Generalized Extending Method for Construction of q-ary Low Correlation Zone Sequence Sets

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Abstract— **In this paper, a new extending method of q-ary low correlation zone(LCZ) sequence sets is proposed, which is a generalization of binary LCZ sequence set by Kim, Jang, No, and Chung. Using this method, q-ary LCZ sequence set with parameters (N, M, L, ϵ) is extended as a q-ary LCZ sequence set with parameters $(pN, pM, p\lfloor(L+1)/p\rfloor - 1, p\epsilon)$, where p is prime and $p|q$.**

I. Introduction

In the quasi-synchronous code-division multiple-access(QS-CDMA) system, the low correlation zone (LCZ) sequence sets are adopted to reduce the interference between users. Since LCZ sequence sets have very low cross-correlation around the origin, QS-CDMA system can perform better in the environment that allows more time delay than CDMA systems.

There are many proposed LCZ sequence sets such as [2]–[6]. Recently, Kim, Jang, No, and Chung proposed flexible binary LCZ sequence sets whose LCZ can be controlled by two parameters f_0 and f [6]. In this paper, a new extending method of q-ary LCZ sequence sets is proposed, which is a generalization of binary LCZ sequence set by Kim, Jang, No, and Chung. Using this method, q-ary LCZ sequence set with parameters (N, M, L, ϵ) is extended as a q-ary LCZ sequence set with parameters $(pN, pM, p\lfloor(L+1)/p\rfloor - 1, p\epsilon)$, where p is prime and $p|q$. When $L \equiv p-1 \mod p$, we have a new extended q-ary LCZ sequence set with parameters $(pN, pM, L, p\epsilon)$, which preserves optimality of LCZ sequence set.

II. Extending Method of q-ary LCZ Sequence Sets

Let $s_i(t)$ and $s_j(t)$ be q-ary sequences of period N and ω_q a complex qth root of unity, $\omega = e^{2\pi/q}$. The correlation function $R_{i,j}$ of two sequences $s_i(t)$ and $s_j(t)$ is defined as

$$R_{i,j}(\tau) = \sum_{t=0}^{N-1} \omega^{s_i(t)-s_j(t+\tau)} \tag{1}$$

where $0 \le \tau \le N-1$.

Let $\mathcal{S}$ be the q-ary LCZ sequence set with parameters (N, M, L, ϵ) given by

$$\mathcal{S} = \{v_i(t) \mid 0 \le i \le M-1,\ 0 \le t \le N-1\}.$$

Let p be a prime. Let g be a generator of the finite field F_p with p elements. Let $\mathbf{D}$ be a $p \times p$ matrix given as

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & g & 2g & \cdots & (p-1)g \\ 0 & g^2 & 2g^2 & \cdots & (p-1)g^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & g^{p-2} & 2g^{p-2} & \cdots & (p-1)g^{p-2} \\ 0 & 1 & 2 & \cdots & (p-1) \end{bmatrix} = [d_{ij}]$$

where $0 \le i, j \le p-1$.

Using a q-ary LCZ sequence set with parameters (N, M, L, ϵ), a new extended q-ary LCZ sequence set can be constructed as in the following theorem.

Theorem 1: Let p be a prime and $p|q$. Let $\mathcal{T}$ be the q-ary sequence set given as

$$\mathcal{T} = \{s_i(t) \mid 0 \le i \le pM-1,\ 0 \le t \le pN-1\}$$
$$s_i(t) = s_i(px+y) = v_{i-kM}\left(x + y\lfloor\frac{L+1}{p}\rfloor\right) + \frac{q}{p}d_{ky},$$
$$\text{for } kM \le i \le (k+1)M-1$$

where $0 \le x \le N-1$, $0 \le y \le p-1$, and $\lfloor a \rfloor$ denotes the greatest integer less than or equal to a. Then $\mathcal{T}$ is a q-ary LCZ sequence set with parameters $(pN, pM, p\lfloor\frac{L+1}{p}\rfloor - 1, p\epsilon)$.

Proof: By definition of sequence set, it is not difficult to see that the period of the sequences in the set is pN and the size of set is pM. Thus, it is enough to show that the maximum magnitude of correlation values in LCZ is less than or equal to $p\epsilon$ and LCZ is equal to $p\lfloor(L+1)/p\rfloor - 1$.

Let $L+1 = pa+b$, where a is a non-negative integer and $0 \le b \le p-1$. Thus, we have $\lfloor\frac{L+1}{p}\rfloor = a$. According to the size M of $\mathcal{S}$ and the matrix $\mathbf{D}$, we have to consider the following six cases.

Case 1) $0 \le i, j \le M-1$ and $\tau = pk'$;

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In this case, $R_{i,j}(\tau)$ can be rewritten as

$$\begin{aligned} &R_{i,j}(\tau)\\ &=\sum_{y=0}^{p-1}\sum_{x=0}^{N-1}\omega^{v_i(x+ay)-v_j(x+ay+\frac{\tau}{p})}\\ &=\sum_{x=0}^{N-1}\omega^{v_i(x)-v_j(x+\frac{\tau}{p})}+\sum_{x=0}^{N-1}\omega^{v_i(x+a)-v_j(x+a+\frac{\tau}{p})}\\ &+\cdots+\sum_{x=0}^{N-1}\omega^{v_i(x+(p-1)a)-v_j(x+(p-1)a+\frac{\tau}{p})}. \end{aligned} \tag{2}$$

From the property of the LCZ sequence set with parameters (N, M, L, ϵ), it is clear that the magnitude of each summation in (2) is less than or equal to ϵ within the range $-pL < \tau < pL$. Therefore, $|R_{i,j}(\tau)| \le p\epsilon$ within the range $-pL < \tau < pL$.

Case 2) $0 \le i, j \le M-1$ and $\tau = pk'+k''(1 \le k'' \le p-1)$;

In this case, $R_{i,j}(\tau)$ can be rewritten as

$$\begin{aligned} &R_{i,j}(\tau)\\ &=\sum_{y=0}^{p-1}\sum_{x=0}^{N-1}\omega^{v_i(x+ay)-v_j(x+\left\lfloor\frac{y+k''}{p}\right\rfloor+(y\oplus k'')a+\frac{\tau-k''}{p})} \end{aligned} \tag{3}$$

where $\oplus$ denotes addition modulo p.

If $k'' = 1$, we have

$$\begin{aligned} R_{i,j}(\tau)=&\sum_{x=0}^{N-1}\omega^{v_i(x)-v_j(x+a+\frac{\tau-1}{p})}\\ &+\sum_{x=0}^{N-1}\omega^{v_i(x+a)-v_j(x+2a+\frac{\tau-1}{p})}\\ &+\cdots+\sum_{x=0}^{N-1}\omega^{v_i(x+(p-1)a)-v_j(x+1+\frac{\tau-1}{p})}. \end{aligned} \tag{4}$$

From the property of the LCZ sequence set with parameters (N, M, L, ϵ) and in-phase autocorrelation, the magnitude of all summations except the last one is less than or equal to ϵ within the following range $-L < a + \frac{\tau-1}{p} < L$ and of the last one within the range

$$-L<\frac{\tau-1}{p}+1-(p-1)a<L \quad\text{and}\quad \frac{\tau-1}{p}+1<(p-1)a.$$

So, $-L-(p-1)b < \tau < (p-1)(L-b)$. Thus, in the case of $k'' = 1$, $|R_{i,j}(\tau)| \le p\epsilon$ within the range $|\tau| < \min\{L + (p-1)b, (p-1)(L-b)\}$.

Generally, for $1 \le k'' \le p-1$, we have

$$\begin{aligned} -L<k''a+\frac{\tau-k''}{p}<L\\ -L<\frac{\tau-k''}{p}+1-(p-k'')a<L\\ \text{and}\quad \frac{\tau-k''}{p}+1<(p-k'')a. \end{aligned}$$

Thus, $-k''L-(p-k'')b < \tau < \min\{(L-b)(p-k''), (p-k'')L+k''b\}$.

More, $|\tau| < \min\{|k''L+(p-k'')b|, |(L-b)(p-k'')|, |(p-k'')L+k''b|\}$.

When $k'' = p-1$, the range of τ is tightest.

Therefore, $|R_{i,j}(\tau)| \le p\epsilon$ within the range $-(L-b) < \tau < L-b$.

Case 3) $0 \le i \le M-1$, $kM \le j \le (k+1)M-1$, $1 \le k \le (p-1)$ (or $kM \le i \le (k+1)M-1$, $0 \le j \le M-1$), and $\tau = pk'$;

$R_{i,j}(\tau)$ can be rewritten as

$$\begin{aligned} &R_{i,j}(\tau)\\ &=\sum_{y=0}^{p-1}\sum_{x=0}^{N-1}\omega^{v_i(x+ay)-v_{j-kM}(x+ay+\frac{\tau}{p})-d_{ky}}\\ &=\sum_{x=0}^{N-1}\omega^{v_i(x)-v_{j-kM}(x+\frac{\tau}{p})-d_{k0}}\\ &+\sum_{x=0}^{N-1}\omega^{v_i(x+a)-v_{j-kM}(x+a+\frac{\tau}{p})-d_{k1}}\\ &+\cdots+\sum_{x=0}^{N-1}\omega^{v_i(x+(p-1)a)-v_{j-kM}(x+(p-1)a+\frac{\tau}{p})-d_{k(p-1)}}\\ &=\sum_{x=0}^{N-1}\omega^{v_i(x)-v_{j-kM}(x+\frac{\tau}{p})}+\sum_{x=0}^{N-1}\omega^{v_i(x+a)-v_{j-kM}(x+a+\frac{\tau}{p})-g}\\ &+\cdots+\sum_{x=0}^{N-1}\omega^{v_i(x+(p-1)a)-v_{j-kM}(x+(p-1)a+\frac{\tau}{p})-g(p-1)}\\ &=\sum_{x=0}^{N-1}\omega^{v_i(x)-v_{j-kM}(x+\frac{\tau}{p})}\\ &+\omega^{-g}\sum_{x=0}^{N-1}\omega^{v_i(x+a)-v_{j-kM}(x+a+\frac{\tau}{p})}\\ &+\cdots+\omega^{-g(p-1)}\sum_{x=0}^{N-1}\omega^{v_i(x+(p-1)a)-v_{j-kM}(x+(p-1)a+\frac{\tau}{p})}. \end{aligned}$$

Similarly to Case 1), $|R_{i,j}(\tau)| \le p\epsilon$ within the range $-pL < \tau < pL$.

Case 4) $0 \le i \le M-1$, $kM \le j \le (k+1)M-1$, $1 \le k \le (p-1)$ (or $kM \le i \le (k+1)M-1$, $0 \le j \le M-1$), and $\tau = pk' + k''(1 \le k'' \le p-1)$;

We have

$$\begin{aligned} &R_{i,j}(\tau)\\ &=\sum_{y=0}^{p-1}\sum_{x=0}^{N-1}\omega^{v_i(x+ay)}\\ &\times\omega^{-v_{j-kM}(x+\left\lfloor\frac{y+k''}{p}\right\rfloor+(y\oplus k'')a+\frac{\tau-k''}{p})-d_{k(y\oplus k'')}}\\ &=\sum_{y=0}^{p-1}\omega^{-d_{k(y\oplus k'')}}\\ &\times\sum_{x=0}^{N-1}\omega^{v_i(x+ay)-v_{j-kM}(x+\left\lfloor\frac{y+k''}{p}\right\rfloor+(y\oplus k'')a+\frac{\tau-k''}{p})}. \end{aligned}$$

Similarly to Case 2), $|R_{i,j}(\tau)| \leq p\epsilon$ within the range $-(L-b) < \tau < L-b$.

Case 5) $k_1M \leq i \leq (k_1+1)M-1$, $k_2M \leq j \leq (k_2+1)M-1$, $1 \leq k_1, k_2 \leq (p-1)$, and $\tau = pk'$;

We also have

$$\begin{aligned} &R_{i,j}(\tau)\\ &= \sum_{y=0}^{p-1}\sum_{x=0}^{N-1} \omega^{v_{i-k_1M}(x+ay)+d_{k_1y}-v_{j-k_2M}(x+ay+\frac{\tau}{p})-d_{k_2y}}\\ &= \sum_{y=0}^{p-1}\omega^{d_{k_1y}-d_{k_2y}}\sum_{x=0}^{N-1}\omega^{v_{i-k_1M}(x+ay)-v_{j-k_2M}(x+ay+\frac{\tau}{p})}. \end{aligned}$$

Similarly to Case 1), $|R_{i,j}(\tau)| \leq p\epsilon$ within the range $-pL < \tau < pL$.

Case 6) $k_1M \leq i \leq (k_1+1)M-1$, $k_2M \leq j \leq (k_2+1)M-1$, $1 \leq k_1, k_2 \leq (p-1)$, and $\tau = pk' + k''(1 \leq k'' \leq p-1)$;

We also have

$$\begin{aligned} &R_{i,j}(\tau)\\ &= \sum_{y=0}^{p-1}\sum_{x=0}^{N-1} \omega^{v_{i-k_1M}(x+ay)+d_{k_1y}}\\ &\qquad \times \omega^{-v_{j-k_2M}(x+\left\lfloor\frac{y+k''}{p}\right\rfloor+(y\oplus k'')a+\frac{\tau-k''}{p})-d_{k_2(y\oplus k'')}}\\ &= \sum_{y=0}^{p-1}\omega^{d_{k_1y}-d_{k_2(y\oplus k'')}}\\ &\times \sum_{x=0}^{N-1}\omega^{v_{i-k_1M}(x+ay)-v_{j-k_2M}(x+\left\lfloor\frac{y+k''}{p}\right\rfloor+(y\oplus k'')a+\frac{\tau-k''}{p})}. \end{aligned}$$

Similarly to Case 2), $|R_{i,j}(\tau)| \leq p\epsilon$ within the range $-(L-b) < \tau < L-b$.

From the above 6 cases, it is clear that $\mathcal{T}$ is an LCZ sequence set with parameters $(pN, pM, p\lfloor\frac{L+1}{p}\rfloor - 1, p\epsilon)$ (or $(pN, pM, L-b, p\epsilon)$). □

III. Optimality of the Proposed LCZ Sequences Set

In [7], Tang, Fan, and Matsufuji proposed a lower bound on LCZ sequence sets.

Theorem 2: (Tang, Fan, and Matsufuji [7]) Let $\mathcal{S}$ be an LCZ sequence set with parameters (N, M, L, ϵ). Then we have

$$ML - 1 \leq \frac{N-1}{1-\epsilon^2/N}. \tag{5}$$

□

Theorem 3: Let q-ary LCZ sequence set with parameters (N, M, L, ϵ) be optimal with respect to the bound by Tang, Fan, and Matsufuji. Then the extended LCZ sequence set in Theorem 1 is optimal with respect to the bound by Tang, Fan, and Matsufuji if $L \equiv p-1 \mod p$ and sub-optimal, otherwise.

Proof: From Theorem 2, we have

$$L = \lfloor \frac{1}{M}\frac{N^2-\epsilon^2}{N-\epsilon^2}\rfloor.$$

Let $\mathcal{T}'$ be an optimal q-ary LCZ sequence set with parameters $(pN, pM, L', p\epsilon)$ by Theorem 1, such that

$$\begin{aligned} L' &= \lfloor \frac{1}{pM}\frac{p^2N^2-p^2\epsilon^2}{pN-p^2\epsilon^2}\rfloor = \lfloor \frac{1}{M}\frac{N^2-\epsilon^2}{N-p\epsilon^2}\rfloor\\ &= \lfloor \frac{1}{M}\frac{N^2-\epsilon^2}{N-\epsilon^2}\frac{N-\epsilon^2}{N-p\epsilon^2}\rfloor. \end{aligned}$$

Then we have

$$L' \leq p\lfloor (L+1)/p \rfloor - 1 \leq L.$$

When $L \equiv p-1 \mod p$, the proposed LCZ sequence sets are optimal. □

IV. Extending LCZ Sequence Sets by a Composite Number p_1p_2

In this section, extending methods for a composite number will be presented.

Let $\mathcal{S}$ be a q-ary LCZ sequence set with parameters (N, M, L, ϵ). Using Theorem 1, $\mathcal{T}_{p_1}$ and $\mathcal{T}_{p_2}$ are defined by

$$\begin{aligned} \mathcal{T}_{p_1} &= \{g_i(t) \mid 0 \leq i \leq p_1M-1,\ 0 \leq t \leq p_1N-1\}\\ \mathcal{T}_{p_2} &= \{h_i(t) \mid 0 \leq i \leq p_2M-1,\ 0 \leq t \leq p_2N-1\} \end{aligned}$$

where

$$\begin{aligned} g_i(t) &= g_i(p_1x+y)\\ &= v_{i-kM}\left(x + y\lfloor\frac{L+1}{p_1}\rfloor\right) + \frac{q}{p_1}[\mathbf{D}_{p_1}]_{ky},\\ &\qquad \text{for } \ kM \leq i \leq (k+1)M-1 \end{aligned}$$

and

$$\begin{aligned} h_i(t) &= h_i(p_2x+y)\\ &= v_{i-kM}\left(x + y\lfloor\frac{L+1}{p_2}\rfloor\right) + \frac{q}{p_2}[\mathbf{D}_{p_2}]_{ky},\\ &\qquad \text{for } \ kM \leq i \leq (k+1)M-1. \end{aligned}$$

Then $\mathcal{T}_{p_1}$ is a q-ary LCZ sequence set with parameters $(p_1N, p_1M, L_{p_1}(= p_1\lfloor\frac{L+1}{p_1}\rfloor - 1), p_1\epsilon)$ and $\mathcal{T}_{p_2}$ is a q-ary LCZ sequence set with parameters $(p_2N, p_2M, L_{p_2}(= p_2\lfloor\frac{L+1}{p_2}\rfloor - 1), p_2\epsilon)$.

Theorem 4: Let p_1 and p_2 be primes such that $p_1|q$ and $p_2|q$. Let $\mathcal{T}_{p_3}$ be the extended LCZ sequence set by p_2 of q-ary LCZ sequence set $\mathcal{T}_{p_1}$ defined by

$$\mathcal{T}_{p_3} = \{s_i(t) \mid 0 \leq i \leq p_1p_2M-1,\ 0 \leq t \leq p_1p_2N-1\} \tag{6}$$

where

$$\begin{aligned} s_i(t) &= s_i(p_2x+y)\\ &= g_{i-kM}\left(x + y\lfloor\frac{L_{p_1}+1}{p_2}\rfloor\right) + \frac{q}{p_2}[\mathbf{D}_{p_2}]_{ky},\\ &\qquad \text{for } \ kM \leq i \leq (k+1)M-1 \ \text{ and } \ g_i \in \mathcal{T}_{p_1}. \end{aligned}$$

Then $\mathcal{T}_{p_3}$ is a q-ary LCZ sequence set with parameters $(p_1p_2N, p_1p_2M, p_2\lfloor\frac{L_{p_1}+1}{p_2}\rfloor - 1, p_1p_2\epsilon)$. □

TABLE I
COMPARISON OF EXTENDED LCZ SIZE

$p \backslash L$	100	101	102	103	104	105	106	107	108	109
$p_1 = 3$, $p_2 = 5$	94	99	99	99	104	104	104	104	104	104
$p_1 = 5$, $p_2 = 3$	98	98	98	98	104	104	104	104	104	107
$p_1 = 15$	89	89	89	89	104	104	104	104	104	104

The proof can be given similarly to that of Theorem 1 and thus is omitted.

For a composite integer p_1p_2, the extending method can also be described as follows.

Definition 5: Let $A = (a_{ij})$ be an $l \times m$ and $B = (b_{ij})$ an $k \times n$ matrix. Then $A \odot B$ is defined as

$$A \odot B = \begin{bmatrix} a_{11}E + B & a_{12}E + B & \cdots & a_{1m}E + B \\ a_{21}E + B & a_{22}E + B & \cdots & a_{2m}E + B \\ \vdots & \vdots & \ddots & \vdots \\ a_{l1}E + B & a_{l2}E + B & \cdots & a_{lm}E + B \end{bmatrix}$$

where the $k \times n$ matrix

$$E = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}.$$

□

Using $A \odot B$, we can construct the extended LCZ sequence set by p' as in the following theorems.

Theorem 6: Let $\mathcal{S}$ be a set of q-ary LCZ sequence set with parameters (N, M, L, ϵ). Let p_1 and p_2 be primes and $p' = p_1p_2$ such that $p'|q$. Let

$$\mathbf{D}_{p'} = p_2\mathbf{D}_{p_1} \odot p_1\mathbf{D}_{p_2}.$$

Then the extended q-ary LCZ sequence set with parameters $(p'N, p'M, p'\lfloor\frac{L+1}{p'}\rfloor - 1, p'\epsilon)$ can be given as

$$\mathcal{T}_{p'} = \{s_i(t) \mid 0 \le i \le p'M - 1,\ 0 \le t \le p'N - 1\} \qquad (7)$$

where

$$\begin{aligned} s_i(t) &= s_i(p'x + y) \\ &= v_{i-kM}\left(x + y\lfloor\frac{L+1}{p'}\rfloor\right) + \frac{q}{p'}[\mathbf{D}_{p'}]_{ky}, \\ &\qquad \text{for } \ kM \le i \le (k+1)M - 1. \end{aligned}$$

□

The proof can be given similarly to that of Theorem 1 and thus is omitted.

Theorem 7: Let $\mathcal{S}$ be a set of q-ary LCZ sequence set with parameters (N, M, L, ϵ). Let p_1 and p_2 be primes and $p' = p_1p_2$ such that $p'|q$. A LCZ size of $\mathcal{T}_{p_3}$ in (6) is

$$p_2\lfloor\frac{p_1}{p_2}\lfloor\frac{L+1}{p_1}\rfloor\rfloor - 1$$

and that of $\mathcal{T}_{p'}$ in (6) is

$$p'\lfloor\frac{L+1}{p'}\rfloor - 1.$$

The LCZ size of $\mathcal{T}_{p_3}$ is larger than or equal to that of $\mathcal{T}_{p'}$.

Proof: Let $L = xp' + y$.

Case 1) $y = p' - 1$;

$$p'\lfloor\frac{xp'+p'}{p'}\rfloor - 1 = xp' + p' - 1$$
$$p_2\lfloor\frac{p_1}{p_2}\lfloor\frac{xp'+p'}{p_1}\rfloor\rfloor - 1 = xp' + p' - 1.$$

Case 2) $y \ne p' - 1$;

$$p'\lfloor\frac{xp'+y+1}{p'}\rfloor - 1 = xp' - 1$$
$$\begin{aligned} p_2\lfloor\frac{p_1}{p_2}\lfloor\frac{xp'+y+1}{p_1}\rfloor\rfloor - 1 &= xp' + p_2\lfloor\frac{p_1}{p_2}\lfloor\frac{y+1}{p_1}\rfloor\rfloor - 1 \\ &\ge xp' - 1. \end{aligned}$$

□

Table I shows the LCZ size of extended LCZ sequence sets for three different methods.

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REFERENCES

[1] R. De Gaudenzi, C. Elia, and R. Viola, “Bandlimited quasi-synchronous CDMA: A novel satellite access technique for mobile and personal communication systems,” *IEEE J. Sel. Areas Commun.*, vol. 10, no. 2, pp. 328–343, Feb. 1992.

[2] B. Long, P. Zhang, and J. Hu, “A generalized QS-CDMA system and the design of new spreading codes,” *IEEE Trans. Veh. Technol.*, vol. 47, pp. 1268–1275, Nov. 1998.

[3] X. H. Tang and P. Z. Fan, “A class of pseudonoise sequences over GF(p) with low correlation zone,” *IEEE Trans. Inf. Theory*, vol. 47, no. 4, pp. 1644–1649, May 2001.

[4] Sang-Hyo Kim, Ji-Woong Jang, Jong-Seon No, and Habong Chung, “New constructions of quaternary low correlation zone sequences,” *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1469–1477, Apr. 2005.

[5] J.-W. Jang, J.-S. No, H. Chung, and X. Tang, “New sets of optimal p-ary low correlation zone sequences,” *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 815-821, February 2007.

[6] Young-Sik Kim, Ji-Woong Jang, Jong-Seon No, and Habong Chung “New design of low correlation zone sequence sets,” *IEEE Trans. Inf. Theory*, vol. 52, no. 10, pp. 4607–4616, Oct. 2006.

[7] X. H. Tang, P. Z. Fan, and S. Matsufuji, “Lower bounds on correlation of spreading sequence set with low or zero correlation zone,” *Electron. Lett.*, vol. 36, no. 6, pp. 551–552, Mar. 2000.

[8] H. Torii, M. Nakamura, and N. Suehiro, “A new class of zero-correlation zone sequences,” *IEEE Trans. Inf. Theory*, vol. 50, no. 3, pp. 559–565, Mar. 2004.