Selection Scheme of the Optimal Tone Set in TR Method for PAPR [1]

Dae-Woon Lim [O], Hyung-Suk Noh, Seok-Joong Heo, and Jong-Seon No
School of Electrical Engineering and Computer Science, Seoul National University
Email: jsno@snu.ac.kr

Abstract

In tone reservation (TR) scheme, it is known that the randomly generated set of peak reduction tones (PRT's) performs better than the contiguous PRT set and interleaved PRT set in the PAPR reduction of orthogonal frequency division multiplexing (OFDM). It is known that finding the optimal PRT set corresponds to the secondary peak minimization problem in TR scheme. But, the problem is NP-hard because it must be optimized over all possible discrete sets. Thus, it cannot be solved for practical values of N. In this paper, a new search algorithm for the optimal PRT set is proposed based on the fact that the minimization problem of the secondary peak can be related with the minimization of the variance. From the simulation results, it is shown that the PRT set obtained by the proposed algorithm has the best PAPR reduction performance for OFDM signals.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) system has become the promising solution for the next generation mobile radio communication systems to meet the demand for high data rate services. Multiplexing a serial data symbol stream into a large number of orthogonal tones makes the OFDM signals spectral bandwidth efficient. It has been shown that the performance of OFDM system over frequency selective fading channels is better than that of the single carrier modulation system [12]. The major drawback of the OFDM system is its large envelope fluctuation. Since most practical systems are peak-power limited, designing the system to operate in a perfectly linear region often implies operating at power levels well below the maximum power.

Recently lots of works [1], [4]–[8], [13]–[16] have been done in developing a scheme to reduce the peak to average power ratio (PAPR). The simple and widely used scheme is clipping the signal to limit the PAPR below a threshold level, but it causes both in-band distortion and out of band radiation [10]. Block coding [1], the encoding of an input data into a codeword with low PAPR is another well-known technique to reduce PAPR, but it incurs the transmission rate loss.

In [14], Tellado and Cioffi proposed tone reservation (TR) scheme for PAPR reduction. The basic idea is to reserve a small number of peak reduction tones (PRT's). The PAPR reduction performance of TR scheme increases in proportion to the number of reserved tones. It is known that the randomly generated PRT set performs better than the contiguous tone set and interleaved tone set. The criteria for the optimal set is the secondary peak value of the time domain kernel, which is obtained by inverse fast Fourier-transforming (IFFT-ing) the characteristic sequence of the PRT set. The secondary peak of the kernel is not expressed in a closed form and the complexity of computing the secondary peak is very high due to IFFT and the calculation of the power of the time domain kernel. The problem of the optimal PRT set is NP-hard because it must be optimized over all possible discrete tone sets and cannot be solved for practical values of N.

In this paper, a new algorithm for selecting the optimal PRT set is proposed based on the numerical analysis that the PRT sets with the minimum secondary peak exist in the PRT sets with the minimum or near minimum variance.

The paper is organized as follows: in Section II, the TR scheme and the proposed algorithm for selecting the optimal set of reserved tones are described. The simulation results are shown in Section III and concluding remarks are given in Section IV.

2. New Selection Scheme of PRT Set

The discrete time domain OFDM signal sequence $\mathbf{a} = [a_0 a_1 \cdots a_{N-1}]$ of N subcarriers can be expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi \frac{n}{N} t}, \quad 0 \le t \le N-1$$

where $\mathbf{A} = [A_0 A_1 \cdots A_{N-1}]$ is an input symbol sequence and t is a discrete time index. The PAPR of

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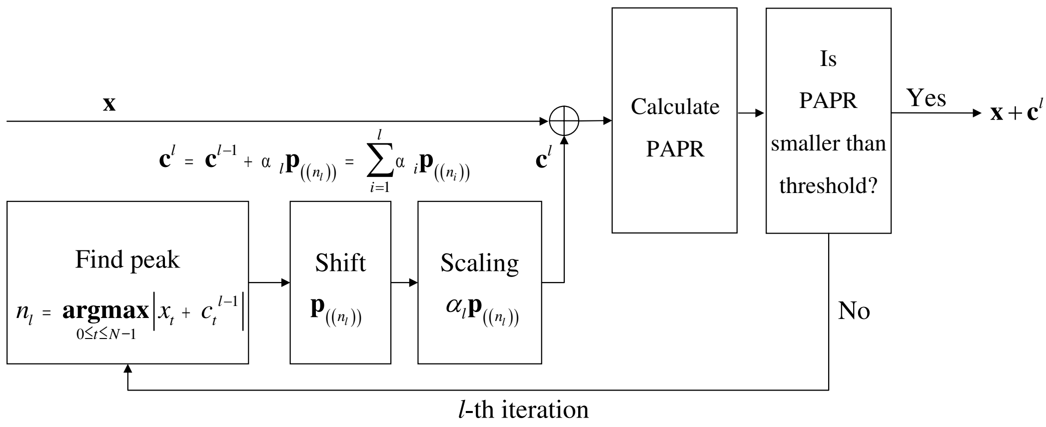


Figure 1: A block diagram of a tone reservation scheme.

an OFDM signal, defined as the ratio of the maximum to the average power of signal [5], can be expressed as

$$\text{PAPR}_{\mathbf{a}} = \frac{\underset{0\le t\le N-1}{\text{Max}} |a_t|^2}{\text{E}\left[|a_t|^2\right]}$$

where $\text{E}[\cdot]$ denotes the expected value.

In the TR scheme [14], some tones are reserved to generate a PAPR reduction signal. The reserved tones are not used for data transmission. Let $\mathcal{R} = \{i_0, i_1, \cdots, i_{W-1}\}$ denote the ordered set of the positions of the reserved tones and $\mathcal{R}^{\mathcal{C}}$ the complement set of $\mathcal{R}$ in $\mathcal{N} = \{0, 1, \cdots, N-1\}$, where W is the number of reserved tones. The input symbol A_n of the TR scheme is expressed as

$$A_n = X_n + C_n = \begin{cases} C_n, & n \in \mathcal{R} \\ X_n, & n \in \mathcal{R}^{\mathcal{C}} \end{cases}$$

where X_n is a data symbol and C_n a PAPR reduction symbol. Then, the OFDM signal a_t is composed of the data signal x_t and the PAPR reduction signal c_t as

$$\begin{aligned} a_t &= x_t + c_t \\ &= \frac{1}{\sqrt{N}} \sum_{n\in\mathcal{R}^{\mathcal{C}}} X_n e^{j2\pi \frac{n}{N} t} + \frac{1}{\sqrt{N}} \sum_{n\in\mathcal{R}} C_n e^{j2\pi \frac{n}{N} t}, \\ &\qquad\qquad\qquad\qquad 0 \le t \le N-1. \end{aligned}$$

A block diagram of the TR scheme is shown in Fig. 1. The PAPR reduction signal sequence $\mathbf{c} = [c_0 c_1 \cdots c_{N-1}]$ is the sampled version of c_t. Let p_t be the time domain signal defined as

$$p_t = \frac{1}{\sqrt{N}} \sum_{n\in\mathcal{R}} P_n e^{j2\pi \frac{n}{N} t}$$

where $\mathbf{P} = [P_0 P_1 \cdots P_{N-1}]$ is a frequency domain kernel for PAPR reduction with $P_n = 0$ for $n \in \mathcal{R}^{\mathcal{C}}$. Then, the sampled version of p_t becomes the time domain kernel $\mathbf{p} = [p_0 p_1 \cdots p_{N-1}]$ which is used to compute the PAPR reduction signal sequence $\mathbf{c}$ iteratively [15]. That is, the l-th iteration $\mathbf{c}^l$ is expressed as

$$\mathbf{c}^l = \sum_{i=1}^{l} \alpha_i \mathbf{p}_{((n_i))} \tag{1}$$

where $\mathbf{p}_{((n_i))}$ denotes a circular shift of $\mathbf{p}$ by n_i and α_i a complex scaling factor which is computed according the threshold level and the maximum peak value at i-th iteration. The circular shift n_i is determined as

$$n_i = \underset{0\le t\le N-1}{\text{argmax}} |x_t + c_t^{i-1}|.$$

By the shift property of Fourier transform, $\mathbf{P}_{n_i}^T = \mathbf{Q}\mathbf{p}_{((n_i))}^T$ also has zero value in $\mathcal{R}^{\mathcal{C}}$, where $\mathbf{Q}$ is FFT matrix. If the maximum number of iteration is reached or a desired peak-power is obtained, the iteration stops. For simplicity, we assumed that the only one maximum peak of OFDM signal is reduced at each iteration in (1).

The PAPR reduction performance depends on the the kernel $\mathbf{p}$ and the best performance can be achieved when the kernel $\mathbf{p}$ is a discrete impulse, where a maximum peak is canceled without generating the secondary peak at each iteration. In order for $\mathbf{P}$ to be a discrete impulse, $\mathbf{P}$ should have nonzero values in $\mathcal{R} = \mathcal{N}$. As the number of reserved tones becomes larger, the PAPR reduction performance increases but the data transmission rate decreases. Therefore, the kernel should be designed to be as close as possible to a discrete impulse and still satisfy $W \ll N$. The typical number of reserved tones is less than 15 percent of N.

It is known that the optimal choice of $\mathbf{P}$ corresponds to selecting $\mathbf{p}$ such that the secondary peaks or sidelobes of $\mathbf{p}$ are minimized [15]. When the PRT set $\mathcal{R}$ is given [16] shows that the optimal frequency domain kernel $\mathbf{P}$ is obtained as

$$P_n = \begin{cases} 1, & n \in \mathcal{R} \\ 0, & n \in \mathcal{R}^{\mathcal{C}}. \end{cases}$$

Then, the optimal frequency domain kernel $\mathbf{P}$ corresponds to the characteristic sequence of $\mathcal{R}$. Note the maximum peak of $\mathbf{p}$ is always p_0 since $\mathbf{P}$ is a $\{0,1\}$ sequence.

The PAPR reduction performance of TR scheme depends on the selection of the PRT set $\mathcal{R}$. Let $\mathcal{R}^{opt}$ be the optimal PRT set given as [15]

$$\mathcal{R}^{opt} = \underset{\{i_o,i_1,\cdots,i_{W-1}\}}{\operatorname{argmin}} \| |p_1||p_2|\cdots|p_{N-1}| \|_\infty$$

where $\|\cdot\|_v$ denotes v-norm and ∞-norm refers to the maximum value.

It is known that this problem is NP-hard since the kernel $\mathbf{p}$ must be optimized over all possible discrete sets $\mathcal{R}$. Thus it cannot be solved for practical values of N. Since the computational complexity of evaluating the secondary peak of the PRT set by IFFT is very high, a brute force search over all the PRT sets takes too much time for large N. For $N = 128$ and $W = 10$, the number of the candidate PRT sets is $\binom{128}{10} \simeq 2.27 \times 10^{14}$. Thus, we have to find the efficient method for the secondary peak minimization of $\mathbf{p}$ with low computational complexity.

Let $\mathbf{y} = \{y_0 y_1 y_2 \cdots y_{N-1}\}$, $\sum_{t=0}^{N-1} y_t = \gamma$, and $0 \le y_t \le y_0$, $1 \le t \le N-1$. Suppose that y_0 is a fixed value and y_ts, $1 \le t \le N-1$ are variable values. Then, it follows that

$$\underset{1\le t\le N-1}{\operatorname{Max}} y_t \ge \frac{1}{N-1}\sum_{t=1}^{N-1} y_t = \frac{\gamma - y_0}{N-1}.$$

It is clear that $\underset{1\le t\le N-1}{\operatorname{Max}} y_t$ is minimized when $y_1 = y_2 = \cdots = y_{N-1}$. The variance $\sigma_{\mathbf{y}}^2$ of $\mathbf{y}$ can be given as

$$\begin{aligned}
\sigma_{\mathbf{y}}^2 &= \frac{1}{N}\sum_{t=0}^{N-1} y_t^2 - (\frac{1}{N}\sum_{t=0}^{N-1} y_t) \\
&= \frac{1}{N}\sum_{t=1}^{N-1} y_t^2 + \frac{1}{N}y_0^2 - (\frac{\gamma}{N})^2 \\
&\ge \frac{1}{N(N-1)}(\sum_{t=1}^{N-1} y_t)^2 + \frac{1}{N}y_0^2 - (\frac{\gamma}{N})^2 \\
&= \frac{1}{N(N-1)}(\gamma - y_0)^2 + \frac{1}{N}{y_0}^2 - (\frac{\gamma}{N})^2
\end{aligned}$$

where the inequality holds from the Caushy-Schwartz inequality. Thus, the variance $\sigma_{\mathbf{y}}^2$ of $\mathbf{y}$ can also be minimized when $y_1 = y_2 = \cdots = y_{N-1}$. Let $y_0 = |p_0|^2$ and $y_t = |p_t|^2$, $1 \le t \le N-1$. Generally, y_t could not have the same value because it is generated by IFFT-ing the characteristic sequence of the PRT set in TR scheme. Therefore, in this situation, it is not easy to prove that the minimization of the second peak is equivalent to the minimization of $\sigma_{\mathbf{y}}^2$. But, the numerical analysis gives us that the minimization problem of the secondary peak of $\mathbf{p}$ can be related with the minimization of the variance. In the numerical analysis, the variance and the secondary peak are calculated for all the PRT sets $\mathcal{R}$ for $N = \{8, 16, 32, 64\}$. The numerical results show that the PRT sets with the variance σ^2 have the different secondary peaks raging from α to β and α statistically decreases as σ^2 decreases. From the numerical results, it is worth to mention that the PRT sets with the minimum secondary peak exist in the PRT sets with the minimum or near minimum variance although the PRT set with the minimum variance does not guarantee the minimum secondary peak.

Now, we are going to evaluate the variance ${\sigma_{\mathbf{p}}}^2$ of $\mathbf{p}$ and to show that ${\sigma_{\mathbf{p}}}^2$ can be calculated only with additive operation and without IFFT of $\mathbf{P}$. The power spectrum of p_t [5] is expressed as

$$\begin{aligned}
|p_t|^2 &= p_t p_t^* \\
&= \frac{1}{N}R_0 + \frac{1}{N}\sum_{\tau=1}^{N-1}(R_\tau e^{j2\pi\frac{\tau}{N}t} + R_\tau^* e^{-j2\pi\frac{\tau}{N}t})
\end{aligned} \tag{2}$$

where R_τ denotes aperiodic autocorrelation defined by

$$R_\tau = \sum_{n=0}^{N-1-\tau} P_n P_{n+\tau}^*. \tag{3}$$

The time average μ of $|p_t|^2$ is given as

$$\mu = \frac{1}{N}\sum_{t=0}^{N-1}|p_t|^2 = \frac{1}{N}R_0. \tag{4}$$

Using (2) and (4), the variance σ^2 of $|p_t|^2$ is obtained as

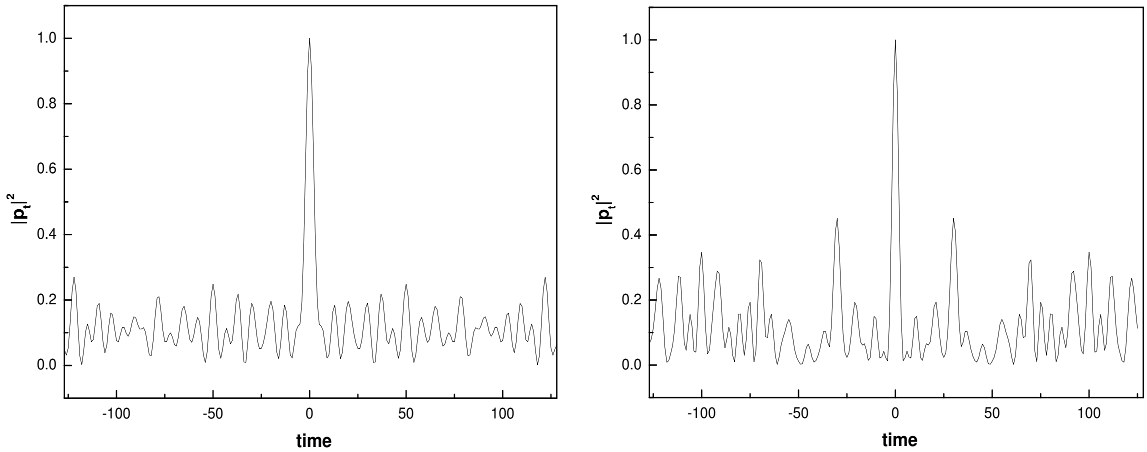
$$\begin{aligned}
{\sigma_{\mathbf{p}}}^2 &= \frac{1}{N}\sum_{t=0}^{N-1}(|p_t|^2 - \mu)^2 \\
&= \frac{1}{N^2}\sum_{\tau=1}^{N-1}(2|R_\tau|^2 + R_\tau R_{N-\tau} + R_\tau^* R_{N-\tau}^*)
\end{aligned} \tag{5}$$

Since P_n is binary, (5) can be rewritten as

$${\sigma_{\mathbf{p}}}^2 = \frac{2}{N^2}\sum_{\tau=1}^{N-1}(R_\tau^2 + R_\tau R_{N-\tau}). \tag{6}$$

For a binary P_n, (3) can be calculated without multiplication. The multiplication in (6) can be avoided using a small size of a table that contains $R_{\tau_1}^2$ and $R_{\tau_1}R_{\tau_2}$ for $0 \le R_{\tau_1}, R_{\tau_2} \le W$.

Since the PRT sets with the minimum secondary peak exist in the PRT sets with the minimum or near minimum variance, we propose a new search algorithm for finding the optimal PRT set. The proposed algorithm takes two steps. The first step is to select the PRT sets with the minimum or near minimum variance from all PRT sets. The second step is to find the PRT sets with the minimum secondary peak by IFFT-ing the PRT sets chosen from the first step. For the second peak evaluation, complex multiplications and additions are needed to compute the power of the time



(a) (b)

Figure 2: The power $|p_t|^2$ of the time domain kernel (a) the proposed PRT set and (b) the random PRT set.

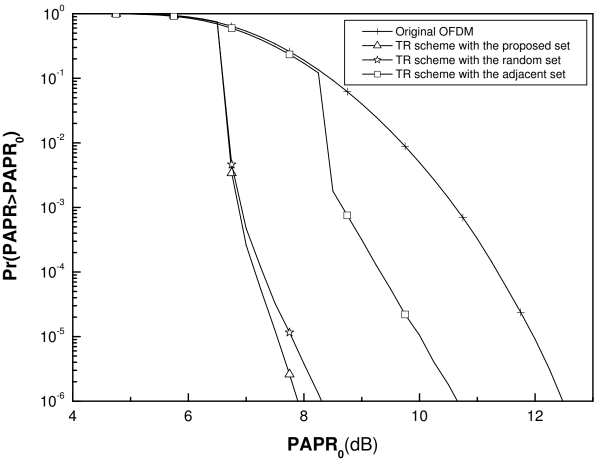


Figure 3: The CCDF of the original OFDM signal and the OFDM signal with TR scheme for various PRT sets.

domain kernel and comparison to select the secondary peak of the kernel. Since only integer additions are needed to compute the variance using the aperiodic autocorrelation, the computational complexity of the proposed algorithm for selecting the optimal PRT set via two stages can be reduced.

3. Simulation Results

Simulations for the proposed algorithm of searching the optimal PRT set are performed for the OFDM system of the *IEEE* standard 802.11a for wireless local area network (WLAN). The OFDM system specified in *IEEE* 802.11a has 64 tones with BPSK, QPSK, 16-QAM, and 64-QAM constellations. The number of used tones for data symbol and reserved tones is 52 and the remaining 12 tones are set to zero to shape the power spectral density of the transmit signal. Among 52 tones, the 8 tones can be used for PRT set and thus the redundancy is about 15%. The 10^7 input data symbol sequences for IFFT are randomly generated with uniform distribution. The complex baseband OFDM signal is oversampled by a factor of four which is sufficient to represent the analog signal [11]. The PAPRs of three different PRT sets, namely the proposed set, the random set, and the contiguous set are evaluated for the purpose of comparison. The proposed set is $\{8, 23, 31, 34, 43, 44, 48, 50\}$, the random set $\{6, 13, 21, 29, 31, 40, 54, 57\}$, and the contiguous set $\{6, 7, 8, 9, 10, 11, 12, 13\}$.

The power $|p_t|^2$ of the time domain kernel for the proposed set and the random set is shown in Fig. 2. The secondary peak power for the optimal set is 0.27 and the random set 0.45 when the first peak power is normalized to 1.

Fig. 3 illustrates the probability that the PAPR of the OFDM signal exceeds the given value. In TR scheme, a threshold level affects the PAPR reduction performance and it is evaluated from the simulation results. The threshold level for the proposed and random set is 4.7 (6.7dB) and for the contiguous set 7.0 (8.5dB). The proposed set has the best PAPR reduction performance and it reduces the PAPR by 5dB at 10^{-6} while the adjacent set reduces the PAPR only by 2dB at 10^{-6}.

4. Conclusion

A new algorithm for selecting the optimal PRT set is proposed, which is composed of two steps. The first step is to select the PRT sets with the minimum or near minimum variance of the time domain kernel from all PRT sets. The second step is to find the PRT sets with the minimum secondary peak by IFFT-ing the PRT sets chosen by the first step. Since only integer additions are needed to compute the variance, the computational complexity of the variance calculation can be much smaller than the secondary peak evaluation if we design the dedicated hardware for the variance calculation. From the simulation results, it is shown that the proposed set has the best PAPR reduction performance.

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