A New SLM OFDM System with Low Complexity for PAPR Reduction

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Abstract

In this paper, we introduce a new selected mapping (SLM) orthogonal frequency division multiplexing (OFDM) scheme with low computational complexity. The proposed SLM scheme transforms an input symbol sequence into a set of the time domain signals by multiplying the phase sequences to the signal after a certain intermediate stage of inverse fast Fourier transform (IFFT). Then, the OFDM signal with the lowest peak to average power ratio (PAPR) is selected for transmission. The new SLM OFDM scheme reduces the computational complexity while it shows almost the same performance of PAPR reduction as that of the conventional SLM OFDM scheme.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) system has been proposed as a standard for the next generation mobile radio communication system. OFDM signals have efficient spectral bandwidth and the performance of the OFDM system over frequency selective fading channels is better than that of single carrier modulation. One of the major drawbacks of OFDM system is that the OFDM signal has high peak to average power ratio (PAPR). The high PAPR brings on signal distortion in the nonlinear high power amplifier (HPA) and the signal distortion induces the degradation of bit error rate (BER).

Various techniques [1]~[5] have been proposed for reducing the PAPR. The simple and widely used method is clipping the signal to limit the PAPR below a threshold level, but it causes both in-band distortion and out-of-band radiation. Block coding [2] reduces PAPR by encoding an input data into a codeword with low PAPR, but it increases the redundancy.

In [4], selected mapping (SLM) and partial transmit sequence (PTS) were proposed to lower the PAPR with relatively small increase in redundancy, but without any signal distortion. In SLM, alternative input symbol sequences are generated by multiplying the pre-determined sequences, called phase sequences, to an input symbol sequence. Then, the OFDM signal with the lowest PAPR is selected for transmission after inverse fast Fourier transform (IFFT). In PTS, the input symbol sequence is partitioned into a number of disjoint sub-blocks. IFFT is applied to each sub-block and the signals of subblocks are summed after they are multiplied by distinct rotating factors. Then, the PAPR is computed for each set of rotating factors and compared.

It is known that SLM is more advantageous than PTS if the amount of redundancy is limited but the computational complexity of SLM is larger than that of PTS. The computational complexity of the conventional SLM is increased in proportion to the number of phase sequences. It is mainly the increase in the computational complexity that restricts the implementation of SLM for the OFDM system with large carriers.

This paper is organized as follows: in Section II, the definition of PAPR and the conventional SLM OFDM scheme are described. Section III introduces a new SLM OFDM scheme and discusses the computational complexity issue. The simulation results are shown in Section IV, and finally, the concluding remarks are given in Section V.
II. CONVENTIONAL SLM OFDM SCHEME

In the discrete time domain, an OFDM signal \( a = [a_0, a_1, \cdots, a_{N-1}] \) of \( N \) carriers can be expressed as

\[
a(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi n t/N}, \quad 0 \leq t \leq N-1
\]

where \( A = [A_0, A_1, \cdots, A_{N-1}] \) is an input symbol sequence and \( t \) is a discrete time index.

The PAPR of an OFDM signal, defined as the ratio of the maximum power to the average power of the signal, is expressed as

\[
PAPR(a) = \frac{\text{Max} |a(t)|^2}{E[|a(t)|^2]}
\]

where \( E[\cdot] \) denotes the expected value.

In the conventional SLM OFDM scheme [4], alternative symbol sequences \( A^{(u)} = [A_0^{(u)}, A_1^{(u)}, \cdots, A_{N-1}^{(u)}] \), \( 1 \leq u \leq U \), are generated by multiplying the phase sequences \( P^{(u)} = [P_0^{(u)}, P_1^{(u)}, \cdots, P_{N-1}^{(u)}] \), \( 1 \leq u \leq U \), to the input symbol sequence \( A \). We use the expression \( A^{(u)} = A \cdot P^{(u)} \) to represent the component-wise multiplication, i.e., \( A_n^{(u)} = A_n P_n^{(u)} \), \( 0 \leq n \leq N-1 \).

Each symbol of the phase sequences should have unit magnitude to preserve the power and constellation of the input symbol sequence. The first phase sequence \( P^{(1)} \) is usually all one sequence \( I_0 \).

For the ease of implementation, \( P_s^{(u)} \) is usually selected from \( \{\pm 1\} \). The OFDM signal \( a^{[u]} = \text{IFFT} \{A^{(u)}\} \) with the lowest PAPR is selected for transmission, where \( \tilde{u} \) is expressed as

\[
\tilde{u} = \arg \min_{u \in U} \{ \text{PAPR}(a^{(u)}) \}
\]

III. NEW SLM OFDM SCHEME

A. New SLM OFDM Scheme

Unlike the conventional SLM scheme where different phase sequences are multiplied to an input symbol sequence, in the proposed scheme, they are multiplied to the ‘so-called’ intermediate signal, the partially IFFT-ed input symbol sequence.

Fig. 1 shows the block diagram of the new SLM OFDM scheme. In this scheme, the \( N = 2^s \) point IFFT based on decimation-in-time algorithm is partitioned into two parts. The first part is IFFT of the first \( k \) stages and the second part is IFFT of the remaining \( n-k \) stages.

A set of OFDM signals is generated by multiplying different phase sequences \( P^{(u)} \), \( 1 \leq u \leq U \) to the intermediate signal \( \hat{a}_{\text{data}} \) after \( k \)-th stage of IFFT. Compared to the conventional SLM scheme, the computational load of the new scheme is much relieved since the intermediate signal \( \hat{a}_{\text{data}} \) is used in common.

The information on which phase sequence is used in the transmitter must be conveyed to the receiver in SLM scheme. In the conventional SLM scheme, this information, represented as an index symbol sequence \( A_{\text{index}} \) is added to the data symbol sequence \( A_{\text{data}} \) to form the input symbol sequence \( A \), i.e., \( A = A_{\text{data}} + A_{\text{index}} \). Usually the index is encoded for error detection and correction.

![Fig. 1. Block diagram of the new SLM OFDM scheme.](image_url)
For example, in M-QAM signalling, when encoder code rate is \( R \) and the number of phase sequence is \( U \), the number of index symbols to transmit is \( \lceil \log_M U/R \rceil \), where \( \lceil x \rceil \) denotes the smallest integer exceeding or equal to \( x \). Thus \( \lceil \log_M U/R \rceil \) elements of data \( A_{\text{data}} \) are set to zero to reserve the index information and \( N - \lceil \log_M U/R \rceil \) elements of \( A_{\text{index}} \) are set to zero.

In the proposed SLM scheme, the index signals \( a_{\text{index}}^{(u)} = \text{IFFT}\{A_{\text{index}}^{(u)}\}, 1 \leq u \leq U \) are stored in the memory. Then, the index signal \( a_{\text{index}}^{(u)} \) is added after IFFT of \( A_{\text{data}} \). The new SLM OFDM signal \( a^{(u)} \) can be written as

\[
\begin{align*}
    a^{(u)} &= \text{IFFT}_{n-1}\{P^{(u)} \cdot \text{IFFT}_{i}\{A_{\text{data}}^{(u)}\}\} + \text{IFFT}_{n-1}\{A_{\text{index}}^{(u)}\} \\
    &= \text{IFFT}_{n-1}\{P^{(u)} \cdot \hat{a}_{\text{data}}\} + \hat{a}_{\text{index}}^{(u)}
\end{align*}
\]

where IFFT\(_i\) indicates IFFT from \( i \) stage to \( j \) stage and the size of IFFT is \( N = 2^n \).

**B. Phase Sequences of New SLM OFDM Scheme**

As mentioned previously, in our proposed scheme, the phase sequences are multiplied to the intermediate signals of IFFT while the phase sequences are multiplied to the input symbol sequences in the conventional SLM OFDM scheme. Certainly, the computational complexity of the new SLM OFDM scheme is reduced as the intermediate stage \( k \) for multiplication with the phase sequences approaches to the last stage \( n \). As in the conventional SLM OFDM scheme, we put the restriction on the phase sequences to be \( \{\pm 1\} \) sequences. From the numerical analysis, we can see that the performance of PAPR reduction degrades as \( k \) approaches \( n \). This trade-off between the computational complexity and the performance of PAPR reduction is considered in finding the optimal stage \( k \) from the simulation results.

For \( N = 2^n \), an \( N \) point IFFT can be computed from two \( N/2 \) point transforms, the \( N/2 \) point IFFT from two \( N/4 \) point transforms and so on. According to the successive doubling method, the transform at the \( k \)-th stage consists of \( 2^{n-k} \) blocks, where each block corresponds to \( N/2^{n-k} = 2^k \) point IFFT. Thus, if we define \( T_i \) to be \( N \times N \) symmetric matrix representing \( i \)-th stage of IFFT, then the \( N \) point IFFT-ed signal \( a \) can be expressed as

\[
a^T = T_{n-1} \cdots T_1 a^T.
\]

Multiplying a phase sequence \( Q \) to the input symbol sequence \( A \) corresponds to multiplying an \( N \times N \) diagonal matrix \( \hat{Q} \) whose diagonal entries form the phase sequence \( Q \). Therefore, the output signal \( a_Q \) corresponding to the phase sequence \( Q \) in the conventional SLM scheme can be expressed as

\[
a_Q^T = T_{n-1} \cdots T_1 \hat{Q} a^T,
\]

\[Q(\bullet) \rightarrow +1 +1 -1 -1 +1 +1 +1 -1 -1 +1 -1 -1 -1 -1 -1 \]

\[R(\bullet) \rightarrow +1 +1 +1 +1 +1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 \]

\[k = 1 \]

\[2 \text{ point transform} \]

\[4 \text{ point transform} \]

\[8 \text{ point transform} \]

\[16 \text{ point transform} \]

Fig. 2. An example of the phase sequence generation and the successive doubling method when \( N = 16 \) and \( k = 1 \) are used.
The computational complexity reduction ratio (CCRR) is defined as

$$\frac{N_n \cdot U_n}{N_n \cdot U_n - C_{\text{CCRR}}}$$

where $$N_n$$ represents the number of carriers and $$U_n$$ the number of data symbols. The CCRR measures the reduction in computational complexity compared to a conventional SLM OFDM scheme.

The table below presents the CCRR for various values of $$N_n$$ and $$U_n$$.

<table>
<thead>
<tr>
<th>$$N_n, U_n$$</th>
<th>$$N=256 (n=8)$$</th>
<th>$$N=1024 (n=10)$$</th>
<th>$$N=2048 (n=11)$$</th>
<th>$$N=8192 (n=13)$$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$$U=4$$</td>
<td>$$U=8$$</td>
<td>$$U=16$$</td>
<td>$$U=4$$</td>
</tr>
<tr>
<td>3</td>
<td>47%</td>
<td>55%</td>
<td>59%</td>
<td>53%</td>
</tr>
<tr>
<td>4</td>
<td>38%</td>
<td>44%</td>
<td>47%</td>
<td>45%</td>
</tr>
<tr>
<td>5</td>
<td>28%</td>
<td>33%</td>
<td>35%</td>
<td>38%</td>
</tr>
<tr>
<td>6</td>
<td>19%</td>
<td>22%</td>
<td>23%</td>
<td>30%</td>
</tr>
</tbody>
</table>

C. Computational Complexity

The computational complexity reduction ratio (CCRR) of the new SLM OFDM scheme over the conventional SLM OFDM scheme is defined as

$$\text{CCRR} = \frac{N_n \cdot U_n}{N_n \cdot U_n - C_{\text{CCRR}}}$$
CCRR = \left(1 - \frac{\text{Complexity of new SLM}}{\text{Complexity of conventional SLM}} \right) \times 100

= \left(1 - \frac{1}{U} \right)^k \times 100\% .

Table 1 gives CCRR of the new SLM OFDM scheme over the conventional SLM OFDM scheme with typical values of $U$, $k$, and $n$.

IV. SIMULATION RESULTS

Simulations are performed for the OFDM system of the IEEE standard 802.16 [6] for mobile wireless metropolitan area network (WMAN). The OFDM system specified in IEEE 802.16 has 2048 carriers with QPSK, 16-QAM, and 64-QAM constellation. The number of used carriers is 1702. Of the remaining 346 carriers, 345 carriers are set to zero to shape the power spectral density of the transmit signal and one carrier is used for DC. The 100,000 input symbol sequences are generated randomly with uniform distribution. Figure 3 illustrate the probability that the PAPR of the OFDM signal exceeds the given threshold. The new SLM OFDM scheme with 2048 carriers has almost the same performance compared to the conventional SLM OFDM scheme when $n-k$ is 5. By the simulation results, we can say that the optimal value for $n-k$ does not depend on the number of carriers and the optimal value is 5 when the number of carriers is between 256 and 8192. Fig. 3 shows a comparison of the performance between the conventional SLM OFDM scheme and the new SLM OFDM scheme with $n-k=5$ and 16-QAM constellation. As one can see, the new SLM OFDM has almost the same performance of PAPR reduction as that of the conventional SLM OFDM scheme. In the case of $n-k=5$, the new SLM OFDM system reduces the computational complexity by 41% ~ 51% as the number of sequences $U$ increases from 4 to 16.

V. CONCLUSION

The computational complexity of the conventional SLM OFDM scheme is increased in proportion to the number of phase sequences. A new SLM OFDM scheme with low computational complexity has been proposed and its performance is analyzed in reference to the standard of IEEE 802.16 for mobile WMAN. The simulation results show that the new SLM OFDM scheme with 2048 carriers reduces the computational complexity by 51% for $n-k=5$, while it has almost the same performance of PAPR reduction as that of the conventional SLM OFDM scheme. The computational complexity reduction ratio increases as the number of carriers increases. Thus, the
proposed scheme is appropriate for the high data rate OFDM systems.

REFERENCES


