New Constructions of Optimal Binary Low Correlation Zone Sequences[1]

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Abstract

In this paper, we propose new methods of constructing optimal binary low correlation zone (LCZ) sequences. In the new method, we devise a column sequence set of length $2^{m+1}-1$ from a binary sequence of period 2^m-1 having ideal autocorrelation property and this column sequence set is used to construct binary LCZ sequence sets of period 2^n-1 when $(m+1)|n$. The new method gives us the optimal sets with respect to the bound by Tang, Fan, and Matsufuji.

1. Introduction

Unlike the conventional code division multiple access (CDMA) systems, in the quasi-synchronous CDMA system [1] where maintaining synchronization within a few chips is feasible even in the reverse link due to the relatively small transmission delay, the most important property of the sequences used for reducing multiple access interference (MAI) is low correlation property around the origin [5]. The sequence set with this property is called *low correlation zone (LCZ) sequence.* They also have shown that an LCZ sequence set has better performance than other well-known sequence sets with optimal correlation property [5]. For a prime p, Tang and Fan [7] proposed p-ary LCZ sequence sets by extending the alphabet size of each sequence in Long's work [5]. And they also proposed a construction method of p-ary LCZ sequence sets by using interleaved sequences [8]. But for a prime p, no optimal set of p-ary LCZ sequence set has been reported yet.

In this paper, we propose new methods of constructing optimal binary LCZ sequences. In the new method, we devise a column sequence set of length $2^{m+1}-1$ from a binary sequence of period 2^m-1 having ideal autocorrelation property and this column sequence set is used to construct binary LCZ sequence sets of period 2^n-1 when $(m+1)|n$. The new method gives us the optimal sets with respect to the bound by Tang, Fan, and Matsufuji [9].

2. Preliminaries

Let $\mathcal{S}$ be a set of D sequences of period N. If the magnitude of correlation function between any two sequences in $\mathcal{S}$ takes the values less than or equal to ϵ for the offset τ in the range $-Z<\tau<Z$, then $\mathcal{S}$ is called an (N,D,Z,ϵ) LCZ sequence set.

Let p be a prime and F_{p^n} be the finite field with p^n elements. Let $v_i(x)$ and $v_j(x)$ be two p-ary sequences of period p^n-1, defined in $F^*_{p^n}=F_{p^n}\backslash\{0\}$. Then for $\delta\in F^*_{p^n}$, the correlation function between two p-ary sequences $v_i(x)$ and $v_j(x)$ is defined as

$$R_{v_i,v_j}(\delta)=\sum_{x\in F^*_{p^n}}\omega_p^{v_i(x\delta)-v_j(x)}$$

where ω_p is a complex primitive p-th root of unity. We will abuse the notation of the correlation function as $R_{i,j}(\tau)=R_{v_i,v_j}(\alpha^\tau)$ for $\delta=\alpha^\tau$, where α is a primitive element in F_{p^n}.

Let $v(t)$ be a p-ary sequence of period p^n-1. Then $v(t)$ is said to have balance property if number of zero element is one less than that of each nonzero element in one period of the sequence. And if the sequence $v(t)-v(t+\tau)$ is balanced for all $\tau\not\equiv 0 \bmod p^n-1$, then $v(t)$ is said to have difference-balance property.

The trace function $\mathrm{tr}^n_m(\cdot)$ from F_{p^n} to F_{p^m} is defined by

$$\mathrm{tr}^n_m(x)=\sum_{i=0}^{\frac{n}{m}-1}x^{p^{mi}}$$

where $x\in F_{p^n}$ and $m|n$. It is well known that $\mathrm{tr}^n_m(\alpha^t)$ is a p^m-ary m-sequence of period p^n-1, where α is a primitive element in F_{p^n}.

Klapper [4] introduced the d-form function. A d-form function $H(x)$ on F_{p^n} over F_{p^m} is defined as a function satisfying for any $y\in F_{p^m}$ and $x\in F_{p^n}$

$$H(yx)=y^dH(x). \tag{1}$$

Tang and Fan [8] stated the following theorem using

[1] This research was supported by the MIC, Korea, under the ITRC support program and by the MOE, the MOCIE, and the MOLAB, Korea, through the fostering project of the Lab. of Excellency.

the interleaved sequence [2], which can be used for the construction of an LCZ sequence set.

Theorem 1 [8] Let m and n be integers such that $m|n$. Let $f(y)$ and $g(y)$ be cyclically distinct sequences of period p^m-1 from F_{p^m} to F_p and the function $h(x)$ from F_{p^n} to F_{p^m} be a 1-form function over F_{p^m} with balance and difference-balance property. If we set $f(0)=g(0)=0$, then the correlation function $R_{f,g}(\delta)$ between $f(h(x))$ and $g(h(x))$ is given as

$$\begin{aligned}R_{f,g}(\delta) &= \sum_{x\in F^*_{p^n}} \omega_p^{f(h(\delta x))-g(h(x))}\\ &= \begin{cases} p^{n-m}\big(C_{f,g}(\delta)+1\big)-1, & \text{if } \delta\in F_{p^m}\\ p^{n-2m}(I(f)+1)(\bar{I}(g)+1)-1, & \text{if } \delta\notin F_{p^m}\end{cases}\end{aligned}$$

where $I(f) = \sum_{y\in F^*_{p^m}} \omega_p^{f(y)}$, $C_{f,g}(\delta) = \sum_{y\in F^*_{p^m}} \omega_p^{f(\delta y)-g(y)}$, and $\bar{I}(\cdot)$ denotes complex conjugate of $I(\cdot)$. □

In the above theorem, $f(\cdot)$ and $g(\cdot)$ are called the *column sequences* of period p^m-1 in the two dimensional representation of the sequences $f(h(\cdot))$ and $g(h(\cdot))$ of period p^n-1, respectively.

It is clear that $I(f)=-1$ corresponds to the balance property of the column sequence $f(y)$ defined on $F^*_{p^m}$ if p is a prime. If the column sequences are balanced, we have

$$R_{f,g}(\delta)=-1, \quad \text{for } \delta\notin F_{p^m}.$$

In order to have $R_{f,g}(1)=-1$, we have to have $C_{f,g}(1)=-1$, which means that the in-phase cross-correlation function of each pair in the column sequence set has the value -1.

Property 2 Let $\mathcal{A}$ be the set of sequences of period p^m-1 satisfying the following properties:

i) All the sequences in the set $\mathcal{A}$ are cyclically distinct.

ii) Each sequence in the set $\mathcal{A}$ has the balance property.

iii) In-phase cross-correlation value of each pair of the sequences in the set $\mathcal{A}$ is always -1.

□

Theorem 1 tells us that if we have the sequence set $\mathcal{A}$ satisfying Property 2, then the $(p^n-1, |\mathcal{A}|, (p^n-1)/(p^m-1), 1)$ p-ary LCZ sequence set can be constructed.

In the subsequent sections, we propose methods of constructing the column sequence sets satisfying Property 2, some of which are of the maximum size.

3. New Optimal Binary LCZ Sequence Sets

In this section, for integers n and m such that $(m+1)|n$, we construct the optimal binary LCZ sequence set of period 2^n-1 by using binary sequences of period 2^m-1 with ideal autocorrelation.

The following lemma can be easily stated without proof.

Lemma 3 Let $m_1(t)$ and $m_2(t)$ be two cyclically distinct p-ary sequences with linear span L_1 and L_2, respectively. The maximum run lengths of the symbol 0 and the symbol a, $1\le a\le p-1$, for the difference sequence $m_1(t)-m_2(t)$ are less than or equal to L_1+L_2-1 and L_1+L_2, respectively.

□

Using two binary sequences with ideal autocorrelation, we can construct a set of column sequences satisfying Property 2 as in the following theorem.

Theorem 4 Let $m_1(t)$ and $m_2(t)$ be two binary sequences, not necessarily distinct, of period 2^m-1 with ideal autocorrelation. Let L_1 and L_2 be the linear spans of the sequences $m_1(t)$ and $m_2(t)$, respectively, such that $L_1+L_2+\max(L_1,L_2) < 2^m-1$ and in addition, $L_1=L_2<2^{m-1}$, if m_1 and m_2 are cyclically equivalent. Define the new sequences $s_i(t)$, $0\le i\le 2^{m+1}-2$ of period $2^{m+1}-1$ such that

i) for $0\le i\le 2^m-2$

$$s_i(t)=\begin{cases} m_1(t+i), & 0\le t\le 2^m-2\\ 0, & t=2^m-1\\ m_2(t-1-i), & 2^m\le t\le 2^{m+1}-2\end{cases} \tag{2}$$

ii) for $2^m-1\le i\le 2^{m+1}-3$

$$s_i(t)=\begin{cases} m_1(t+i), & 0\le t\le 2^m-2\\ 1, & t=2^m-1\\ m_2(t-1-i)+1, & 2^m\le t\le 2^{m+1}-2\end{cases} \tag{3}$$

and

$$s_{2^{m+1}-2}(t)=\begin{cases} 0, & 0\le t\le 2^m-2\\ 1, & 2^m-1\le t\le 2^{m+1}-2.\end{cases}$$

Then the set of sequences $s_i(t)$ satisfies Property 2.

Proof: From the definition of $s_i(t)$, it is clear that $s_i(t)$ is balanced and it is also easy to see that the in-phase cross-correlation $C_{s_i,s_j}(1)$ between $s_i(t)$ and $s_j(t)$ takes the value -1.

Certainly the last sequence $s_{2^{m+1}-2}(t)$ is cyclically distinct to every other sequence. What we are going to show is that for any i, j, $0\le i,j\le 2^{m+1}-3$, and τ, $s_j(t)=s_i(t+\tau)$ implies that $i=j$ and $\tau=0$.

Case 1) $0\le i,j\le 2^m-3$ and $0\le\tau\le 2^m-2$

It is not difficult to see that $s_i(t+\tau)$ can be expressed as:

$$s_i(t+\tau) = \begin{cases} m_1(t+i+\tau), & 0 \le t \le 2^m - 2 - \tau \\ 0, & t = 2^m - 1 - \tau \\ m_2(t-1-i+\tau), & \\ \qquad 2^m - \tau \le t \le 2^{m+1} - 2 - \tau & \\ m_1(t+i+\tau-1), & \\ \qquad 2^{m+1} - 1 - \tau \le t \le 2^{m+1} - 2. & \end{cases} \tag{4}$$

Assume $s_j(t) = s_i(t+\tau)$ for all t. From (2) and (4), we have

$$m_1(t+j) + m_1(t+i+\tau) = 0, \quad 0 \le t \le 2^m - 2 - \tau \tag{5}$$

$$m_1(t+j) + m_2(t-1-i+\tau) = 0, \quad 2^m - \tau \le t \le 2^m - 2 \tag{6}$$

$$m_2(t-1-j) + m_2(t-1-i+\tau) = 0, \quad 2^m \le t \le 2^{m+1} - 2 - \tau \tag{7}$$

$$m_2(t-1-j) + m_1(t+i+\tau-1) = 0, \quad 2^{m+1} - 1 - \tau \le t \le 2^{m+1} - 2. \tag{8}$$

Equations (5) and (7) tell us that

$$m_1(t+j) = m_1(t+i+\tau)$$

and

$$m_2(t-1-j) = m_2(t-1-i+\tau)$$

for consecutive $2^m - 1 - \tau$ values of t. Thus if $\tau < 2^m - \max(L_1, L_2)$, i.e., $2^m - 1 - \tau \ge \max(L_1, L_2)$, then we have $j = i - \tau = i + \tau$, which further tells us that $i = j$ and $\tau = 0$. Note that in this case (6) and (8) become meaningless. If $\tau \ge 2^m - \max(L_1, L_2)$, then (6) and (8) tell us that

$$m_1(t+j) = m_2(t-1-i+\tau)$$

and

$$m_2(t-1-j) = m_1(t+i+\tau-1)$$

for consecutive $\tau - 1$ and τ values of t, respectively, which is impossible from Lemma 3 unless $m_1(t)$ and $m_2(t)$ are cyclically equivalent, since $\tau - 1 \ge L_1 + L_2$. Thus, satisfying (6) and (8) at the same time means that $m_1(t)$ and $m_2(t)$ are cyclically equivalent and $i + j = \tau - 1 = -\tau$, which further implies $\tau = 2^{m-1}$. But $\tau = 2^{m-1}$ is not in the range $\tau \ge 2^m - \max(L_1, L_2)$, since $\max(L_1, L_2) < 2^{m-1}$.

Case 2) $0 \le i, j \le 2^m - 3$ and $2^m \le \tau \le 2^{m+1} - 2$

In this case, $s_i(t+\tau)$ can be expressed as:

$$s_i(t+\tau) = \begin{cases} m_2(t-1-i+\tau), & 0 \le t \le 2^{m+1} - 2 - \tau \\ 0, & t = 2^m - 1 - \tau \\ m_1(t+i+\tau), & \\ \qquad 2^{m+1} - 1 - \tau \le t \le 2^{m+1} + 2^m - 3 - \tau & \\ m_2(t-2-i+\tau), & \\ \qquad 2^{m+1} + 2^m - 1 - \tau \le t \le 2^{m+1} - 2. & \end{cases} \tag{9}$$

Assume $s_j(t) = s_i(t+\tau)$ for all t. From (2) and (9), we have

$$\begin{aligned} m_1(t+j) + m_2(t-1-i+\tau) &= 0, \\ 0 \le t \le 2^{m+1} &- 2 - \tau \\ m_1(t+j) + m_1(t+i+\tau) &= 0, \\ 2^{m+1} - 1 - \tau \le t &\le 2^m - 2 \\ m_2(t-1-j) + m_1(t+i+\tau) &= 0, \\ 2^m \le t \le 2^{m+1} &+ 2^m - 3 - \tau \\ m_2(t-1-j) + m_2(t-2-i+\tau) &= 0, \\ 2^{m+1} + 2^m - 1 - \tau \le t &\le 2^{m+1} - 2. \end{aligned}$$

Similarly to Case 1), we can deduce that $s_j(t) = s_i(t+\tau)$ implies $i = j$ and $\tau = 0$.

Case 3) $2^m - 1 \le i, j \le 2^{m+1} - 3$ and $0 \le \tau \le 2^m - 2$

In this case, $s_i(t+\tau)$ can be expressed as:

$$s_i(t+\tau) = \begin{cases} m_1(t+i+\tau), & 0 \le t \le 2^m - 2 - \tau \\ 1, & t = 2^m - 1 - \tau \\ m_2(t-1-i+\tau) + 1, & \\ \qquad 2^m - \tau \le t \le 2^{m+1} - 2 - \tau & \\ m_1(t+i+\tau-1), & \\ \qquad 2^{m+1} - 1 - \tau \le t \le 2^{m+1} - 2. & \end{cases} \tag{10}$$

Assume $s_j(t) = s_i(t+\tau)$ for all t. From (3) and (10), we have

$$m_1(t+j) + m_1(t+i+\tau) = 0, \quad 0 \le t \le 2^m - 2 - \tau \tag{11}$$

$$m_1(t+j) + m_2(t-1-i+\tau) = 1, \quad 2^m - \tau \le t \le 2^m - 2 \tag{12}$$

$$m_2(t-1-j) + m_2(t-1-i+\tau) = 0, \quad 2^m \le t \le 2^{m+1} - 2 - \tau \tag{13}$$

$$m_2(t-1-j) + m_1(t+i+\tau-1) = 1, \quad 2^{m+1} - 1 - \tau \le t \le 2^{m+1} - 2. \tag{14}$$

Using the similar argument in Case 1), we can derive contradiction for some of (11)–(14) from Lemma 3.

Case 4) $2^m - 1 \le i, j \le 2^{m+1} - 3$ and $2^m \le \tau \le 2^{m+1} - 2$

In this case, $s_i(t+\tau)$ can be expressed as:

$$s_i(t+\tau) = \begin{cases} m_2(t-1-i+\tau) + 1, & 0 \le t \le 2^{m+1} - 2 - \tau \\ 0, & t = 2^m - 1 - \tau \\ m_1(t+i+\tau), & \\ \qquad 2^{m+1} - 1 - \tau \le t \le 2^{m+1} + 2^m - 3 - \tau & \\ m_2(t-2-i+\tau) + 1, & \\ \qquad 2^{m+1} + 2^m - 1 - \tau \le t \le 2^{m+1} - 2. & \end{cases} \tag{15}$$

Assume $s_j(t) = s_i(t+\tau)$ for all t. From (3) and (15), we have

$$\begin{aligned} &m_1(t+j) + m_2(t-1-j+\tau) = 1, \\ &\qquad 0 \le t \le 2^{m+1} - 2 - \tau \qquad (16) \\ &m_1(t+j) + m_1(t+i+\tau) = 0, \\ &\qquad 2^{m+1} - 1 - \tau \le t \le 2^m - 2 \qquad (17) \\ &m_2(t-1-j) + m_1(t+i+\tau) = 1, \\ &\qquad 2^m \le t \le 2^{m+1} + 2^m - 3 - \tau \qquad (18) \\ &m_2(t-1-j) + m_2(t-2-j+\tau) = 0, \\ &\qquad 2^{m+1} + 2^m - 1 - \tau \le t \le 2^{m+1} - 2. \qquad (19) \end{aligned}$$

Using the similar argument in Case 1), we can derive contradiction for some of (16)–(19) using Lemma 3.

Case 5) $2^m - 1 \le i \le 2^{m+1} - 3$, $0 \le j \le 2^m - 2$, and $0 \le \tau \le 2^m - 2$

In this case, $2^m - 1 \le i \le 2^{m+1} - 3$, $s_i(t+\tau)$ can be expressed as:

$$s_i(t+\tau) = \begin{cases} m_1(t+i+\tau), & 0 \le t \le 2^m - 2 - \tau \\ 1, & t = 2^m - 1 - \tau \\ m_2(t-1-i+\tau) + 1, & 2^m - \tau \le t \le 2^{m+1} - 2 - \tau \\ m_1(t+i+\tau-1), & 2^{m+1} - 1 - \tau \le t \le 2^{m+1} - 2. \end{cases} \qquad (20)$$

Assume $s_j(t) = s_i(t+\tau)$ for all t. From (2) and (20), we have

$$\begin{aligned} &m_1(t+j) + m_1(t+i+\tau) = 0, \\ &\qquad 0 \le t \le 2^m - 2 - \tau \qquad (21) \\ &m_1(t+j) + m_2(t-1-i+\tau) = 1, \\ &\qquad 2^m - \tau \le t \le 2^m - 2 \qquad (22) \\ &m_2(t-1-j) + m_2(t-1-i+\tau) = 1, \\ &\qquad 2^m \le t \le 2^{m+1} - 2 - \tau \qquad (23) \\ &m_2(t-1-j) + m_1(t+i+\tau-1) = 0, \\ &\qquad 2^{m+1} - 1 - \tau \le t \le 2^{m+1} - 2. \qquad (24) \end{aligned}$$

Similarly to Case 1), we can also derive contradiction for some of (21)–(24) using Lemma 3.

Case 6) $2^m - 1 \le i \le 2^{m+1} - 3$, $0 \le j \le 2^m - 2$, and $2^m \le \tau \le 2^{m+1} - 2$

In this case, $s_i(t+\tau)$ can be expressed as:

$$s_i(t+\tau) = \begin{cases} m_2(t-1-i+\tau) + 1, & 0 \le t \le 2^{m+1} - 2 - \tau \\ 0, & t = 2^m - 1 - \tau \\ m_1(t+i+\tau), & 2^{m+1} - 1 - \tau \le t \le 2^{m+1} + 2^m - 3 - \tau \\ m_2(t-2-i+\tau) + 1, & 2^{m+1} + 2^m - 1 - \tau \le t \le 2^{m+1} - 2. \end{cases} \qquad (25)$$

Assume $s_j(t) = s_i(t+\tau)$ for all t. From (2) and (25), we have

$$\begin{aligned} &m_1(t+j) + m_2(t-1-i+\tau) = 1, \\ &\qquad 0 \le t \le 2^{m+1} - 2 - \tau \qquad (26) \\ &m_1(t+j) + m_1(t+i+\tau) = 0, \\ &\qquad 2^{m+1} - 1 - \tau \le t \le 2^m - 2 \qquad (27) \\ &m_2(t-1-j) + m_1(t+i+\tau) = 0, \\ &\qquad 2^m \le t \le 2^{m+1} + 2^m - 3 - \tau \qquad (28) \\ &m_2(t-1-j) + m_2(t-2-i+\tau) = 1, \\ &\qquad 2^{m+1} + 2^m - 1 - \tau \le t \le 2^{m+1} - 2. \qquad (29) \end{aligned}$$

Using the similar argument in Case 1), we derive contradiction for some of (26)–(29) using Lemma 3.

Case 7) $\tau = 2^m - 1$

In this case, it is straightforward that $s_i(t+\tau) \neq s_j(t)$, $0 \le i, j \le 2^{m+1} - 3$.

From the above 7 cases, we proved that $s_i(t)$ and $s_j(t)$ are cyclically distinct for all i and j. Thus, we proved that the set of sequences $s_i(t)$ satisfies Property 2. □

Note that the conditions for linear spans of $m_1(t)$ and $m_2(t)$ in Theorem 4 are sufficient not necessary. Even though we cannot directly apply Theorem 4 to the Legendre sequence with $m_1(t) = m_2(t)$, whose linear span is half of the period, we can see that all $s_i(t)$ constructed from Legendre sequence are also cyclically distinct because the run lengths of 1 in the difference sequence of Legendre sequence and its cyclic shift cannot exceed $2^{m-1} - 1$.

Using Theorem 1 and the column sequence sets in Theorem 4, we can construct the binary LCZ sequence sets as in the following theorem.

Theorem 5 Let n and m be integers such that $(m+1)|n$ and $T = (2^n - 1)/(2^{m+1} - 1)$. Let α be a primitive element in F_{2^n} and $\beta = \alpha^T$ be a primitive element in $F_{2^{m+1}}$. Let $h(x)$ from F_{2^n} to $F_{2^{m+1}}$ be a 1-form function over $F_{2^{m+1}}$ with balance and difference-balance property, i.e., either a 2^{m+1}-ary m-sequence, a 2^{m+1}-ary GMW sequence, or a 2^{m+1}-ary generalized GMW sequence. Let $f_i(\beta^t) = s_i(t)$, where $s_i(t)$ is the binary sequence defined in Theorem 4. Then the sequence set $\mathcal{B}$ defined by

$$\mathcal{B} = \{v_i(t) = f_i(h(\alpha^t)) \mid 0 \le i \le 2^{m+1} - 2, \quad 0 \le t \le 2^n - 2\}$$

is a binary LCZ sequence set with parameters $(2^n - 1, 2^{m+1} - 1, T, 1)$. □

Tang, Fan, and Matsufuji [9] derived the lower bound on LCZ sequences using the Welch bound [10].

Using Tang-Fan-Matsufuji bound given as

$$DZ - 1 \le \frac{N-1}{1 - \epsilon^2/N} \qquad (30)$$

we can check the optimality of our binary LCZ sequence set $\mathcal{B}$.

Corollary 6 The binary LCZ sequence set $\mathcal{B}$ in Theorem 5 is optimal with respect to the Tang-Fan-Matsufuji bound.

Proof: The proof is straightforward. By substituting $N = 2^n - 1$, $D = 2^{m+1} - 1$, and $\epsilon = 1$ in (30), we have

$$(2^{m+1} - 1)Z - 1 \leq \frac{2^n - 2}{1 - 1/(2^n - 1)}$$

and thus

$$Z \leq \frac{2^n}{2^{m+1} - 1}.$$

Since Z is an integer, we have

$$Z \leq \left\lfloor \frac{2^n}{2^{m+1} - 1} \right\rfloor = \frac{2^n - 1}{2^{m+1} - 1} = T.$$

Clearly, $\mathcal{B}$ is optimal with respect to the Tang-Fan-Matsufuji bound. □

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