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New Stopping Criteria for Decoding LDPC Codes in H-ARQ Systems

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Abstract

In this paper, we propose a combination of stopping criteria for BP decoding of LDPC codes effective on both high and low SNR and scalable to variable rate and length codes. The proposed algorithms combine three distinct criteria for decoding including conventional syndrome stopping. Each criterion is extremely simple and shall not be a burden to overall system. With these criteria, it is shown that the decoding complexity of H-ARQ system with AMC scheme can be reduced.

1. INTRODUCTION

Discovery of turbo codes motivated the revisit to LDPC codes which also show near Shannon limit performance with moderate decoding complexity. Even though the decoding complexity of these powerful channel codes came down to practical level, the iterative decoder still requires massive computation so that it is one of the principal power consumers in the communication receivers.

In order to alleviate the computational burden of iterative decoding, several stopping criteria have been proposed for turbo codes [1]. It turned out that those stopping criteria work very effectively. With regard to this, LDPC codes have an advantage of having an inherent method of stopping iterative decoding, which stops the decoding if the syndrome of the tentative codeword is zero. This syndrome stopping is virtually optimal in the sense of detecting successful decoding.

By using stopping criteria, the average number of iterations can be substantially reduced at high signal to noise ratio (SNR) or at low word error rate (WER) without any practical loss of performance. Since the ordinary bit error rate for reliable connection is necessarily very low, the computational complexity of decoding LDPC codes can be kept to be low by the adaptive scheduling to channel soundness at the transmitter.

Typically low SNR region where the decoder may frequently hit the predetermined maximum number of iterations has not been of the concern. However, we encounter a problem when hybrid automatic repeat request (H-ARQ) systems are applied. In H-ARQ systems, codewords are retransmitted when they are not successfully decoded and NAKed to the transmitter. Frequent failures of decoding imply that the decoder reaches the maximum number of iterations frequently and thus the decoding complexity increases where we define "decoding complexity" in H-ARQ systems by counting the average number of iterations per codeword transmission.

To avoid this problem, we want to know if the received codeword is good (enough to be successfully decoded) or not in the early stage of the decoding procedure. A stopping method which is very effective at even low SNR under AWGN channel was proposed assuming perfect SNR estimation [2]. The stopping can cause the performance degradation in fast-varying fading channel where the assumptions may not hold. A stopping criterion based on the convergence of mean magnitude (CMM) [3] may be effective for the H-ARQ system under fast fading environment. Taking H-ARQ systems into consideration, a simple stopping criteria [4] was proposed which is very similar to our proposed algorithm in part. But the explicit explanation on the benefit of the complexity reduction at low SNR of H-ARQ systems has not been addressed so far.

In this paper, we propose a combination of stopping criteria for belief propagation (BP) decoding of LDPC codes effective on both high and low SNR and scalable to variable rate and length codes. The proposed algorithms combine three distinct stopping criteria including conventional syndrome stopping. Each

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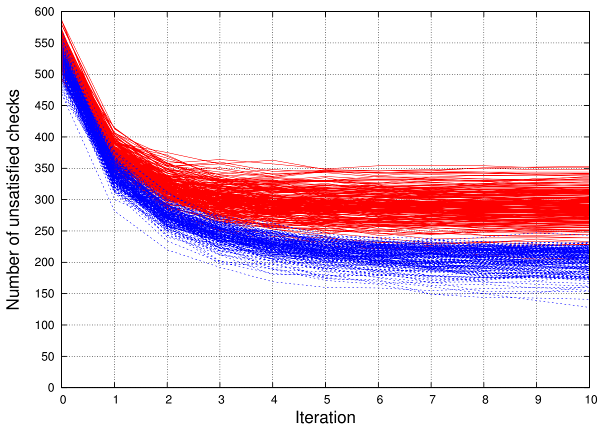

Fig. 1: Iteration vs. $w(\mathbf{s}_i)$ of $1,000$ error frames at $E_b/N_o = -0.5$dB (solid) and 0.0dB (dashed) each. (IEEE802.16e LDPC code with $N = 2,304, R = 0.5$, max. iterations = 50).

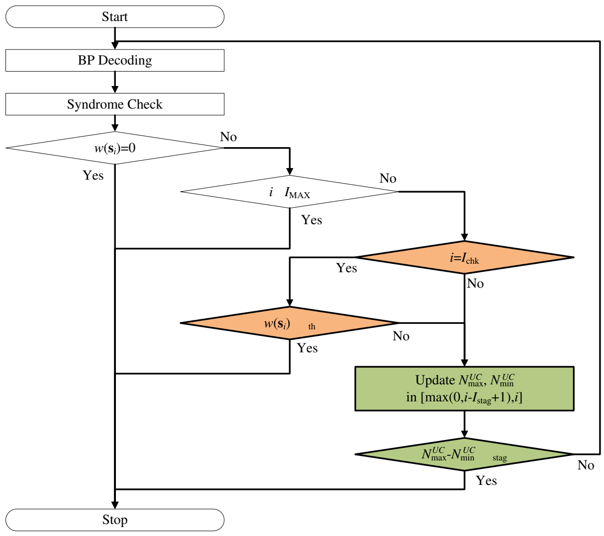

Fig. 2: Flowchart of the proposed stopping criteria.

criterion is extremely simple and shall not be a burden to overall system. It is shown that the proposed scheme reduces the decoding complexity by 70-80% in the H-ARQ system with adaptive modulation and coding (AMC) scheme.

The paper is organized as follows: in Section 2, after taking a look at some interesting phenomena, we propose an effective stopping criteria for iterative BP decoding of LDPC codes in H-ARQ system to reduce the unnecessary computations. The simulation results are shown in Section 3 and concluding remarks are given in Section 4.

2. NEW STOPPING CRITERIA

In this section, we give the illustrations of two major observations, and propose two stopping criteria based on the observations.

2.1. Stagnancy Check

From the observation that the number of unsatisfied check nodes in almost all error frames converges to fixed values after some iterations as in Fig. 1, general form of the stopping criterion to detect a stagnant state of BP decoder can be derived as follows. Let the (tentative) syndrome after the i-th iteration be denoted by $\mathbf{s}_i$. Then, the weight of the syndrome, $w(\mathbf{s}_i)$, equals the number of unsatisfied check nodes after the i-th iteration of decoding. In addition, let I_{stag} be threshold for stagnant length of $w(\mathbf{s}_i)$ to declare a failure. At the i-th iteration stage, if the gap between the maximum and the minimum values of $w(\mathbf{s}_i)$ recorded during the past I_{stag} iterations is less than or equal to a given value δ_{stag}, the decoding shall be stopped. In other words, decoder stops the decoding when the following condition is satisfied.

$$N_{\text{max}}^{UC} - N_{\text{min}}^{UC} \leq \delta_{\text{stag}},$$

where N_{max}^{UC} and N_{min}^{UC} are the maximum and the minimum values of $w(\mathbf{s}_i)$ in last I_{stag} iterations, respectively. Although restricted to relatively low SNR region, $w(\mathbf{s}_i)$s of error frames converge to non-zero value rather than oscillate in most cases. Thus a tighter condition (e.g, $\delta_{\text{stag}} = 0$) will be enough to show good performance of the proposed stopping criterion, and we consider only the case of $\delta_{\text{stag}} = 0$ in this paper. This implies that we just check whether $N_{\text{max}}^{UC} = N_{\text{min}}^{UC}$ holds or not throughout the past I_{stag} iterations.

2.2. Early BP Convergence Speed Check

Another observation on error frames in Fig. 1 brings a stopping criterion which can make dramatic reduction on decoding complexity especially at low SNR region where most of frames are erroneous. Following algorithm can be derived from the observation that lower the SNR region, higher the average of $w(\mathbf{s}_i)$ which error frames tend to produce. After the syndrome check process at $i = I_{\text{chk}}$, if $w(\mathbf{s}_i)$ is smaller than predetermined threshold level (δ_{th}), the frame will be appraised as healthy enough, and the decoding shall be continued. Otherwise, the frame will be regarded as one at very low SNR region where almost all frames are doomed to fail at last, and the decoding shall be stopped. This

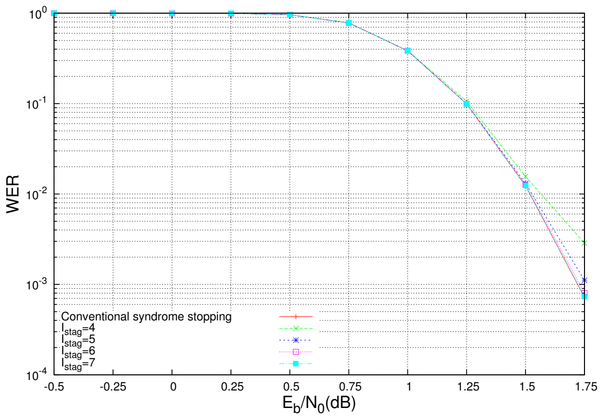

Fig. 3: WER performance of an LDPC code (IEEE802.16e, $n = 2304$, $R = 1/2$) with stagnancy check criterion in AWGN channel.

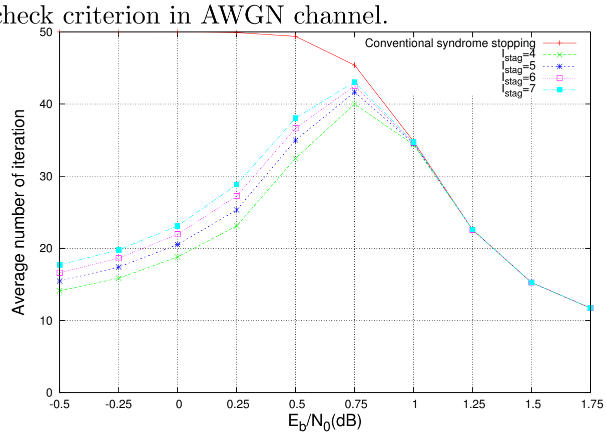

Fig. 4: Average number of iterations of BP decoding for an LDPC code (IEEE802.16e, $n = 2304$, $R = 1/2$) with stagnancy check criterion in AWGN channel.

algorithm can be used to separate out the frames received at very low SNR region.

To reduce the decoding complexity further, we want to distinguish very bad frames before the iterative decoder is stuck to stagnancy because the “stagnancy check” stopping is yet to begin at early stage of decoding (e.g., the $1^{st} \sim 6^{th}$ iteration). However, the “early BP convergence speed check” algorithm can be applied in first few iterations where the cycle-free assumption holds, and at very low SNR region, this difference greatly decreases the unnecessary computations which cannot be effectively removed by “stagnancy check” algorithm (or other existing algorithms).

2.3. Combining Stopping Criteria

Since both of proposed stopping criteria exploit $w(\mathbf{s}_i)$ which is already used in conventional syndrome stopping criterion, the following combining method

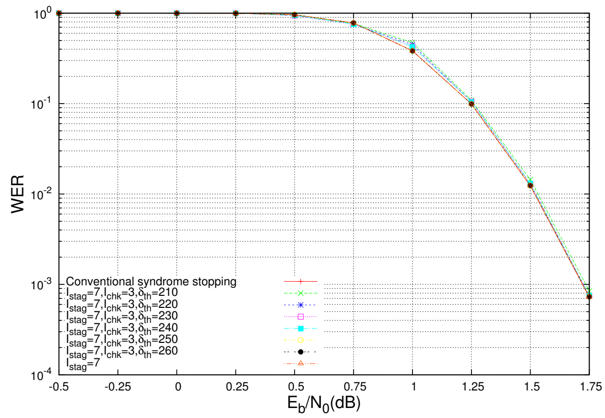

Fig. 5: WER performance of an LDPC code (IEEE802.16e, $n = 2304$, $R = 1/2$) with the proposed criteria in AWGN channel.

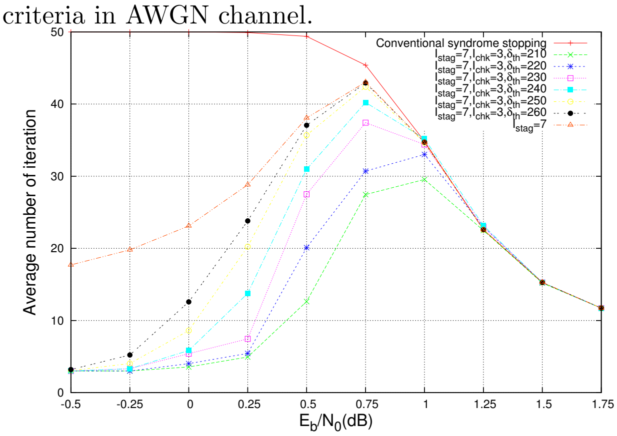

Fig. 6: Average number of iterations of BP decoding for an LDPC code (IEEE802.16e, $n = 2304$, $R = 1/2$) with the proposed criteria in AWGN channel.

barely introduces additional complexity.

- step 1: $i \leftarrow 0$
- step 2: if $i \geq$ maximum number of iterations, stop decoding
- step 3: run the i-th iteration of BP decoding
- step 4: if $w(\mathbf{s}_i) = 0$, stop decoding
- step 5: if $i = I_{\text{chk}}$ and $w(\mathbf{s}_i) > \delta_{\text{th}}$, stop decoding
- step 6: update $N_{\max}^{UC}$, $N_{\min}^{UC}$ in the duration $[\max(0, i - I_{\text{stag}} + 1), i]$
- step 7: if $N_{\max}^{UC} - N_{\min}^{UC} \leq \delta_{\text{stag}}$, stop decoding
- step 8: $i \leftarrow i + 1$
- step 9: Go to step 2

Note that the only metric needed for early stopping is $w(\mathbf{s}_i)$ in the above decoding process. Flowchart of the combined criteria is given in Fig. 2.

Table 1: AMC scheme in IEEE802.16e standard.

Modulation	Code		# of check nodes
	Length	Rate	
QPSK	576	1/2	288
		3/4	144
16QAM	1152	1/2	576
		3/4	288
64QAM	1728	1/2	864
		2/3	576
		3/4	432
		5/6	288

Table 2: Threshold values for proposed stopping criteria.

Modulation	Rate	δ_{th}					
		100%	10%	15%	20%	25%	30%
QPSK	1/2	288	28	43	57	72	86
	3/4	144	14	21	28	36	43
16QAM	1/2	576	57	86	115	144	172
	3/4	288	28	43	57	72	86
64QAM	1/2	864	86	129	172	216	259
	2/3	576	57	86	115	144	172
	3/4	432	43	64	86	108	129
	5/6	288	28	43	57	72	86

3. NUMERICAL RESULTS

In this section, numerical results to assess the effectiveness of the new stopping criteria are given. First, system with no H-ARQ protocol is simulated and then simplified version of AMC scheme from IEEE802.16e[5] is applied for the simulation on H-ARQ system. The maximum number of iterations for iterative BP decoding is set to 50 for the entire simulations.

3.1. AWGN channel

Although our main interest is of H-ARQ systems working in fading channel, verification on the proposed criterion over the AWGN channel may produce meaningful results. A half rate, $n = 2304$ LDPC code from IEEE802.16e standard is used. Firstly, numerical results only with the "stagnancy check" algorithm is depicted in Fig. 3 and Fig. 4. The sole parameter I_{stag} is chosen to reduce the number of average iterations without significant loss of performance.

As we can see, $I_{\text{stag}} = 7$ is enough to remove the useless computations without significant degradation in WER performance especially in the range below 1.0dB, and this improvement is more noticeable as the number of maximum iteration grows.

Only with this stagnancy check algorithm, though it may quite well operate in some cases, time to reach

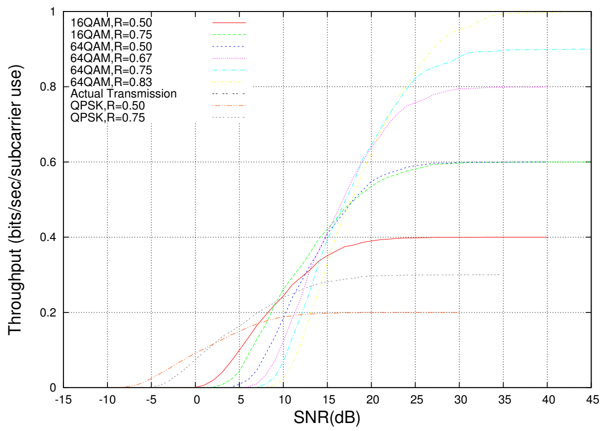

Fig. 7: Throughput performance for the given H-ARQ scheme in IEEE802.16e.

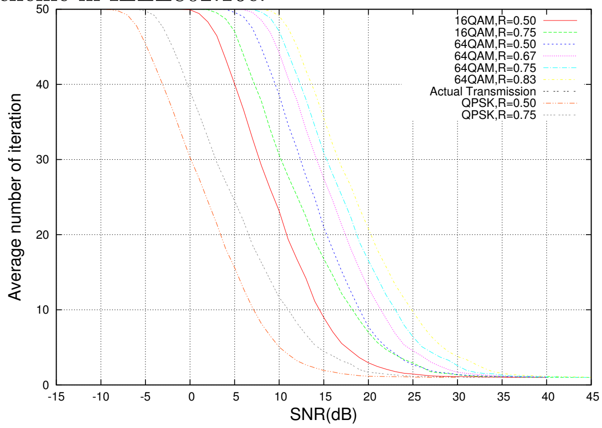

Fig. 8: Average number of iteration for the given H-ARQ scheme in IEEE802.16e.

the "stagnant state" bounded by I_{stag}. This means that the number of average iteration cannot decrease below I_{stag}, and for all the parameters in Fig. 4 the number of average iteration is greater than 10.

As shown in Fig. 5, we can find adequate parameters for combined stopping algorithm to avoid unnecessary iterations of decoding with negligible performance degradation. In Fig. 6, as mentioned earlier in Section 2.2, "stagnancy check" algorithm lowers the number of average iterations over low-to-mid range of SNR region while "early BP convergence speed check" algorithm intensively makes a big drop at the very low SNR region.

3.2. H-ARQ System with AMC Scheme

To verify the effectiveness of proposed algorithms on H-ARQ scheme, PC-based simulations are performed with following parameters. Type-I H-ARQ system with a maximum of four transmissions per one codeword is

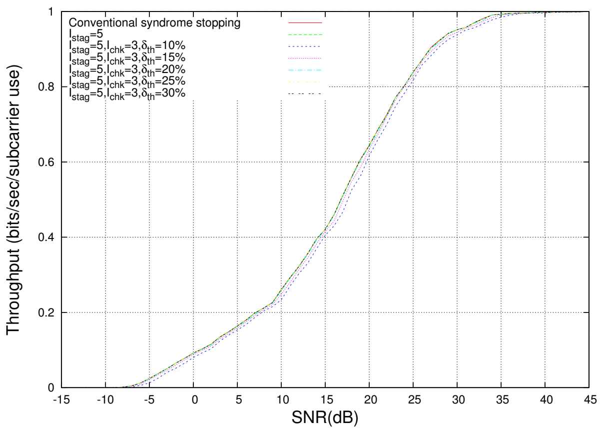

Fig. 9: Throughput performances. Solid line is the maximum achievable throughput.

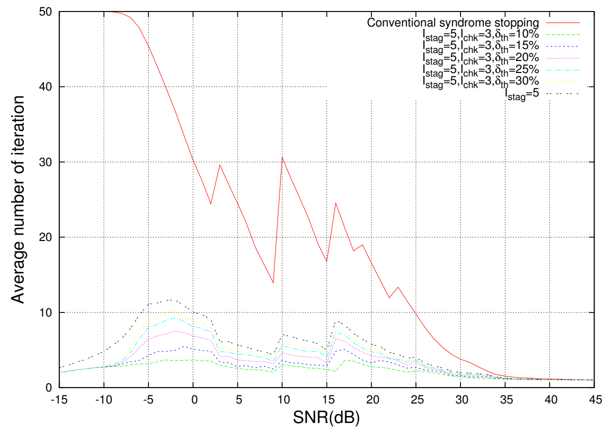

Fig. 10: Average number of iteration per transmission.

used. AMC scheme which includes various types of codes with different lengths and different rates is given in Table 1. 288 orthogonal frequency-division multiplexing (OFDM) subcarriers are allocated as resource for transmission and 3 equi-power path fading channel is assumed. Applying these settings to the system which uses the conventional syndrome stopping criterion only, solid thick lines in Fig. 7 and Fig. 8 are obtained as the overall results.

For all the codes in this syetem, δ_{th}s are determined by the same principle (i.e. the percentage of threshold value on the number of entire check nodes) as in Table 2. From Figs. 9 and 10, it is observed that large scale reduction of average number of iteration took place without noticeable performance degradation. Moreover, if small and restricted amount of degradation is allowed, just $3 \sim 4$ ($\simeq I_{\text{chk}}$) iterations are enough over the extremely wide range of SNR. Note that all these results are attained by single threshold decision rule for different codes, which supports the scalability of the proposed algorithms.

4. CONCLUSIONS

In this paper, we propose a combination of stopping criteria for BP decoding of LDPC codes. Its effectiveness and scalability was proved by numerical analysis. It is also shown that the decoding complexity of given H-ARQ system is reduced by 70-80% with the proposed scheme. Needless to say, the proposed methods are well compatible with other types of message passing decodings such as ones in [6].

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