}}$. Type II is when nonzero $u$ is equal to $v$. In this case, there are one or two symbol errors between $\mathbf{X}$ and $\hat{\mathbf{X}}$. Type III is when $u$ and $v$ are nonzero and distinct. In this case, there are two symbol errors between $\mathbf{X}$ and $\hat{\mathbf{X}}$. According to the above three types, (15) can be rewritten as

$$P_I(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+(\gamma_s/4)u}\right)^{2L_r}d\theta, \quad (16)$$
$$P_{II}(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+(\gamma_s/4)u}\right)^{4L_r}d\theta,$$
$$P_{III}(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = P(\mathbf{X}\rightarrow\hat{\mathbf{X}}).$$

From (16), we can see that QO-STBC does not achieve full diversity. Unlike the Jafarkhani and TBH schemes, Sharma and Papadias scheme with constellation rotation has full diversity and does not have the exact PEP of type I.

Using the results in [9, Appendix 5A], we can derive the closed-form expressions of the three types of exact PEPs for QO-STBCs as follows.

$$P_I(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{2}\left[1-\sqrt{\frac{\gamma_s u}{4+\gamma_s u}}\sum_{k=0}^{2L_r-1}\binom{2k}{k}\left(\frac{1}{4+\gamma_s u}\right)^k\right]$$

$$P_{II}(\mathbf{X}\rightarrow\hat{\mathbf{X}}) = \frac{1}{2}\left[1-\sqrt{\frac{\gamma_s u}{4+\gamma_s u}}\sum_{k=0}^{4L_r-1}\binom{2k}{k}\left(\frac{1}{4+\gamma_s u}\right)^k\right]$$

Table 1: Distribution of $u$ and $v$ for QPSK.

| $u$ | $v$ | Type | 1 Symbol Error | 2 Symbol Errors | Freq |
|---|---|---|---|---|---|
| 0 | 8 | I | | 16 | 16 |
| 0 | 16 | I | | 4 | 4 |
| 8 | 0 | I | | 16 | 16 |
| 16 | 0 | I | | 4 | 4 |
| 2 | 2 | II | 64 | | 64 |
| 4 | 4 | II | 32 | 32 | 64 |
| 8 | 8 | II | | 8 | 8 |
| 2 | 10 | III | | 32 | 32 |
| 10 | 2 | III | | 32 | 32 |

$$\begin{aligned} & P_{III}(\mathbf{X} \to \hat{\mathbf{X}}) \\ = & \frac{(u/v)^{2L_r-1}}{2(1-u/v)^{4L_r-1}} \Bigg[ \sum_{k=0}^{2L_r-1} \left(\frac{v}{u}-1\right)^k B_k I_k(\frac{\gamma_s}{4}v) \\ & -\frac{u}{v} \sum_{k=0}^{2L_r-1} \left(1-\frac{u}{v}\right)^k C_k I_k(\frac{\gamma_s}{4}u) \Bigg] \end{aligned}$$

where

$$A_k = (-1)^{2L_r-1+k} \frac{\binom{2L_r-1}{k}}{(2L_r-1)!} \prod_{\substack{n=1 \\ n\neq k+1}}^{2L_r} (4L_r - n),$$

$$B_k = \frac{A_k}{\binom{4L_r-1}{k}}, \qquad C_k = \sum_{n=0}^{2L_r-1} \frac{\binom{k}{n}}{\binom{4L_r-1}{n}} A_n$$

and

$$I_k(c) = 1 - \sqrt{\frac{c}{1+c}} \left[ 1 + \sum_{n=1}^{k} \frac{(2n-1)!!}{n! 2^n (1+c)^n} \right].$$

The double factorial !! denotes the product of only odd integers from 1 to $2k-1$ and note that by convention, $\binom{k}{n} = 0$ for $n > k$.

## 4. Union Bound on Symbol Error Probability

In this section, we derive the union bound on the SEP of QO-STBCs given by

$$\text{SEP} \leq \frac{1}{n_s} \sum_{\mathbf{X}} \left[ \sum_{\hat{\mathbf{X}} \neq \mathbf{X}} P(\mathbf{X} \to \hat{\mathbf{X}}) d_H(\mathbf{X}, \hat{\mathbf{X}}) \right] p(\mathbf{X})$$

where $n_s$ is the number of symbols of $\mathbf{X}$ in PEP expression, $d_H(\mathbf{X}, \hat{\mathbf{X}})$ is the number of different symbols between $\mathbf{X}$ and $\hat{\mathbf{X}}$, and $p(\mathbf{X})$ is the probability that $\mathbf{X}$ is transmitted.

We will derive the union bound on the SEP of $4 \times 4$ QO-STBCs in (2), (3), and (4) by considering QPSK and single receive antenna. We assume that the information symbols are equiprobable. Since only $x_1$ and $x_4$ appear in (11), the union bound is given as

$$\begin{aligned} \text{SEP} & \leq \frac{1}{2} \sum_{x_1, x_4} \Big[ \sum_{(\hat{x}_1, \hat{x}_4) \neq (x_1, x_4)} P_{14}(\mathbf{X} \to \hat{\mathbf{X}}) \\ & \quad \times d_H(\mathbf{X}, \hat{\mathbf{X}}) \Big] p(\mathbf{X}) \\ & \leq \frac{1}{32} \sum_{x_1, x_4} \Big[ \sum_{(\hat{x}_1, \hat{x}_4) \neq (x_1, x_4)} P_{14}(\mathbf{X} \to \hat{\mathbf{X}}) \\ & \quad \times d_H(\mathbf{X}, \hat{\mathbf{X}}) \Big] \end{aligned} \qquad (17)$$

where $P_{14}$ means the PEP for only $x_1$ and $x_4$ of $\mathbf{X}$. Table 1 shows the distribution of $u$ and $v$ and the corresponding PEP types. The last column "Freq" denotes the total number of $(u, v)$ pairs. Using Table 1, (17) can be evaluated.

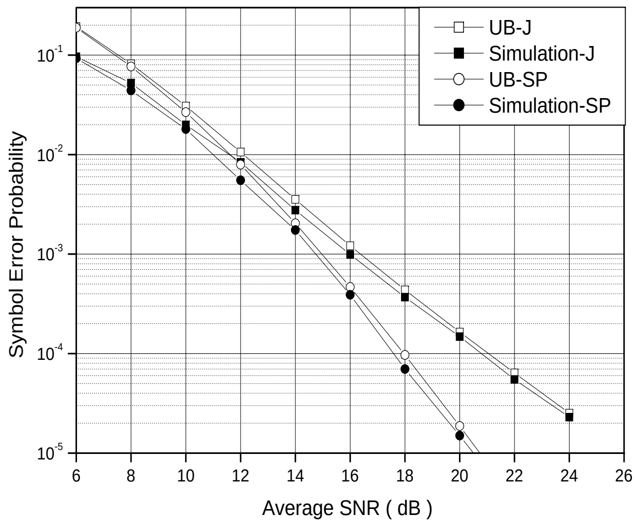


Figure 1: Comparison of union bounds and simulation results for single receive antenna with QPSK.

Figure 1 compares the union bound and the SEP obtained by simulation. In Figure 1, "UB" means the union bound and "Simulation" means the SEP obtained by simulation. Also, "J" means the Jafarkhani scheme and "SP" means the Sharma and Papadias scheme. Note that Jafarkhani and TBH have the same performance and only the results for Jafarkhani scheme are included. In low SNR region, the union bound is loose, whereas in high SNR region, it becomes tight. Figure 1 also shows that Sharma and Papadias scheme achieves full diversity contrary to Jafarkhani scheme.

## 5. Conclusion

We derived the exact pairwise error probability of QO-STBCs using the moment generating function. We classified the pairwise error probabilities into three types and found the closed-form expressions for them. We also derived the union bound on the symbol error probability, which turns out to be tight compared to the simulation results.

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