Quaternary Low Correlation Zone Sequence Set With Flexible Parameters

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***Abstract*— In this paper, we proposed a new quaternary low correlation zone(LCZ) sequence set with parameters $(2(2^n-1), M, L, 2)$. The new LCZ sequence set is constructed from the binary sequence with ideal autocorrelation of period 2^n-1. The proposed construction method corresponds to the generalization of the construction method of binary LCZ sequence set by using binary sequence with ideal autocorrelation proposed by Kim, Jang, No, and Chung [1].**

I. Introduction

In the quasi-synchronous code-division multiple-access (QS-CDMA) system proposed by Gaudenzi, Elia, and Vilola [2], multiple chip time delay among different users is allowed, which gives more flexibility in designing the wireless communication system. In order to reduce the interference between users in the QS-CDMA system, the low correlation zone (LCZ) sequence sets are adopted. Due to the low correlation property around origin, QS-CDMA system can outperform in the environment that allows more time delay than CDMA systems.

There are many research results on the LCZ sequence sets such as [3], [4], [5]. Recently, flexible binary LCZ sequence sets whose LCZ can be controlled by two parameters f_0 and f was proposed by Kim, Jang, No, and Chung [1]. In this paper, we proposed a new quaternary low correlation zone(LCZ) sequence set with parameters $(2(2^n-1), M, L, 2)$. The new LCZ sequence set is constructed from the binary sequence with ideal autocorrelation of period 2^n-1. The proposed construction method corresponds to the generalization of the construction method of binary LCZ sequence set by using binary sequence with ideal autocorrelation proposed by Kim, Jang, No, and Chung [1].

II. Preliminaries

In this section, we introduce some definitions and notations.

Let q be a positive integer. And let $s_u(t)$ and $s_v(t)$ be q-ary sequences with period L. Then the correlation $R_{u,v}(\tau)$ is defined as

$$R_{u,v}(\tau)=\sum_{t=0}^{L-1}\omega_q^{s_u(t)-s_v(t+\tau)}$$

where $0\le\tau\le L-1$ is phase shift and ω_q is a complex primitive q-th root of unity. In the case of $s_u(t)=s_v(t)$, we call $R_{u,v}(\tau)$ autocorrelation of $s_u(t)$. And the case of $s_u(t)\neq s_v(t)$, we call $R_{u,v}(\tau)$ cross-correlation between the sequence $s_u(t)$ and $s_v(t)$.

Let n be a positive integer and $N=2\times(2^n-1)$. Let Z_N be the set of integers modulo N, i.e., $Z_N=\{0,1,\cdots,N-1\}$. Let $a(t)$ be a binary sequence with ideal autocorrelation of period 2^n-1. Let D_u be the characteristic set of $a(t-u)$, i.e.,

$$D_u=\{t\mid s(t-u)=1,\ 0\le t\le 2^n-2\}=D_0+u$$

where $u\in Z_{2^n-1}$, $D_0+u=\{d+u\mid d\in D_0\}$, and + means addition modulo 2^n-1. Let $\overline{D}_u=Z_{2^n-1}\backslash D_u$. From the balancedness of $s(t)$, it is clear that

$$\begin{aligned}|D_u| &= 2^{n-1},\\ |\overline{D}_u| &= 2^{n-1}-1.\end{aligned}$$

From the pairwise balancedness of the sequence that has ideal autocorrelation, for $u\neq v$, we have

$$\begin{aligned}|D_u\cap D_v| &= 2^{n-2}\\ |D_u\cap\overline{D}_v| &= 2^{n-2}\\ |\overline{D}_u\cap D_v| &= 2^{n-2}\\ |\overline{D}_u\cap\overline{D}_v| &= 2^{n-2}-1.\end{aligned}$$

If $u=v$, it is clear that

$$\begin{aligned}|D_u\cap D_v| &= 2^{n-1}\\ |D_u\cap\overline{D}_v| &= 0\\ |\overline{D}_u\cap D_v| &= 0\\ |\overline{D}_u\cap\overline{D}_v| &= 2^{n-1}-1.\end{aligned}$$

By the Chinese remainder theorem, we can represent $Z_N\cong Z_2\otimes Z_{2^n-1}$ under the isomorphism $\phi:\zeta\longmapsto(\zeta \bmod 2,\zeta \bmod 2^n-1)$, where $\otimes$ means direct product. For convenience, we will use the notations $\zeta\in Z_N$ interchangeably with $(\zeta \bmod 2,\zeta \bmod 2^n-1)$ throughout this paper.

III. Design of New LCZ Sequence Set

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Theorem 1: Let $s_u(t)$, $1 \le u < 2^{n-1}$, be the quaternary sequence defined by

$$s_u(t) = \begin{cases} 0, & \text{if } t \in \{0\} \otimes \overline{D}_u \\ 1, & \text{if } t \in \{1\} \otimes \overline{A}_u \\ 2, & \text{if } t \in \{0\} \otimes D_u \\ 3, & \text{if } t \in \{1\} \otimes A_u \end{cases}$$

where $A_u \in \{D_{1-u}, \overline{D}_{1-u}\}$. Let $\mathcal{U}_1$ and $\mathcal{U}_2$ be sets of sequences such that $A_u = D_{1-u}$ and $A_u = \overline{D}_{1-u}$, respectively.

Then the correlation $R_{u,v}(\tau)$ between $s_v(t)$ and $s_u(t)$ is calculated as follows:

For $s_u(t), s_v(t) \in \mathcal{U}_1$;

$$R_{u,v}(\tau) = \begin{cases} 2^{n+1}-2, & u = v \text{ and } \tau = 0 \\ 2^n j, & u = v \text{ and } \tau = (1, 2u-1) \\ -2^n j, & u = v \text{ and } \tau = (1, 1-2u) \\ 0, & u = v,\ \tau_1 = 1 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{2u-1, 1-2u\} \\ -2, & u = v,\ \tau_1 = 0 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{0\} \\ 2^n - 2, & u \ne v \text{ and } \\ & \tau \in \{(0, u-v), (0, v-u)\} \\ 2^n j, & u \ne v \text{ and } \tau = (1, u+v-1) \\ -2^n j, & u \ne v \text{ and } \tau = (1, 1-u-v) \\ 0, & u \ne v,\ \tau_1 = 1 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{1-u-v, u+v-1\} \\ -2, & u \ne v,\ \tau_1 = 0 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{u-v, v-u\}. \end{cases}$$

For $s_u(t) \in \mathcal{U}_1$ and $s_v(t) \in \mathcal{U}_2$;

$$R_{u,v}(\tau) = \begin{cases} (2^n-2)j, & u = v \text{ and } \\ & \tau \in \{(1, 1-2u), (1, 2u-1)\} \\ 0, & u = v \text{ and } \tau_1 = 0 \\ -2j, & u = v,\ \tau_1 = 1 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{2u-1, 1-2u\} \\ 2^n, & u \ne v \text{ and } \tau = (0, v-u) \\ -2^n, & u \ne v \text{ and } \tau = (1, u-v) \\ (2^n-2)j, & u \ne v, \text{ and } \\ & \tau \in \{(1, 1-u-v), (1, u+v-1)\} \\ 0, & u \ne v,\ \tau_1 = 0 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{u-v, v-u\} \\ -2j, & u \ne v,\ \tau_1 = 1 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{1-u-v, u+v-1\}. \end{cases}$$

For $s_u(t), s_v(t) \in \mathcal{U}_2$;

$$R_{u,v}(\tau) = \begin{cases} 2^{n+1}-2), & u = v \text{ and } \tau = 0 \\ 2^n j, & u = v \text{ and } \tau = (1, 1-2u) \\ -2^n j, & u = v \text{ and } \tau = (1, 2u-1) \\ 0, & u = v,\ \tau_1 = 1 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{2u-1, 1-2u\} \\ -2, & u = v,\ \tau_1 = 0 \text{ and } \tau_2 \in Z_{2^n-1} \backslash \{0\} \\ 2^n - 2, & u \ne v \text{ and } \\ & \tau \in \{(0, u-v), (0, v-u)\} \\ 2^n j, & u \ne v \text{ and } \tau = (1, u+v-1) \\ -2^n j, & u \ne v \text{ and } \tau = (1, 1-u-v) \\ 0, & u \ne v,\ \tau_1 = 1 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{1-u-v, u+v-1\} \\ -2, & u \ne v,\ \tau_1 = 0 \text{ and } \\ & \tau_2 \in Z_{2^n-1} \backslash \{u-v, v-u\} \end{cases}$$

where j is complex 4th primitive root of unity.

Proof :

From the definition, it is clear that $u + v \not\equiv 1 \bmod 2^n - 1$. Let $\tau = (\tau_1, \tau_2)$, where $\tau_1 \in Z_2$ and $\tau_2 \in Z_{2^n-1}$. Then we have

$$\begin{aligned} R_{u,v}(\tau) &= \sum_{t=0}^{N-1} \omega_4^{s_u(t)-s_v(t+\tau)} \\ &= \sum_{t=0}^{N-1} \omega_4^{s_u(t-\tau)-s_v(t)} \\ &= |\{\tau_1\} \otimes \overline{D}_{u+\tau_2} \cap \{0\} \otimes \overline{D}_v| \\ &\quad +|\{1+\tau_1\} \otimes \overline{A}_{u-\tau_2} \cap \{1\} \otimes \overline{A}_v| \\ &\quad +|\{\tau_1\} \otimes D_{u+\tau_2} \cap \{0\} \otimes D_v| \\ &\quad +|\{1+\tau_1\} \otimes A_{u-\tau_2} \cap \{1\} \otimes A_v| \\ &\quad +\omega_4 \times |\{\tau_1\} \otimes \overline{D}_{u+\tau_2} \cap \{1\} \otimes A_v| \\ &\quad +\omega_4 \times |\{1+\tau_1\} \otimes \overline{A}_{u-\tau_2} \cap \{0\} \otimes \overline{D}_v| \\ &\quad +\omega_4 \times |\{\tau_1\} \otimes D_{u+\tau_2} \cap \{1\} \otimes \overline{A}_v| \\ &\quad +\omega_4 \times |\{1+\tau_1\} \otimes A_{u-\tau_2} \cap \{0\} \otimes D_v| \\ &\quad -|\{\tau_1\} \otimes \overline{D}_{u+\tau_2} \cap \{0\} \otimes D_v| \\ &\quad -|\{1+\tau_1\} \otimes \overline{A}_{u-\tau_2} \cap \{1\} \otimes A_v| \\ &\quad -|\{\tau_1\} \otimes D_{u+\tau_2} \cap \{0\} \otimes \overline{D}_v| \\ &\quad -|\{1+\tau_1\} \otimes A_{u-\tau_2} \cap \{1\} \otimes \overline{A}_v| \\ &\quad -\omega_4 \times |\{\tau_1\} \otimes \overline{D}_{u+\tau_2} \cap \{1\} \otimes \overline{A}_v| \\ &\quad -\omega_4 \times |\{1+\tau_1\} \otimes \overline{A}_{u-\tau_2} \cap \{0\} \otimes D_v| \\ &\quad -\omega_4 \times |\{\tau_1\} \otimes D_{u+\tau_2} \cap \{1\} \otimes A_v| \\ &\quad -\omega_4 \times |\{1+\tau_1\} \otimes A_{u-\tau_2} \cap \{0\} \otimes \overline{D}_v| \end{aligned}$$

where ω_4 is 4th complex primitive root of unity. That means $\omega_4 = j$.

If $\tau_1 = 0$, then $R_{u,v}(\tau)$ is calculated as

$$\begin{aligned} R_{u,v}(\tau) &= |\overline{D}_{u+\tau_2} \cap \overline{D}_v|) + |\overline{A}_{u-\tau_2} \cap \overline{A}_v| \\ &\quad + |D_{u+\tau_2} \cap D_v| + |A_{u-\tau_2} \cap A_v| \\ &\quad - |\overline{D}_{u+\tau_2} \cap D_v| - |\overline{A}_{u-\tau_2} \cap A_v| \\ &\quad - |D_{u+\tau_2} \cap \overline{D}_v| - |A_{u-\tau_2} \cap \overline{A}_v|. \end{aligned} \tag{1}$$

In the case of $\tau_1 = 1$, we have

$$\begin{aligned} R_{u,v}(\tau) &= \omega_4 \times \left\{|\overline{D}_{u+\tau_2} \cap A_v| + |\overline{A}_{u-\tau_2} \cap \overline{D}_v|\right\} \\ &\quad + \omega_4 \times \left\{|D_{u+\tau_2} \cap \overline{A}_v| + |A_{u-\tau_2} \cap D_v|\right\} \\ &\quad - \omega_4 \times \left\{|\overline{D}_{u+\tau_2} \cap \overline{A}_v| + |\overline{A}_{u-\tau_2} \cap D_v|\right\} \\ &\quad - \omega_4 \times \left\{|D_{u+\tau_2} \cap A_v| + |A_{u-\tau_2} \cap \overline{D}_v|\right\}. \end{aligned} \tag{2}$$

Case 1) $s_u(t)$, $s_v(t) \in \mathcal{U}_1$:

In this case, $A_u = D_{1-u}$ and $A_v = D_{1-v}$. From the fact that characteristic set of the binary sequence with ideal autocorrelation is difference set, for $\tau_1 = 0$, we have

$$|\overline{D}_{u+\tau_2} \cap \overline{D}_v| = \begin{cases} 2^{n-1} - 1, & \text{if } \tau = (0, v-u) \\ 2^{n-2} - 1, & \text{otherwise} \end{cases} \tag{3}$$

$$|\overline{D}_{1-u+\tau_2} \cap \overline{D}_{1-v}| = \begin{cases} 2^{n-1} - 1, & \text{if } \tau = (0, u-v) \\ 2^{n-2} - 1, & \text{otherwise} \end{cases} \tag{4}$$

$$|D_{u+\tau_2} \cap D_v| = \begin{cases} 2^{n-1}, & \text{if } \tau = (0, v-u) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{5}$$

$$|D_{1-u+\tau_2} \cap D_{1-v}| = \begin{cases} 2^{n-1}, & \text{if } \tau = (0, u-v) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{6}$$

$$|\overline{D}_{u+\tau_2} \cap D_v| = \begin{cases} 0, & \text{if } \tau = (0, v-u) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{7}$$

$$|\overline{D}_{1-u+\tau_2} \cap D_{1-v}| = \begin{cases} 0, & \text{if } \tau = (0, u-v) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{8}$$

$$|D_{u+\tau_2} \cap \overline{D}_v| = \begin{cases} 0, & \text{if } \tau = (0, v-u) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{9}$$

$$|D_{1-u+\tau_2} \cap \overline{D}_{1-v}| = \begin{cases} 0, & \text{if } \tau = (0, u-v) \\ 2^{n-2}, & \text{otherwise.} \end{cases} \tag{10}$$

For $\tau_1 = 1$, we have

$$|\overline{D}_{u+\tau_2} \cap D_{1-v}| = \begin{cases} 0, & \text{if } \tau = (1, 1-u-v) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{11}$$

$$|\overline{D}_{1-u+\tau_2} \cap \overline{D}_v| = \begin{cases} 2^{n-1} - 1, & \text{if } \tau = (1, u+v-1) \\ 2^{n-2} - 1, & \text{otherwise} \end{cases} \tag{12}$$

$$|D_{u+\tau_2} \cap \overline{D}_{1-v}| = \begin{cases} 0, & \text{if } \tau = (1, 1-u-v) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{13}$$

$$|D_{1-u+\tau_2} \cap D_v| = \begin{cases} 2^{n-1}, & \text{if } \tau = (1, u+v-1) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{14}$$

$$|\overline{D}_{u+\tau_2} \cap \overline{D}_{1-v}| = \begin{cases} 2^{n-1} - 1, & \text{if } \tau = (1, 1-u-v) \\ 2^{n-2} - 1, & \text{otherwise} \end{cases} \tag{15}$$

$$|\overline{D}_{1-u+\tau_2} \cap D_v| = \begin{cases} 0, & \text{if } \tau = (1, u+v-1) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{16}$$

$$|D_{u+\tau_2} \cap D_{1-v}| = \begin{cases} 2^{n-1}, & \text{if } \tau = (1, 1-u-v) \\ 2^{n-2}, & \text{otherwise} \end{cases} \tag{17}$$

$$|D_{1-u+\tau_2} \cap \overline{D}_v| = \begin{cases} 0, & \text{if } \tau = (1, u+v-1) \\ 2^{n-2}, & \text{otherwise.} \end{cases} \tag{18}$$

i) $u \neq v$;

Applying (3)–(18) to (1) and (2), we have

$$R_{u,v}(\tau) = \begin{cases} 2^n - 2, & \text{if } \tau = (0, u-v) \text{ and } (0, v-u) \\ 2^n j, & \text{if } \tau = (1, u+v-1) \\ -2^n j, & \text{if } \tau = (1, 1-u-v) \\ -2, & \text{if } \tau_1 = 0 \text{ and} \\ & \quad \tau_2 \in Z_{2^n-1} \backslash \{u-v, v-u\} \\ 0, & \text{if } \tau_1 = 1 \text{ and} \\ & \quad \tau_2 \in Z_{2^n-1} \backslash \{1-u-v, u+v-1\}. \end{cases} \tag{19}$$

ii) $u = v$;

In this condition, $u - v = v - u = 0$. Similarly, applying (3)–(18) to (1) and (2) and replacing v by u, we have

$$R_{u,v}(\tau) = \begin{cases} 2 \times (2^n - 1), & \text{if } \tau = 0 \\ 2^n j, & \text{if } \tau = (1, 2u-1) \\ -2^n j, & \text{if } \tau = (1, 1-2u) \\ -2, & \text{if } \tau_1 = 0 \text{ and } \tau_2 \in Z_{2^n-1} \backslash \{0\} \\ 0, & \text{if } \tau_1 = 1 \text{ and} \\ & \quad \tau_2 \in Z_{2^n-1} \backslash \{1-2u, 2u-1\}. \end{cases} \tag{20}$$

Case 2) $s_u(t) \in \mathcal{U}_1$ and $s_v(t) \in \mathcal{U}_2$:

In this case, $A_u = D_{1-u}$ and $A_v = \overline{D}_{1-v}$. Similar to Case 1), applying (3)–(18) to (1) and (2), we get

i) $u \neq v$;

$$R_{u,v}(\tau) = \begin{cases} 2^n, & \text{if } \tau = (0, v-u) \\ -2^n, & \text{if } \tau = (1, u-v) \\ (2^n - 2)j, & \text{if } \tau \in \{(1, 1-u-v), \\ & \qquad (1, u+v-1)\} \\ 0, & \text{if } \tau_1 = 0 \text{ and} \\ & \quad \tau_2 \in Z_{2^n-1} \backslash \{u-v, v-u\} \\ -2j, & \text{if } \tau_1 = 1 \text{ and} \\ & \quad \tau_2 \in Z_{2^n-1} \backslash \{1-u-v, \\ & \qquad u+v-1\}. \end{cases} \tag{21}$$

ii) $u = v$;

$$R_{u,v}(\tau)=\begin{cases}(2^n-2)j, & \text{if } \tau\in\{(1,1-2u),(1,2u-1)\}\\ 0, & \text{if } \tau_1=0\\ -2j, & \text{if } \tau_1=1 \text{ and}\\ & \tau_2\in Z_{2^n-1}\backslash\{2u-1,1-2u\}.\end{cases} \quad (22)$$

Case 3) $s_u(t)$, $s_v(t)\in\mathcal{U}_2$:

In this case, $A_u=\overline{D}_{1-u}$ and $A_v=\overline{D}_{1-v}$. Similarly, applying (3)–(18) to (1) and (2), we have

i) $u\neq v$;

$$R_{u,v}(\tau)=\begin{cases}2^n-2, & \text{if } \tau\in\{(0,u-v),(0,v-u)\}\\ 2^n j, & \text{if } \tau=(1,u+v-1)\\ -2^n j, & \text{if } \tau=(1,1-u-v)\\ 0, & \text{if } \tau_1=1 \text{ and}\\ & \tau_2\in Z_{2^n-1}\backslash\{1-u-v,u+v-1\}\\ -2, & \text{if } \tau_1=0 \text{ and}\\ & \tau_2\in Z_{2^n-1}\backslash\{u-v,v-u\}.\end{cases} \quad (23)$$

ii) $u=v$;

$$R_{u,v}(\tau)=\begin{cases}2\times(2^n-1), & \text{if } \tau=0\\ 2^n j, & \text{if } \tau=(1,1-2u)\\ -2^n j, & \text{if } \tau=(1,2u-1)\\ 0, & \text{if } \tau_1=1 \text{ and}\\ & \tau_2\in Z_{2^n-1}\backslash\{2u-1,1-2u\}\\ -2, & \text{if } \tau_1=0\text{and}\\ & \tau_2\in Z_{2^n-1}\backslash\{0\}.\end{cases} \quad (24)$$

From (19)–(24), we prove the theorem.

□

In [6], Tang, Fan, and Matsufuji proposed a lower bound on LCZ sequence sets as follows.

Theorem 2 (Tang, Fan, and Matsufuji [6]): Let $\mathcal{S}$ be and LCZ sequence set with parameters (N,M,L,ϵ). Then we have

$$ML-1\leq\frac{N-1}{1-\epsilon^2/N}. \quad (25)$$

□

Note that the locations of sidelobes are symmetric with respect to the origin. In terms of the distances from the origin to the sidelobes, there are at most two distinct distances. Let $L_{u,v}$ denote the distance to the nearest sidelobes from the origin in $R_{u,v}(\tau)$. According to the result in [1], $L_{u,v}$ can be determined as in the following lemma.

Lemma 3 (Kim, Jang, No, and Chung[1]): For $s_u(t),s_v(t)\in\mathcal{U}_1\cup\mathcal{U}_2$, $1\leq v\leq u<2^{n-1}$, $L_{u,v}$ is given as

$$L_{u,v}=\begin{cases}\frac{N}{2}-u-v+1, & \text{if } u-v \text{ is odd}\\ u-v, & \text{if } u-v \text{ is even and } u\neq v\\ 2u-1, & \text{if } u=v.\end{cases} \quad (26)$$

□

Lemma 3 shows that the LCZ of a set of sequences $s_u(t)$ is solely dependent on the index values u's regardless of whether the sequence $s_u(t)$ is in $\mathcal{U}_1$ or $\mathcal{U}_2$.

Thus what we are going to do now is to choose an index set $I\subset\{1,2,\cdots,2^{n-1}-1\}$ and construct the set of sequences

$$W_I=\{s_u(t)\in\mathcal{U}_1\mid u\in I\}\cup\{s_u(t)\in\mathcal{U}_2\mid u\in I\} \quad (27)$$

so that W_I becomes a good LCZ sequence set.

From Lemma 3, the LCZ of the set W_I is the minimum of three values: $2^n-(u+v)$ for odd $|u-v|$, $|u-v|$ for nonzero even $|u-v|$, and $2u-1$ for $u=v$ such that $u,v\in I$.

Thus, to maintain a given parameter L, the index set I must have following properties.

i) the indices in I should be greater than or equal to $\frac{(L+1)}{2}$,
ii) sum of two indices should be less than or equal to 2^n-L unless their differences are even,
iii) difference of two indices should not be even numbers less than L.

At the same time, for a given L, we want to make the size of I as large as possible.

From these constraints, we can formulate fairly complex optimal design problem. The solution for this problem seems somewhat complicated, but aforementioned constraints implicitly lead us to consider an index set I which forms an arithmetic progression with odd value of common difference.

Theorem 4: Let q and r be the quotient and the remainder of 2^{n-1}, respectively, when divided by f, i.e., $2^{n-1}=qf+r$. Let I be the set of indices defined by

$$I=\left\{f_0+mf\mid m=1,2,\cdots,\lfloor\frac{2^{n-1}-f_0}{f}\rfloor\right\}.$$

Then W_I from (27) becomes a quaternary LCZ sequence set with parameters $(2^{n+1}-2,M,L,2)$, where M and L are given as

$$M=2q$$

and if $f_0=0$,

$$L=\begin{cases}f+2r, & \text{for } f\geq 2r+1\\ 2f-1, & \text{for } f<2r+1\end{cases} \quad (28)$$

and if $f_0\neq 0$,

$$L=\begin{cases}f+2r-2f_0, & \text{for } f\geq 2r-2f_0\\ 2f, & \text{for } f<2r-2f_0.\end{cases} \quad (29)$$

Proof: From Lemma 3 and the fact that f is odd, L is the smallest value of $2f$, $2f+2f_0-1$, and $2^n-(u+v)$, where u and v are the largest and the second largest elements in I. Since $u+v=2qf-f+2f_0$, we have $2^n-(u+v)=f+2r-2f_0$. Therefore, we have

$$L=\min\{2f,2f+2f_0-1,f+2r-2f_0\}. \quad (30)$$

We can obtain (28) and (29) from (30). □

Note that if f is even, then from Lemma 3, LCZ of the sequence set W_I becomes

$$L = \min_{u,v \in I, u \neq v} (u - v) = f.$$

But if f is odd, then from Theorem 4, LCZ is greater than f, which is the reason why we make the common difference f odd.

Now, we can easily obtain the following corollary.

Corollary 5: The product of set size and LCZ in Theorem 4 is given as

$$ML = \begin{cases} N - M(f-2r) - 4r + 2, \\ \qquad \text{for } f \geq 2r+1 \text{ and } f_0 = 0 \\ N - M - 4r + 2, \\ \qquad \text{for } f < 2r+1 \text{ and } f_0 = 0 \\ N - M(f - 2r + 2f_0) - 4r + 2, \\ \qquad \text{for } f \geq 2(r - f_0) \text{ and } f_0 \neq 0 \\ N - 4r + 2, \\ \qquad \text{for } f < 2(r - f_0) \text{ and } f_0 \neq 0. \end{cases} \quad (31)$$

□

By comparing the result in (31) with the bound by Tang, Fan, and Matsufuji[6], one can easily see that our construction corresponding to the first three cases of the inequality (31) cannot be optimal. But most of the sets are very nearly optimal, although we cannot find any which is optimal. This observation motivates us to consider a little modification to the index set I. In the following construction method, we allow two distinct values $f+2$ and f for the difference values between adjacent indices in I.

The set size M and LCZ L are given as in the following theorem.

Theorem 6: Let q and r be the quotient and the remainder of $2^{n-1} - 1$, respectively, when divided by $2(f+1)$, i.e., $2^{n-1} - 1 = 2q(f+1) + r$. Let I be the set of indices defined by

$$I = \{u_j \mid j = 0, 1, 2, \cdots, J, \ u_0 = f + 2 - f_0, \\ u_{2k+1} - u_{2k} = f, \ u_{2k+2} - u_{2k+1} = f + 2\}$$

Then W_I from (27) becomes a quaternary LCZ sequence set with parameters $(2^{n+1} - 2, M, L, 2)$, where M and L are given as

$$M = \begin{cases} 4q, & \text{for } 0 \leq r < f + 2 - f_0 \\ 4q + 2, & \text{for } f + 2 - f_0 \leq r < 2f + 2 \end{cases}$$

and

$$L = \begin{cases} 2r + f + 2 + 2f_0, \\ \qquad \text{for } 0 \leq r < \frac{f - 3f_0}{2} \\ 2f + 2 - f_0, \\ \qquad \text{for } \frac{f-3f_0}{2} \leq r < f + 2 - f_0 \\ \qquad\qquad \text{and } \frac{3f+2-3f_0}{2} \leq r < 2f + 2 \\ 2r - f + 2f_0, \\ \qquad \text{for } f + 2 - f_0 \leq r < \frac{3f+2-3f_0}{2}. \end{cases} \quad (32)$$

Proof: From Lemma 3 and the fact that f is odd, L is the smallest value of $2f+2$, $2(f+2-f_0)-1$, and $2^n - (u+v)$, where u and v are the largest and the second largest elements in I.

If $0 \leq r < f + 2 - f_0$, then $|I| = 2q$ and $u + v = 4q(f+1) - f - 2f_0$. Thus, $2^n - (u+v) = f + 2 + 2r + 2f_0$. Therefore, we have

$$L = \min\{2f+2, 2f+3-2f_0, f+2+2r+2f_0\}. \quad (33)$$

If $f + 2 - f_0 \leq r < 2f + 2$, then $|I| = 2q + 1$ and $u+v = 4q(f+1)+f+2-2f_0$. Thus $2^n-(u+v) = 2r-f+2f_0$. Therefore, we have

$$L = \min\{2f+2, 2f+3-2f_0, 2r-f+2f_0\}. \quad (34)$$

We can obtain (32) from (33) and (34). □

Now, we can easily obtain the following corollary.

Corollary 7:

$$ML = \begin{cases} N - M(f - 2r - 2f_0) - 4r - 2, \\ \qquad \text{for } 0 \leq r < \frac{f-3f_0}{2} \\ N - Mf_0 - 4r - 2, \\ \qquad \text{for } \frac{f-3f_0}{2} \leq r < f + 2 - f_0 \\ N - M(3f - 2r - 2f_0 + 2) - 4(r - f) + 2, \\ \qquad \text{for } f + 2 - f_0 \leq r < \frac{3f+2-3f_0}{2} \\ N - Mf_0 + 2 - 4(r - f), \\ \qquad \text{for } \frac{3f+2-3f_0}{2} \leq r < 2f + 2. \end{cases}$$

□

Note that the sets from Theorems 4 and 6 can have many different L and M for the same period. Therefore we can have the flexibility in the construction of LCZ sequence sets and a tradeoff between the set size and the LCZ of the set.

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