On the Decoding of LDPC Codes in IEEE 802.16e Standards for Improving the Convergence Speed

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Abstract

In this paper, the modified iterative decoding algorithm by partitioning check nodes is applied to low-density parity-check (LDPC) codes in IEEE 802.16e standards, which gives us the improvement for convergence speed of decoding. Also, the new method of check node partitioning which is suitable for decoding of the LDPC codes in IEEE 802.16e system is proposed. The improvement of convergence speed in decoding reduces the number of iterations and thus the computational complexity of the decoder. The decoding method by partitioning check nodes can be applied to the LDPC codes whose decoder cannot be implemented in the fully parallel processing.

1. INTRODUCTION

Low-density parity-check (LDPC) codes which were first introduced by Gallager [1] in the 1960’s show the good performance approaching closely the Shannon’s theoretical limit for various channels. Also, the probabilistic iterative decoding based on low density of a parity check matrix is easily implemented in comparison with the past. The contemporary technology of hardware enables LDPC codes to be used in practice; recent technologies have contributed to achieve the reliability required by today’s high-speed digital systems. Thus, recently, the researchers of error correcting codes have been attracted by LDPC codes. In practice, LDPC codes draw attention as standards of various fields such as communication, broadcast, and storage.

The various studies related to LDPC codes can be divided into two parts; one is the method of code design with efficient encoding structure and the other is the method of decoding with low computational complexity. To decrease encoding complexity, the efficient encoding structure is designed in finite length and block-type codes using protograph codes [2]. LDPC codes in IEEE 802.16e systems [3] have the efficient encoding structure in the same manner.

Belief propagation (BP) [4], [5] and min-sum approximation (MSA) [6] iterative decoding algorithm are well-known decoding methods of LDPC codes. But, in the BP and MSA iterative decoding algorithm, a large number of iterations which cause high computational complexity are demanded to recover the reliable information. Thus, we would pay attention to a modified decoding algorithm [7] in order to improve the convergence speed, which means reducing the computational complexity of the decoder. It can be used to implement the practical decoder in the wireless communication systems.

The paper is organized as follows: in Section II, after a brief review of conventional decoding algorithm, we introduce a modified decoding algorithm to improve the convergence speed and propose the method of check node non-repetition partitioning which is suitable for decoding of LDPC codes in IEEE 802.16e system. The simulation results are shown in Section III and concluding remarks are given in Section IV.

2. DECODING METHOD FOR IMPROVING CONVERGENCE SPEED

A brief review of conventional BP and MSA decoding algorithm is provided to understand the modified decoding algorithm for improving convergence speed. See [5] and [6] in detail.

The iterative decoding algorithm whose examples are BP and MSA decoding algorithm is the procedure of error correcting in which each variable and check node of an LDPC code interchange messages iteratively using message updating operations. In BP algorithm,
we assume that all messages are used in log-likelihood ratio (LLR) values. Then, in case of a check node, each output message is represented as the form of complicated \( \tanh \) function. Similarly, in a variable node, each output message is the summation of all incoming messages including channel output message except for the incoming message on the edge where the output message will be sent. However, in BP algorithm, the check node updating operation is so complicated that it cannot be implemented easily. To solve this computational complexity, MSA decoding algorithm is introduced. As a result of MSA decoding, the check node updating operation can be simplified with a little degradation of performance.

### 2.1. Decoding Algorithm with Improved Convergence Speed

Now, we consider the modified message-passing iterative decoding algorithm [7] to improve the convergence speed of performance. Each iteration in the conventional iterative decoding algorithm consists of two steps. At the first step, all variable nodes carry out the updating calculation of messages and send them to all neighboring check nodes. Similarly, at the second step, all check nodes calculate the messages by updating operation and send them to all neighboring variable nodes. At each step, all the operations are performed simultaneously.

For convenience, in the modified decoding algorithm, we assume that the check nodes are partitioned into \( p \) subsets. The messages from variable nodes to the check nodes in the first subset are updated and then the messages from the check nodes in the first subset to their neighboring variable nodes are updated. This procedure corresponds to one iteration for the first subset of check nodes. The decoding process is sequentially applied to the remaining \( p - 1 \) subsets of check nodes, which corresponds to one iteration of decoding. In other words, one iteration in the modified decoding algorithm means the above sequential message updating and passing for all variable nodes and all subsets of the check nodes. Therefore, it is clear that the amount of computation for one iteration in the modified decoding algorithm is the same as that of one iteration in the conventional decoding algorithm.

### 2.2. New Method of Check Node Partitioning

How to partition the subsets of check nodes has influence on the convergence speed of the modified decoding algorithm. Here, it is described that the efficient method of check node partitioning can improve the convergence speed in the decoding. The simplest method is to divide sequentially all check nodes into subsets which contain the same number of check nodes. It is called sequential partitioning.

We would like to propose the non-repetition partitioning as the efficient method of check node partitioning. Row permutations in the parity check matrix of LDPC codes do not change the characteristic of codes. Thus, it is possible to change the ordering of rows in parity check matrix. Using this fact, we can construct the equivalent parity check matrix according to the following partitioning criterion. The partitioning criterion for subsets of check nodes is as follows: Within the same subset of rows, each column contains the component ‘1’ less than or equal to one. In other words, variable nodes connected by check nodes within a subset have the only connection with this subset. Note that the number of check nodes within each subset of check nodes can be different and for convenience of implementation, it is determined as the minimum number of subsets satisfying the partitioning criterion.

The reason why the above criterion is adopted is that for the present iteration, the updating messages from check nodes in a subset to variable nodes are sent and sequentially the variable nodes send updated messages to check nodes in another subset. But other variable nodes that are not connected with the check nodes in a subset and thus do not update the messages from the subset send the messages used at previous iteration. For example, let \( S_i \), \( 1 \leq i \leq p \), be the \( i \)-th subset of check nodes. During the \( l \)-th iteration in the modified decoding algorithm, the \( l \)-th updated messages from \( S_1, S_2, \cdots, S_{l-1} \) and the \( (l - 1) \)-st updated messages from the remaining subsets are used for the \( l \)-th message updating from the variable nodes to the check nodes in \( S_l \). Therefore, in terms of propagating many internal updating messages through edges of graph corresponding to LDPC codes, the above criterion – variable nodes connected by check nodes within a subset have the single connection with this subset – is optimum. After all, the messages interchanged more frequently among nodes cause faster convergence speed of performance.

### 3. SIMULATION RESULTS

In this section, the modified iterative decoding algorithm by partitioning check nodes is applied to LDPC codes in IEEE 802.16e standards and is examined in terms of the convergence speed of decoding performance. We consider an additive white Gaussian noise (AWGN) channel and the maximum number of iterations is limited to 50 times.

In Fig. 1, it is shown the block-type \( 12 \times 24 \) parity
check matrix of a rate 1/2 LDPC code in the IEEE 802.16e standards [3] using protograph codes. Each blank in the block-type parity check matrix denotes a $z \times z$ all-zero matrix, and each block with a shift value represents a $z \times z$ circular right shifted matrix of a $z \times z$ identity matrix. In particular, each block with a 0 value denotes a $z \times z$ identity matrix. Therefore the LDPC codes with various code lengths can be defined by the adjustment of the shift values according to $z$.

When the sequential partitioning with the number of subsets of check nodes $p = 6$ is applied to the LDPC code, each subset of check nodes corresponding to the $(1, 2), (3, 4), (5, 6), (7, 8), (9, 10)$, or $(11, 12)$-th row in $12 \times 24$ matrix is fixed. Also, for the non-repetition partitioning with $p = 6$, each subset corresponding to the $(1, 10), (2, 11), (3, 5), (4, 6), (7, 9)$, or $(8, 12)$-th row in matrix is determined according to the criterion of non-repetition partitioning. First of all, we compare the decoding performance by sequential partitioning with that by non-repetition partitioning applying each to modified decoding algorithm by partitioning check nodes. Fig. 2 shows the frame error rate (FER) performance for LDPC codes in the IEEE 802.16e standards with rate 1/2 and code length 2304 according to iterations applying sequential and non-repetition partitioning.

For the case of sequential partitioning, the convergence speed of performance is faster as $p$ increases. But the modified decoding with the large $p$ value cannot be implemented due to latency. Because the performance by non-repetition partitioning with $p = 6$ approaches to that by sequential partitioning with $p = 1152$ which corresponds to the maximum number of check node partitioning, non-repetition partitioning is optimum for both performance and implementation. In particular, the performance is rapidly improved by non-repetition partitioning for the small number of iterations.

Now, we apply the modified iterative decoding algorithm using non-repetition partitioning to LDPC codes with various lengths and rates in IEEE 802.16e standards and confirm the improvement of convergence speed of it. In the case of rates 2/3A, 3/4A, and 3/4B LDPC codes, each subset corresponds to each row block in the block-type parity check matrix and thus the numbers of subsets of check nodes are $p = 8$, $p = 6$, and $p = 6$, respectively. Also, in the case of rate 2/3B LDPC codes, for the non-repetition partitioning with $p = 4$, each subset corresponding to the $(1, 4), (2, 7), (3, 6)$, or $(5, 8)$-th row block in matrix can be selected.

![Figure 1: Block-type parity check matrix of LDPC codes with rate 1/2 in IEEE 802.16e standards.](image1)

![Figure 2: The FER performance for LDPC codes with rate 1/2 and code length 2304 according to iterations applying sequential and non-repetition partitioning.](image2)

![Figure 3: The FER performance according to iterations applying modified BP decoding method by non-repetition partitioning to rate 1/2 LDPC codes.](image3)
First, we start with modified decoding method based on check node updating operation of BP decoding. Fig. 3, Fig. 4, and Fig. 5 show the FER performance of LDPC codes with rate $1/2$, $2/3$, and $3/4$, respectively. Next, we consider the modified decoding method based on check node updating operation for MSA decoding algorithm by partitioning the check nodes [6]. Optimum normalization constant $\gamma = 1/\alpha$ less than 1 and offset constant $\beta$ for each rate are determined through the brutal search to obtain the performance of MSA decoding as a reference (Table 1).

Table 1: Optimum values of $\gamma$ and $\beta$ in min-sum approximation decoding for LDPC codes in IEEE 802.16e standards.

<table>
<thead>
<tr>
<th>Rate</th>
<th>$\gamma$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2$</td>
<td>0.83</td>
<td>0.43</td>
</tr>
<tr>
<td>$2/3$A</td>
<td>0.82</td>
<td>0.44</td>
</tr>
<tr>
<td>$3/4$A</td>
<td>0.76</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Although MSA decoding decreases the computational complexity by simple check node updating operation, its performance approaches to that of BP decoding within 0.1dB. For optimum $\gamma$ and $\beta$ for LDPC codes with various rates and lengths, similar characteristic is appeared. Fig. 6 shows the FER performance of rate $1/2$ LDPC codes using modified MSA decoding algorithm with non-repetition check node partitioning and optimum $\gamma$.

From the above simulation results, we know that the performance by non-repetition partitioning with 25 iterations is similar to that by conventional (BP and MSA) decoding ($p = 1$) with 50 iterations for various lengths and rates. Therefore, we can confirm the improvement for convergence speed in the modified decoding algorithm with non-repetition check node partitioning. Note that the amount of computation for one iteration in the modified decoding algorithm is the same as that for one iteration in the conventional decoding algorithm. Because the same performance of decoding is guaranteed by less iterations, the computational complexity of modified decoding algorithm by non-repetition partitioning can be reduced by half.
4. CONCLUSIONS

In this paper, the modified iterative BP and MSA decoding algorithm by partitioning check nodes was applied to LDPC codes in IEEE 802.16e standards and we confirmed the improvement in convergence speed of performance. Because the same performance of decoding is guaranteed by less iterations, the computational complexity can be reduced by half. Also, the method of non-repetition check node partitioning which is suitable for LDPC codes in IEEE 802.16e system is proposed. The decoding method by partitioning check nodes can be applied to the systems which cannot be implemented in the fully parallel processing as an efficient sequential processing method. The modified iterative decoding method of LDPC codes using the proposed check node partitioning method can be used to implement the practical decoder in the wireless communication systems.

References


