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On the Phase Sequences for Selected Mapping OFDM System

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Abstract

The peak to average power ratio (PAPR) distribution of orthogonal frequency division multiplexing (OFDM) symbol sequences is evaluated and it is shown that an OFDM symbol sequence with short period is expected to have high PAPR. The PAPR relationship of two input symbol sequences with Hamming distance D is also derived. Using these two results, we derive two conditions for the optimal phase sequence set of the selected mapping (SLM) OFDM system and propose rows of a cyclic Hadamard matrix constructed by an m-sequence as a near optimal phase sequence set for the SLM OFDM system, which has the best performance to reduce the PAPR of OFDM signals with small redundancy and no signal distortion.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) system is one of the strong candidates for the standard of the next generation mobile radio communication system. It has been accepted as a standard for the wireless local area networks (WLAN) and mobile wireless metropolitan area networks (WMAN). In the OFDM system, parallel data symbols are transmitted using the orthogonal subcarriers. It is known that the OFDM system is efficient in respect of spectral bandwidth and its performance over frequency selective fading channels is better than that of a single carrier modulation. One of the major drawbacks of OFDM system is that it has high peak to average power ratio (PAPR). Due to the nonlinearity of high power amplifier (HPA), the high PAPR brings on signal distortion in the nonlinear HPA, which induces the degradation of bit error rate (BER).

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Lots of researches [1]~[3], [5], [7]~[9], [11], [13], [14] have been devoted to reduce the PAPR of the OFDM signals. The methods for the PAPR reduction can be classified into two categories. First, there are deterministic methods that limit the PAPR of the OFDM signal below a threshold level. Clipping and block coding belong to this category.

The second category is based on probabilistic approach. These methods statistically improve the characteristic of the PAPR of the OFDM signal without signal distortion. Selected mapping (SLM) and partial transmit sequence (PTS) [7] are included in this category. In SLM, a set of alternative input symbol sequence is generated from a given input symbol sequence by being multiplied by the pre-determined sequence called phase sequences. Then the one with the lowest PAPR in the set is selected for transmission.

In the SLM OFDM system, it was shown from the simulation results that randomly generated phase sequence set has the best performance for PAPR reduction [11]. In [12], Ohkubo and Ohtsuki have proposed design criteria for the phase sequences of SLM OFDM system. They claimed that the sequence set having a low average and a large variance of PAPRs of its members is good as a phase sequence set in SLM. But their criteria can not be considered sufficient. Moreover, it is difficult to design a phase sequence set safisfying their criteria in a systematic manner. This motivates us to approach this problem in different view point.

In this paper, we propose two conditions for the optimal phase sequence set of the SLM OFDM system. First, the phase sequences are to be orthogonal to each other. Second, the phase sequences should not be periodic or similar to periodic sequences. The paper is organized as follows: inSection II, the OFDM system model and the definition of the PAPR are described. The PAPR distribution of periodic input symbol sequences is evaluated in Section III and the PAPR relationship of two input symbol sequences with Hamming distance D is derived in Section IV. In Section V, we derive two conditions for the optimal phase sequence set and propose rows of a cyclic Hadamard matrix constructed by an m-sequence as a near optimal phase sequence set for the SLM OFDM system. Finally, concluding remarks are given in Section VI.

II. AN OFDM SYSTEM

The discrete time domain OFDM signal $\boldsymbol{a} = [a_0 a_1 \cdots a_{N-1}]$ of N carriers can be expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi \frac{n}{N} t}, \quad 0 \le t \le N-1$$

where $\boldsymbol{A} = [A_0 A_1 \cdots A_{N-1}]$ is an input symbol sequence and t is a discrete time index. The PAPR of the OFDM signal is defined as the ratio of the maximum power divided by the average power of the signal. An alternative measure of the envelope variation of the OFDM signal is the crest factor ζ, which is defined as the ratio of the maximum value to the root mean square value of the signal envelope as follows:

$$\mathrm{PAPR}_{\boldsymbol{a}} = \zeta_{\boldsymbol{a}}^2 \triangleq \frac{\underset{0 \le t \le N-1}{\mathrm{Max}} |a_t|^2}{\mathrm{E}\left[|a_t|^2\right]}$$

where $\mathrm{E}[\cdot]$ denotes the expected value.

Let $\boldsymbol{A} = [A_0 A_1 \cdots A_{N-1}]$ and $\boldsymbol{B} = [B_0 B_1 \cdots B_{N-1}]$ be input symbol sequences of length N with M-PSK or M-QAM constellation, and let $\boldsymbol{a} = [a_0 a_1 \cdots a_{N-1}]$ and $\boldsymbol{b} = [b_0 b_1 \cdots b_{N-1}]$ denote the OFDM signals of input symbol sequences $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. Let $B_n = cA_n$, where c is a nonzero complex number. Then, it is clear that $\mathrm{PAPR}_{\boldsymbol{b}} = \mathrm{PAPR}_{\boldsymbol{a}}$.

In the SLM OFDM system alternative symbol sequences $\boldsymbol{A}^u = [A_0^u A_1^u \cdots A_{N-1}^u]$, $1 \le u \le U$, are generated by multiplying the phase sequences $\boldsymbol{S}^u = [S_0^u S_1^u \cdots S_{N-1}^u]$, $1 \le u \le U$, to the input symbol sequence $\boldsymbol{A} = [A_0 A_1 \cdots A_{N-1}]$. We use the expression $\boldsymbol{A}^u = \boldsymbol{A} \cdot \boldsymbol{S}^u$ to represent the componentwise multiplication, i.e., $A_n^u = A_n S_n^u$, $0 \le n \le N-1$. Each symbol of the phase sequences should have unit magnitude to preserve the power and the first phase sequence $\boldsymbol{S}^1$ is usually all one sequence $\boldsymbol{1}_N$. For the ease of implementation, S_n^u is usually selected from $\{\pm 1\}$. The OFDM signal $\boldsymbol{a}^{\tilde{u}} = \mathrm{IFFT}\{\boldsymbol{A}^{\tilde{u}}\}$ with the lowest PAPR is selected for transmission, where $\tilde{u}$ is expressed as

$$\tilde{u} = \underset{1 \le u \le U}{\mathrm{argmin}} \left(\mathrm{PAPR}_{\boldsymbol{a}^u}\right).$$

If U alternative symbol sequences are statistically independent, the probability that the crest factor ζ exceeds the threshold value ζ_0 can be written as

$$\mathrm{Pr}_{\text{SLM-OFDM}}\{\zeta > \zeta_0\} = \left(\mathrm{Pr}_{\text{OFDM}}\{\zeta > \zeta_0\}\right)^U. \qquad (1)$$

The computational complexity of the SLM OFDM system is increased in proportion to the number of phase sequences.

III. PAPR DISTRIBUTION OF PERIODIC INPUT SYMBOL SEQUENCE

Let $\boldsymbol{X} = [X_0 X_1 \cdots X_{N-1}]$ be a symbol sequence of length N, which has nonzero complex values in the interval, $0 \le n \le N/M - 1$ and 0 otherwise, where M is a divisor of N. An input symbol sequence $\boldsymbol{A} = [A_0 A_1 \cdots A_{N-1}]$ of length N is generated by repeating the non-zero value of $\boldsymbol{X}$ M times. Then, A_n is expressed as

$$A_n = \sum_{m=0}^{M-1} X_{n - mN/M}, \quad 0 \le n \le N-1$$

where subscript of X is computed modulo N. Let $\boldsymbol{a} = [a_0 a_1 \cdots a_{N-1}]$ and $\boldsymbol{x} = [x_0 x_1 \cdots x_{N-1}]$ denote the discrete time domain OFDM signals of $\boldsymbol{A}$ and $\boldsymbol{X}$, respectively. According to the shift property of Fourier transform, a_t is given as

$$a_t = x_t \sum_{m=0}^{M-1} e^{j2\pi \frac{m}{M} t} = \begin{cases} M x_t, & t = 0 \bmod M \\ 0, & \text{otherwise.} \end{cases} \qquad (2)$$

In [7], the PAPR distribution of a_t with $M = 1$ is evaluated. We will derive the PAPR distribution for $M \ge 1$. Before evaluating the PAPR distribution of a_t, we calculate the distribution of instantaneous power of x_t. As $\boldsymbol{X}$ has nonzero value only within the interval $0 \le n \le N/M - 1$, x_t is given as

$$x_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N/M} X_n e^{j2\pi \frac{n}{N} t}.$$

For symmetric constellation such as M-PSK or M-QAM, it is legitimate to consider X_n as an independent zero-mean random variable with variance σ_X^2. The average power σ_x^2 of the OFDM signal x_t is calculated using Parseval's theorem as follows: