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Adaptive TR Scheme for OFDM Signals

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Abstract

In the tone reservation (TR) scheme of the orthogonal frequency division multiplexing (OFDM) systems, there exists a trade-off between the peak to average power ratio (PAPR) reduction performance and peak reduction tone (PRT) set size. In this paper, we propose a multi-stage TR scheme for PAPR reduction, which adaptively selects one of several PRT sets according to the PAPR of OFDM signal while the PRT set is fixed for the conventional TR scheme. It is shown that the PAPR reduction performance of the proposed scheme is better than that of the conventional TR scheme when the tone reservation rate (TRR) is almost the same.

1. INTRODUCTION

An orthogonal frequency division multiplexing (OFDM) system suffers from high peak to average power ratio (PAPR) of time domain signal obtained by inverse fast Fourier transform (IFFT). If an OFDM signal has high PAPR, it will cause significant signal distortion such as in-band distortion and out-of-band radiation in a nonlinear high power amplifier (HPA) [1]. There are many techniques to reduce PAPR among which tone reservation (TR) scheme [2] is a technique that has been proposed to reduce the PAPR in discrete multitone (DMT) systems. TR scheme requires a sacrifice in data transmission efficiency because some of subcarriers in an OFDM symbol should be reserved as peak reduction tones (PRTs) which are used only for reducing PAPR without carrying data.

The size of PRT set plays a critical role in TR scheme. To achieve lower PAPR, more subcarriers should be reserved as PRTs which reduces the data transmission efficiency. In this paper, a multi-stage TR scheme is proposed, which can reduce the PAPR of OFDM signal without increasing the tone reservation rate (TRR) [3] which indicates the portion of PRTs in subcarriers.

The rest of this paper is organized as follows. In Section 2, OFDM system and TR scheme are reviewed. A multi-stage TR scheme is proposed in Section 3 and the simulation results are shown in Section 4. Finally, conclusions are given in Section 5.

2. OFDM SYSTEM AND TONE RESERVATION

In an OFDM system, an input data symbol vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]$ in the frequency domain is modulated by N orthogonal subcarriers to generate a discrete time OFDM signal x_k. In other words, a discrete time OFDM signal x_k is obtained by applying the IFFT to $\mathbf{X}$ as

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi \frac{n}{N} k}, 0 \leq k \leq N-1 \qquad (1)$$

where N is the number of subcarriers and k is the discrete time index. The PAPR of an OFDM signal $\mathbf{x} = [x_0, x_1, \ldots, x_{N-1}]$ is defined as

$$\text{PAPR}(\mathbf{x}) = \frac{\max\limits_{0 \leq k \leq N-1} |x_k|^2}{\text{E}[||\mathbf{x}||_2^2]} \qquad (2)$$

where $\text{E}[\cdot]$ denotes the expectation operation and $||\mathbf{x}||_2$ is the second norm of a vector $\mathbf{x}$.

In order to reduce the PAPR of OFDM signal using TR scheme, some subcarriers are reserved as PRT set which is used to generate peak cancelling signal. Let $\mathbf{R} = \{i_0, i_1, \cdots, i_{W-1}\}$ denote the ordered set of indices of PRTs and $\mathbf{R}^C$ be the complement set of $\mathbf{R}$ in $\mathbf{N} =$

$\{0, 1, \cdots, N-1\}$, where W is the size of PRT set. Also, the TRR is defined as

$$\text{TRR} = \frac{W}{N} \times 100[\%]. \tag{3}$$

Note that TRR is closely related with the data transmission efficiency (or throughput).

In the TR scheme, an input symbol A_n in the frequency domain can be written as

$$A_n = X_n + C_n = \begin{cases} C_n, & n \in \mathbf{R} \\ X_n, & n \in \mathbf{R}^{\mathrm{C}} \end{cases} \tag{4}$$

where X_n is an input data symbol and C_n is a symbol assigned for PRT. Here, we assume that $X_n = 0$ for $n \in \mathbf{R}$ and $C_n = 0$ for $n \in \mathbf{R}^{\mathrm{C}}$. Then the discrete time OFDM signal a_k can be rewritten as

$$\begin{aligned} a_k &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} (X_n + C_n) e^{j2\pi \frac{n}{N} k} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi \frac{n}{N} k} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n e^{j2\pi \frac{n}{N} k} \\ &= x_k + c_k \end{aligned} \tag{5}$$

where $0 \le k \le N-1$ and $\mathbf{c} = [c_0, c_1, \ldots, c_{N-1}]$ is called the peak cancelling signal. The peak cancelling signal $\mathbf{c}$ should be designed to reduce the peak of OFDM signal $\mathbf{x}$ efficiently. Several algorithms have been proposed to construct peak cancelling signal [2], [4] and the iterative algorithm is used in this paper.

The PAPR of an OFDM signal $\mathbf{a} = [a_0, a_1, \ldots, a_{N-1}]$ with TR scheme needs to be redefined [2] as

$$\text{PAPR}(\mathbf{a}) = \frac{\max\limits_{0 \le k \le N-1} |x_k + c_k|^2}{\mathrm{E}[||\mathbf{x}||_2^2]}. \tag{6}$$

The PAPR reduction performance of TR scheme mainly depends on the size of PRT set, the maximum number of iterations, and the selection of PRTs [5].

3. MULTI-STAGE TR SCHEME

In this section, we propose a multi-stage TR scheme for PAPR reduction of OFDM signals without reducing the average transmission efficiency compared to the conventional TR scheme.

3.1. Structure of Multi-Stage TR Scheme

In order to achieve low PAPR of OFDM signal using low TRR, we propose a new multi-stage TR scheme which is shown in Fig. 1. The multi-stage TR scheme utilizes the conventional TR schemes in a sequential manner. The first TR block TR_1 is the conventional TR scheme using PRT_1 and γ_1 as its PRT set and threshold level while the second TR block TR_2 uses PRT_2 and γ_2. In a multi-stage TR scheme, the peak of the input OFDM signal is initially reduced by TR_1 using the threshold level γ_1. After processed by TR_1, the OFDM signal is transmitted if the PAPR of the processed OFDM signal is lower than the target PAPR threshold level γ_2. Otherwise, the OFDM signal should be processed by TR_2 for further reduction of PAPR. For this two-stage TR scheme, an additional 1-bit side information should be transmitted to indicate which TR block was used. It is straightforward to construct multi-stage TR scheme with more than 2 stages and we will only focus on two-stage TR scheme shown in Fig. 1.

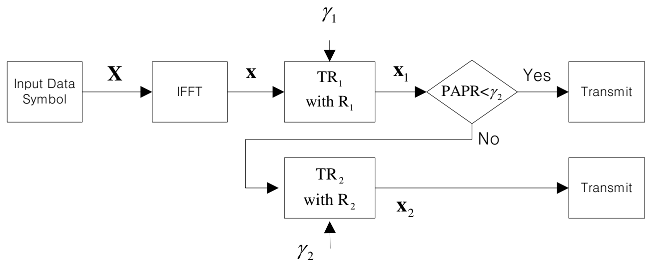

Figure 1: Block diagram of multi-stage TR scheme.

For two PRT sets PRT_1 and PRT_2 in two-stage TR scheme, the ordered sets $\mathbf{R}_1$ and $\mathbf{R}_2$ are used, respectively. The size of PRT_2 is larger than that of PRT_1 and $\mathbf{R}_1 \subset \mathbf{R}_2$. Let W_1 and W_2 denote the size of $\mathbf{R}_1$ and $\mathbf{R}_2$, respectively. The frequency domain kernel $\mathbf{P}_m$ is constructed by assigning 1's in $\mathbf{R}_m$, where $m = 1,\ 2$. Since W_2 is bigger than W_1, sidelobes of $\mathbf{p}_2$ are much lower than those of $\mathbf{p}_1$ and thus $\mathbf{p}_2$ can reduce PAPR more effectively [5].

3.2. Average TRR of Multi-Stage TR Scheme

TRR of the conventional TR scheme is already defined in (3). The average TRR ρ_{avg} of two-stage TR scheme is defined as

$$\begin{aligned} \rho_{avg} &= \Pr(\text{PAPR}_{\mathbf{x}_1} < \gamma_2) \times \rho_1 \\ &\quad + \{1 - \Pr(\text{PAPR}_{\mathbf{x}_1} < \gamma_2)\} \times \rho_2 \end{aligned} \tag{7}$$

where ρ_1 and ρ_2 are the TRR values of TR_1 and TR_2, respectively, and $\text{PAPR}_{\mathbf{x}_1}$ is the PAPR of the OFDM signal $\mathbf{x}_1$ after TR_1 is applied. For the general multi-stage TR scheme, the average TRR can be similarly defined.

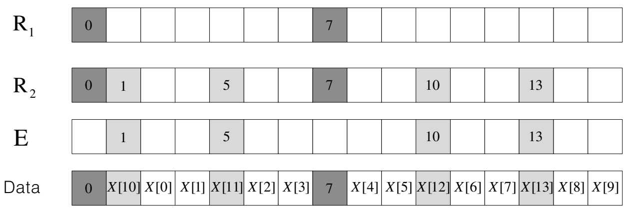

Figure 2: Input data allocation of the multi-stage TR scheme.

Since $\rho_2 > \rho_1$, to minimize ρ_{avg}, it is desirable to select the threshold level γ_1 so that $\Pr(\text{PAPR}_{\mathbf{x}_1} < \gamma_2)$ is quite high (say, higher than 0.9). Then, two-stage TR scheme can reduce the PAPR level of OFDM signals below the target threshold level γ_2 while achieving the average TRR close to ρ_1.

3.3. Selection of Threshold Level

It is important to select γ_1 and γ_2 appropriately to achieve low average TRR as well as good PAPR reduction performance. Usually, we set the threshold level for TR_2 as γ_2 which is also the target PAPR, and the threshold level for TR_1 as γ_1 which is lower than γ_2. These two threshold values can be empirically determined.

3.4. Data Allocation

Since different PRT sets are used for two TR blocks, the number of data subcarriers in OFDM symbol becomes $N - W_1$ or $N - W_2$ if no guardbands and pilot symbols are considered. If the PAPR of the OFDM signal $\mathbf{x}_1$ in TR_1 satisfies the target PAPR γ_2, then the data transmission efficiency is $1-\rho_1$. If $\mathbf{x}_1$ does not satisfy the PAPR requirement, it should go through TR_2 and the data transmission efficiency becomes $1 - \rho_2$ which is lower than $1 - \rho_1$. Once the OFDM signal $\mathbf{x}_1$ passes to TR_2, the input data symbols already assigned to the set $\mathbf{E} = \mathbf{R}_2 \backslash \mathbf{R}_1$ should be overwritten by the peak cancelling signal generated by PRT_2 and these input data symbols in $\mathbf{E}$ should be contained in the next OFDM symbol. Therefore, for the multi-stage TR scheme, $N - W_2$ input data symbols should be allocated to $\mathbf{R}_2^{\mathcal{C}} = \mathbf{N} \backslash \mathbf{R}_2$ and the next $|\mathbf{E}| = W_2 - W_1$ input data symbols should be allocated to $\mathbf{E}$. As an example, the input data allocation of the multi-stage TR scheme with $N = 16$, $W_1 = 2$, and $W_2 = 6$ is given in Fig. 2, where $\mathbf{R}_1 = \{0, 7\}$, $\mathbf{R}_2 = \{0, 1, 5, 7, 10, 13\}$, and $\mathbf{E} = \{1, 5, 10, 13\}$. The first 10 input data symbols, from $X[0]$ to $X[9]$, are assigned to $\mathbf{R}_2^{\mathcal{C}}$ and the next 4 input data symbols, from $X[10]$ to $X[13]$, are assigned to $\mathbf{E}$. If TR_2 is activated, then the input data symbols assigned to $\mathbf{E}$ are overwritten by the peak cancelling signal generated by PRT_2 and those input data symbols in $\mathbf{E}$ should be transmitted in the next OFDM symbol.

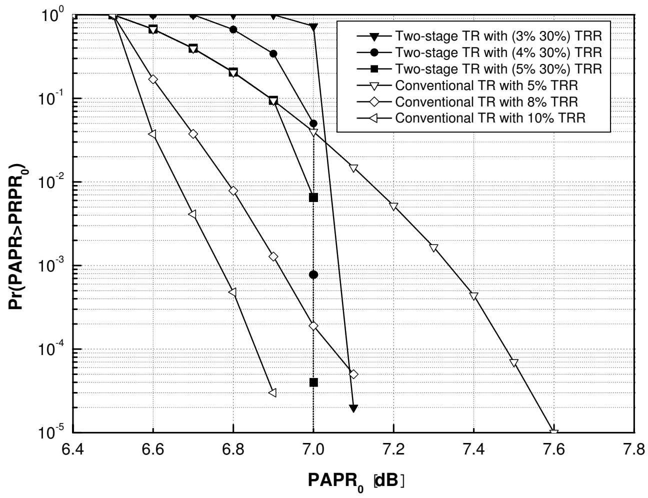

Figure 3: PAPR reduction performance of the conventional TR scheme with 5%, 8%, and 10% TRR and the multi-stage TR scheme with (3%, 30%) TRR, (4%, 30%) TRR, (5%, 30%) TRR when N=1024 and 16-QAM is used.

3.5. Signal Selection in the Second TR Block

The subcarriers in $\mathbf{E}$ are used to carry the input data symbols in the TR_1 block while they are used as a part of PRTs in PRT_2. An OFDM signal $\mathbf{x}_1$ in Fig. 1 can be processed in the TR_2 by the following two methods. The first method is to remove the input data symbols in $\mathbf{E}$ in the frequency domain and take the IFFT to generate the OFDM signal for TR_2. In the second method, the OFDM signal $\mathbf{x}_1$ from TR_1 is directly passed to TR_2, where the peak cancelling signal generated by PRT_2 is applied to $\mathbf{x}_1$. Clearly, the input data symbols in $\mathbf{E}$ are overwritten by the peak cancelling signal. In this paper, we employ the second method. This second method takes advantage of TR_1 which already reduces the peak values of its input signal and does not require an additional IFFT operation. Since the threshold level γ_1 in TR_1 is lower than γ_2 in TR_2, the magnitude of most samples in $\mathbf{x}_1$ are lower than γ_2 and a few iterations are required in TR_2 to achieve the target threshold. Since we do not remove the input data symbols in $\mathbf{E}$, total transmit power may increase a little bit compared to the first method, but it is negligible.

Table 1: TRANSMISSION EFFICIENCY AND AVERAGE NUMBER OF ITERATIONS OF TWO-STAGE TR SCHEME AND CONVENTIONAL TR SCHEME WITH N=1024 AND 16-QAM

	Two-stage TR scheme			Conventional TR scheme		
TRR	(3%, 30%)	(4%, 30%)	(5%, 30%)	5%	8%	10%
Transmission efficiency	0.70	0.92	0.94	0.95	0.92	0.90
Average number of iterations	76.0	43.8	40.7	40.0	40.0	39.9

4. Simulation Results

Numerical analysis is performed for the OFDM system with $N = 1024$ and 16-QAM, where no guardbands and pilot tones are considered. All OFDM signals are four times oversampled. In Fig. 3, the conventional TR scheme reserves 5%, 8%, and 10% of subcarriers as its PRT sets with $|\mathbf{R}| = 51$, 81, and 102, respectively. The threshold level is chosen to $\gamma = 6.5$ dB and the maximum number of iterations is set to 40. For the several multi-stage TR schemes, $|\mathbf{R}_1| = 31$, 41, 51, $|\mathbf{R}_2| = 306$ (which is referred as (3% 30%), (4% 30%), (5% 30%) TRRs) and the threshold levels $\gamma_1 = 7.0$ dB, 6.8 dB, 6.5 dB, respectively, and $\gamma_2 = 7.0$ dB are used. The maximum number of iterations in each TR block is set to 40 and 100000 OFDM symbols are generated. Table 1 lists the transmission efficiency and the average number of iterations for the multi-stage TR schemes and conventional TR schemes. It shows that the multi-stage TR scheme with (5% 30%) TRR has better performance than those with (3% 30%) and (4% 30%) TRRs in terms of the transmission efficiency and the average number of iterations because the multi-stage TR with (3% 30%) and (4% 30%) TRR need more iterations in TR_2 and pass more OFDM signals to TR_2. Fig. 3 shows that the multi-stage TR scheme with (5% 30%) TRR is better than the other two multi-stage TR schemes with (3% 30%) and (4% 30%) TRRs. Among 100000 OFDM symbols, 95984 OFDM symbols (which is about 96%) with (5% 30%) TRR satisfy the required PAPR 7.0 dB only by using TR_1 and the remaining 4016 OFDM symbols (which is about 4%) go through TR_2. Fig. 3 shows that PAPR of the OFDM signals with the multi-stage TR scheme with (5% 30%) TRR does not exceed 7.0 dB at the probability of 10^{-5} while the OFDM signal with the conventional TR schemes with 5% and 8% TRR can have PAPR more than 7.0 dB at the same probability. The average data transmission efficiency is defined as $1 - \rho_{avg}$. The average data transmission efficiency of the conventional TR scheme with 5% TRR is 0.95 while in the multi-stage TR scheme with (5% 30%) TRR has 0.94. But the PAPR reduction performance of the multi-stage TR scheme with (5% 30%) TRR is much better than that of the conventional TR scheme with 5% TRR.

5. CONCLUSIONS

In this paper, a multi-stage TR scheme which adaptively uses PRT sets of different size to TR scheme according to the PAPR level of the OFDM signals has been proposed. The main idea is to select proper threshold level in the first TR block to minimize the probability of activation of the second TR block to increase transmission efficiency without degrading the PAPR performance.

From the simulation results, we can conclude that the multi-stage TR scheme improves PAPR reduction performance significantly while keeping its TRR almost the same as the conventional TR scheme.

The proposed multi-stage TR scheme consists of two TR blocks. However, this configuration can be extended to multiple TR blocks under the same system requirements.

References

[1] M. R. D. Rodrigues and I. J. Wassell, "IMD reduction with SLM and PTS to improve the error-probability performance of nonlinearly distorted OFDM signals," *IEEE Trans. Veh. Technol.*, vol. 55, no. 2, pp. 537–548, Mar. 2006.

[2] J. Tellado, *Peak to average power reduction for multicarrier modulation,* Ph.D. dissertation, Stanford Univ., Stanford, CA, 2000.

[3] J. Tellado, *Multicarrier modulation with low PAR : Application to DSL and Wireless,* Boston, MA: Kluwer Academic, 2000.

[4] B. S. Krongold and D. L. Jones, "An active-set approach for OFDM PAR reduction via tone reservation," *IEEE Trans. Signal Process.*, vol. 52, pp. 495–509, Feb. 2004.

[5] D. W. Lim, H. S. Noh, S. J. Heo, J. S. No, and D. J. Shin, "Construction of the near optimal PRT set using the cyclic difference set in TR scheme," in *Proc. IEEE PIMRC2007*, Athens, Greece, Sep. 2007.