Generalization of No Sequences

Jong-Seon No
Dept. of Electronic Eng., Kon-kuk University
93-1 MoJin-Dong, KwangJin-Gu, Seoul, 133-701, Korea

Abstract-In this paper, GMW sequences and families of No sequences are generalized. Generalized GMW sequences have ideal autocorrelation properties and balance properties and generalized No sequences also have optimal correlation properties in terms of Welch's lower bound. The linear spans of the generalized GMW sequences and generalized No sequences appear to be large although we do not at present have a closed-form expression for the linear span. A count of the numbers of cyclically distinct generalized GMW sequences and generalized No sequences that can be constructed is provided.

I. INTRODUCTION

In this paper, the generalization of GMW sequences and No sequences is introduced. In Section II, GMW sequences are generalized, those ideal full-period autocorrelation properties are derived, and a count of the number of cyclically distinct generalized GMW sequences that can be constructed is provided. It is also shown how the families of No sequences can be generalized in an identical fashion and optimal correlation properties are described in Section III. Here, the number of distinct families of generalized No sequences of given period is described, too.

II. GENERALIZATION OF GMW SEQUENCES

We can define generalized GMW sequences as follows:

Definition 1 : Let n and m_i, $i = 1, 2, ..., d$, be integers satisfying

$$m_d | n \text{ and } m_i | m_{i+1}, \text{ for } 1 \le i \le d-1. \tag{1}$$

A *generalized GMW sequence* is then defined as the multiple trace function sequence of period N given by

$$s_g(t) = tr_1^{m_1}\{[tr_{m_1}^{m_2}\{[tr_{m_2}^{m_3}\{\cdots\{[tr_{m_d}^{n}(\alpha^t)]^{r_d}\}\cdots\}]^{r_2}\}]^{r_1}\}, \tag{2}$$

where α is an element of order $N = 2^n - 1$ and for $1 \le i \le d$,

$$gcd(r_i, 2^{m_i} - 1) = 1, \ 1 \le r_i < 2^{m_i} - 1. \tag{3}$$

The generalized GMW sequence has the ideal full-period autocorrelation values and it can be counted as follows:

Theorem 1 : The number of cyclically different generalized GMW sequences of given period N is given by:

$$N_{gGMW} = \frac{\phi(2^n - 1)}{n} \cdot \prod_{i=1}^{d} \frac{\phi(2^{m_i} - 1)}{m_i}, \tag{4}$$

where $\phi(\cdot)$ is Euler's *phi* function.

III. GENERALIZATION OF NO SEQUENCES

The definition of a generalized No sequence family is given as follows:

Definition 2 : Let n and m_i, $i = 1, 2, ..., d$, be integers satisfying

$$n = 2 \cdot m_d \text{ and } m_i | m_{i+1}, \text{ for } 1 \le i \le d-1. \tag{5}$$

A family of *generalized No sequences*

$$S_g = \{s_i(t) \mid 0 \le t \le N-1, \ 1 \le i \le 2^m\} \tag{6}$$

is a set of multiple trace function sequences defined as

$$s_i(t) = tr_1^{m_1}\{[tr_{m_1}^{m_2}\{[tr_{m_2}^{m_3}\{\cdots\{[tr_{m_d}^{n}(\alpha^{2t}) + \gamma_i \cdot \alpha^{T \cdot t}]^{r_d}\}\cdots\}]^{r_2}\}]^{r_1}\} \tag{7}$$

where $N = 2^n - 1$, γ_i is in $GF(2^{m_d})$, $T = 2^{m_d} + 1$, and for $1 \le i \le d$,

$$gcd(r_i, 2^{m_i} - 1) = 1, \ 1 \le r_i < 2^{m_i} - 1. \tag{8}$$

The full-period correlation function of No sequences are the same as that of Kasami sequences. Counts for the number of cyclically distinct generalized GMW sequences and generalized No sequence families are the same.

REFERENCES

1. J. S. No, "A new family of binary pseudorandom sequences having optimal periodic correlation properties and large linear span," Ph.D. dissertation, University of Southern California, Los Angeles, CA, USA, May, 1988.
2. J. S. No and P. V. Kumar, "A new family of binary pseudorandom sequences having optimal periodic correlation properties and large linear span," *IEEE Trans. Inform. Theory*, vol IT-35, no. 2, pp. 371-379, Mar. 1989.
3. A. Klapper, "d-form sequences: Families of sequences with low correlation values and large linear spans," *IEEE Trans. Inform. Theory*, vol. 41, pp. 423-431, Mar. 1995.