Quadrature Partial Response Signaling Based on Alamouti Code

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Abstract—In this paper, the Alamouti code combined with partial response signaling (PRS) is proposed. The bit error rate of the proposed quadrature PRS (QPRS) Alamouti code is derived for the quasi-static Rayleigh fading and additive white Gaussian noise channel. Furthermore, its performance in a severely bandlimited channel is simulated by using chopping filter and compared with the conventional Alamouti code. The numerical results show that the proposed QPRS Alamouti code outperforms the conventional Alamouti code under the chopping environment.

Index Terms—Alamouti code, maximum *a posteriori* (MAP) decoding, quadrature partial response signaling (QPRS).

I. INTRODUCTION

High rate data transmission is required for the multimedia applications in the next generation wireless communication systems. It is known that the multiple transmit antenna systems with transmit diversity can increase the capacity of the bandlimited channel. The space-time code introduced by Tarokh *et al.* [1] utilizing the multiple transmit antennas and the structured code property is one of the examples. Numerous results on performance analysis for the space-time block codes (STBCs) in the various communication environments have been published. Raju *et al.* [2] analyzed bit error rate (BER) of STBC with QAM in fading channels using log-likelihood ratio (LLR). Kim *et al.* [3] derived the exact BER of various orthogonal STBC using the result in [4]. In [5], the exact pairwise error probability (PEP) using maximum *a posterior* probability (MAP) decoding in viewpoint of minimizing symbol error rate (SER) for orthogonal STBC was derived.

Another way of achieving high rate data transmission is to use the multi-level modulation scheme such as a quadrature amplitude modulation (QAM). But in general, this scheme aggravates the unwanted signal distortion called intersymbol interference (ISI).

The partial response signaling (PRS) system (also called correlative level coding) introduced by Lender is a good alternative to cope with the ISI problem and is suitable for the high rate data transmission [6].

In the PRS systems, it is possible to transmit the high speed data with less sensitivity to the timing error at the sampling instance, although the system itself incurs the performance degradation due to the increase of the effective alphabet size [7]. Wu and Feher [8] suggested the implementation of quadrature PRS (QPRS) and described the spectral efficiency of the QPRS system. It was shown that the PRS systems with chopping filter in the bandlimited channel improve the spectral efficiency so that the data can be transmitted at a rate higher than Nyquist rate at the expense of the BER degradation.

In this paper, QPRS Alamouti code with M-ary QAM (M-QAM) is proposed and the BER of the proposed scheme with 4-QAM is derived as a closed-form expression under symbol-by-symbol MAP decoding. It is also shown through the Monte Carlo simulation that QPRS Alamouti code with M-QAM outperforms the conventional one in the bandlimited communication channel.

This paper is organized as follows. In Section II, Alamouti code combined with class IV PRS for the bandlimited channel is proposed. In Section III, the BER for QPRS Alamouti code with 4-QAM is analyzed when symbol-by-symbol MAP decoding is used. In Section IV, the performance of the proposed QPRS Alamouti code with 4-QAM is compared with that of the conventional Alamouti code with 4-QAM for the case of using chopping filter. Finally, concluding remarks are given in Section V.
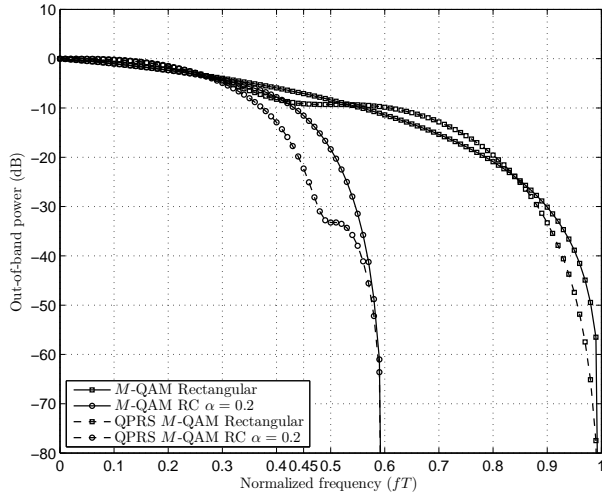
II. SYSTEM MODEL

A. Quadrature PRS

Wu and Feher [8] proposed QPRS, equivalent to the respective PRS's both in the in-phase (I) and the quadrature (Q) channels.

Assume that a raised cosine (RC) filter with roll-off factor α is used for the pulse shaping filter. Fig. 1 compares the out-of-band power of M-QAM and QPRS M-QAM signals. It can be predicted from Fig. 1 that we can mitigate the performance degradation caused in the severely bandlimited channel by using QPRS M-QAM rather than using the conventional M-QAM.

B. STBC: Alamouti Code

Alamouti [10] proposed a simple 2×2 STBC for two transmit antennas, called the Alamouti code in which the

Fig. 1.   Out-of-band power of M-QAM and QPRS M-QAM.

Fig. 2.   Block diagram of QPRS Alamouti code with M-QAM.

signal matrix is given as

$$\mathbf{S}_t = \begin{bmatrix} s_{2t-1} & s_{2t} \\ -s_{2t}^* & s_{2t-1}^* \end{bmatrix}.$$

The signals received with L receive antennas at block time t can be expressed as

$$\begin{bmatrix} r_{2t-1,1} & \cdots & r_{2t-1,L} \\ r_{2t,1} & \cdots & r_{2t,L} \end{bmatrix} = \sqrt{\frac{\gamma}{2}} \begin{bmatrix} s_{2t-1} & s_{2t} \\ -s_{2t}^* & s_{2t-1}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & \cdots & h_{1,L} \\ h_{2,1} & \cdots & h_{2,L} \end{bmatrix}$$
$$+ \begin{bmatrix} w_{2t-1,1} & \cdots & w_{2t-1,L} \\ w_{2t,1} & \cdots & w_{2t,L} \end{bmatrix} \quad (1)$$
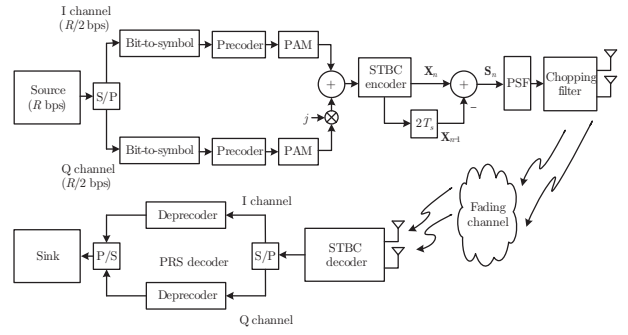
where $r_{t,l}$ denotes the received signal at time t in the lth receive antenna, γ means the average received signal-to-noise ratio (SNR), $h_{i,l}$ denotes the channel coefficient between the ith transmit antenna and the lth receive antenna having zero mean and unit variance complex Gaussian distribution, and $w_{t,l}$ denotes the complex white Gaussian noise with zero mean and 0.5 variance in each dimension, added to the lth receive antenna at time t. It is further assumed that all $h_{i,l}$'s and $w_{t,l}$'s are independent and identically distributed (i.i.d.).

For the sake of tractability, we use the following alternative expression instead of (1) for $l = 1, \cdots, L$

$$\underbrace{\begin{bmatrix} r_{2t-1,l} \\ r_{2t,l}^* \end{bmatrix}}_{\mathbf{r}_{t,l}} = \sqrt{\frac{\gamma}{2}} \underbrace{\begin{bmatrix} h_{1,l} & h_{2,l} \\ h_{2,l}^* & -h_{1,l}^* \end{bmatrix}}_{\mathbf{H}_l} \underbrace{\begin{bmatrix} s_{2t-1} \\ s_{2t} \end{bmatrix}}_{\mathbf{s}_t} + \underbrace{\begin{bmatrix} w_{2t-1,l} \\ w_{2t,l}^* \end{bmatrix}}_{\mathbf{w}_{t,l}}. \quad (2)$$

The above equation can be simplified as $\tilde{\mathbf{r}}_{t,l} = \mathbf{H}_l^H \mathbf{r}_{t,l} = \sqrt{\gamma/2} Z_l \mathbf{s}_t + \tilde{\mathbf{w}}_{t,l}$, where $Z_l = (|h_{1,l}|^2 + |h_{2,l}|^2)$ and $\tilde{\mathbf{w}}_{t,l} = \mathbf{H}_l^H \mathbf{w}_{t,l}$. Note that each element of $\tilde{\mathbf{w}}_{t,l}$ is i.i.d. complex Gaussian random variable. This enables the decoder to do the symbol-by-symbol decoding. Thus, ML decoding rule says

$$\hat{s}_t = \arg\min_{s_t} \left\{ \sum_{l=1}^{L} \frac{|\tilde{r}_{t,l} - \sqrt{\frac{\gamma}{2}} Z_l s_t|^2}{Z_l} \right\}. \quad (3)$$

C. QPRS Alamouti Code

In this subsection, Alamouti code with the QPRS of class IV, called the QPRS Alamouti code is proposed. Fig. 2 shows the block diagram of the proposed QPRS Alamouti code system. An input bit stream is split into I-channel bit stream ($a_{I,t}$) and Q-channel bit stream ($a_{Q,t}$). And then in each channel, every ($\log_2 \sqrt{M}$)-bit block is converted into $\sqrt{M}$-level data symbol ($b_{I,t}$, $b_{Q,t} \in \{0, 1, \cdots, \sqrt{M} - 1\}$). The $\sqrt{M}$-level sequences are precoded to prevent the error propagation in the decoding process as $p_{I,t} = b_{I,t} + p_{I,t-2} \pmod{\sqrt{M}}$ and $p_{Q,t} = b_{Q,t} + p_{Q,t-2} \pmod{\sqrt{M}}$ for $t = 1, 2, \cdots$. And $p_{I,t} = p_{Q,t} = 0$ for $t = -1, 0$. These precoded symbols $p_{I,t}$ and $p_{Q,t}$ are transformed into $\sqrt{M}$-ary PAM symbols $x_{I,t}$ and $x_{Q,t}$, respectively, as

$$x_{I,t} = 2p_{I,t} - (\sqrt{M} - 1) \quad \text{and} \quad x_{Q,t} = 2p_{Q,t} - (\sqrt{M} - 1),$$

which are combined into one M-QAM symbol $x_t = x_{I,t} + jx_{Q,t}$. These M-QAM symbols form an Alamouti code $\mathbf{X}_t = \begin{bmatrix} x_{2t-1} & x_{2t} \\ -x_{2t}^* & x_{2t-1}^* \end{bmatrix}$. And then, using the class IV PRS scheme, QPRS Alamouti code $\mathbf{S}_t$ can be generated as $\mathbf{S}_t = \mathbf{X}_t$ for $t = 1$ and $\mathbf{S}_t = (\mathbf{X}_t - \mathbf{X}_{t-1})/2$ for $t \geq 2$.

The received symbol matrix can be represented by the same form as (1). Usually, the MAP symbol decoding rule says

$$\hat{s}_t = \arg\max_{s_t} \left\{ \ln(\Pr(s_t)) - \sum_{l=1}^{L} \frac{|\tilde{r}_{t,l} - \sqrt{\frac{\gamma}{2}} Z_l s_t|^2}{Z_l} \right\} \quad (4)$$

where $\Pr(s_t)$ is *a priori* probability of s_t.

But, strictly speaking, the optimum symbol-by-symbol detection rule for PRS system must be different from the ordinary MAP detection rule given in (4) because the PRS symbol decoding error does not necessarily imply the data bit error due to enlarged signal constellation. Thus, it is necessary to define the following log *a posteriori* probability ratio (LAPPR) [11],

$$\text{LAPPR}(\mathbf{i}_k) = \log\left(\frac{\Pr(\mathbf{i}_k = 1|\mathbf{r})}{\Pr(\mathbf{i}_k = 0|\mathbf{r})} \right) \quad (5)$$

where $\mathbf{i}_k$ corresponds to kth bit of I-channel component of s_t. From the above metric, the detector declares '$\mathbf{i}_k = 1$' if $\text{LAPPR}(\mathbf{i}_k) \geq 0$, otherwise '$\mathbf{i}_k = 0$'.
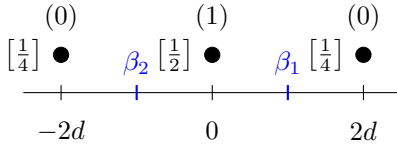
Fig. 3. I-channel illustration of signal constellations of QPRS Alamouti code with 4-QAM ($[\cdot]$ and $(\cdot)$ denote *a priori* and the bit notation, respectively).

Once the symbol decoding is completed, each decoded symbol $\hat{s}_t$ is split into I- and Q-channel symbols, $\hat{s}_{I,t}$ and $\hat{s}_{Q,t}$, and then deprecoded into $\sqrt{M}$-level symbols using the simple class IV PRS decoder as

$$\hat{b}_{I,t} = \hat{s}_{I,t}/2 \pmod{\sqrt{M}} \quad \text{and} \quad \hat{b}_{Q,t} = \hat{s}_{Q,t}/2 \pmod{\sqrt{M}}.$$

Combining Alamouti code with PRS itself does not give rise to an additional decoding complexity. The only increase in decoding complexity is caused by the expansion of the signal constellation from M-QAM to $(2\sqrt{M}-1)^2$ signals in QPRS M-QAM.

III. PERFORMANCE ANALYSIS OF QPRS ALAMOUTI CODE

In this section, we are going to derive the BER of QPRS Alamouti code with M-QAM using the results in [4] and [5]. As mentioned before, the optimum decision boundary for minimizing the bit error probability of the QPRS M-QAM must be determined. Although the case $M = 16$ can be applied, let us only look at the QPRS Alamouti code with 4-QAM.

In the AWGN channel with variance σ^2, using LAPPR, the LAPPR for 2-PAM with class IV PRS in Fig. 3 can be derived as

$$\mathsf{LAPPR}\,(\mathbf{i}) = \log \left[\frac{\frac{1}{4}\exp\left(-\frac{(r+2d)^2}{2\sigma^2}\right) + \frac{1}{4}\exp\left(-\frac{(r-2d)^2}{2\sigma^2}\right)}{\frac{1}{2}\exp\left(-\frac{r^2}{2\sigma^2}\right)} \right].$$

From the above equation, we can have the following two decision boundaries under MAP bit decoding

$$\beta_1 = d + \frac{\sigma^2}{2d}\ln\left(1 + \sqrt{1 - e^{-4d^2/\sigma^2}}\right), \quad \beta_2 = -\beta_1. \quad (6)$$

Using the approximation $\ln(1+\sqrt{1-x^2}) \approx \ln 2 - \frac{1}{4}x^2$, (6) can be approximated to the decision boundary in the MAP symbol decoding

$$\beta_1 \approx \frac{\sigma^2}{2d}\left[\frac{2d^2}{\sigma^2} + \left(\ln 2 - \frac{1}{4}e^{-4d^2/\sigma^2}\right)\right] \approx d + \frac{\sigma^2}{2d}\ln 2. \quad (7)$$

Even when SNR is quite low, the difference between two decision boundaries under MAP symbol decoding and MAP bit decoding is negligible. For this reason, we may argue that MAP symbol decoding is 'near' optimal and thus, we are going to represent the BER for QPRS Alamouti code with M-QAM in terms of the PEP under MAP symbol decoding.

The exact PEP using MAP symbol decoding for STBCs was derived in [5] as (8) in the top of the next page where $d = |s - \hat{s}|$, $\lambda = (1/2)\ln(\Pr(s)/\Pr(\hat{s}))$, $\mathrm{sgn}(x) = |x|/x$ if
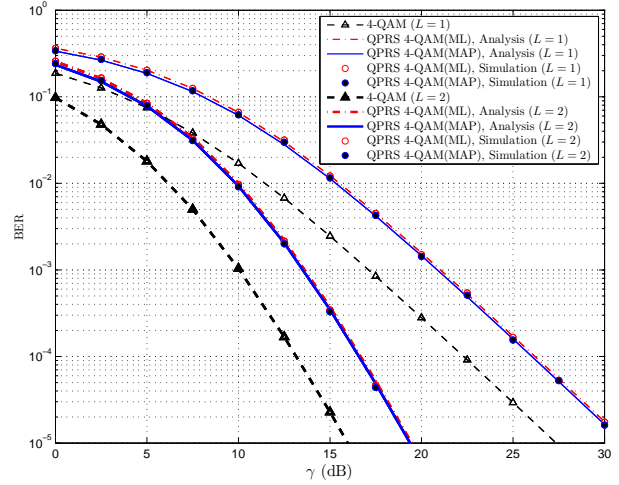

Fig. 4. BERs of QPRS Alamouti code with 4-QAM in the Rayleigh fading channel.

$x \neq 0$ and 0, otherwise, and $\sum_{i=i_L}^{i_U} x_i = 1$ for $i_L > i_U$. Using this result, the BERs of QPRS Alamouti code with 4-QAM are derived under MAP symbol decoding. It is useful to define (8) as $\mathcal{Q}_\gamma(d,\lambda)$ in terms of distance d and log-ratio λ for a fixed SNR γ.

The BER for QPRS Alamouti code with 4-QAM under MAP symbol decoding is given as

$$P_{\mathrm{b}}(\mathbf{S}) = \frac{1}{2}\left\{\mathcal{Q}_\gamma\left(d, -\frac{1}{2}\ln 2\right) + 2\mathcal{Q}_\gamma\left(d, \frac{1}{2}\ln 2\right) - \mathcal{Q}_\gamma\left(3d, \frac{3}{2}\ln 2\right)\right\}. \quad (9)$$

Fig. 4 shows the analytical and numerical results for BERs using ML and MAP symbol decoding instead of MAP bit decoding. As one can see in Fig. 4, the diversity order of QPRS Alamouti code with M-QAM remains to be $2L$. Furthermore, the performance difference between ML and MAP symbol decoding becomes more and more negligible as SNR γ grows. For this reason, we use the ML symbol decoding instead of MAP symbol decoding to perform Monte Carlo simulation with chopping filter in the next section.

IV. NUMERICAL RESULTS WITH CHOPPING

Usually in the bandlimited channel, the frequency spectrum of the transmitted signal should be chopped in order to meet the required emission mask of the transmitted signals. Here, chopping a signal implies that the signal spectrum above f_{chop} is removed. This can be done by the chopping filter in the transmit antenna before signal transmission. The bandwidth of the chopping filter is set to $f_{\mathrm{chop}} = f_N\left(1 - a/100\right)$ where f_N means the Nyquist rate and a, $0 \leq a < 100(\%)$ denotes the chopping ratio. Chopping a signal in the frequency domain usually induces the constellation scattering. The spectral efficiency (in bits/s/Hz) defined as the ratio of the transmitted data rate to the required bandwidth of the system increases as

$$P(s \to \hat{s}) = \frac{1 - \text{sgn}(\lambda)}{2} + \frac{1}{2} e^{-(\lambda + |\lambda|\sqrt{2(\gamma d^2)^{-1}+1})}$$
$$\times \sum_{i=0}^{2L-1} \frac{(-1)^{2L+i-1}}{(2L-i-1)!} \frac{1}{(2+\gamma d^2)^{2L-i-1}} \left\{ \text{sgn}(\lambda) - \sqrt{\frac{\gamma d^2}{2+\gamma d^2}} \sum_{m=0}^{i} \binom{2m}{m} \frac{1}{(2\gamma d^2+4)^m} \right\}$$
$$\times \left\{ \sum_{k=1}^{2L-i-1} \frac{|\lambda|^k}{k!} \sqrt{\left(\frac{2+\gamma d^2}{\gamma d^2}\right)^k} \sum_{p=0}^{k-1} \binom{k}{p} (-1)^{k+p} \prod_{q=0}^{2L-2-i} (k-p-2q) \right\} \tag{8}$$
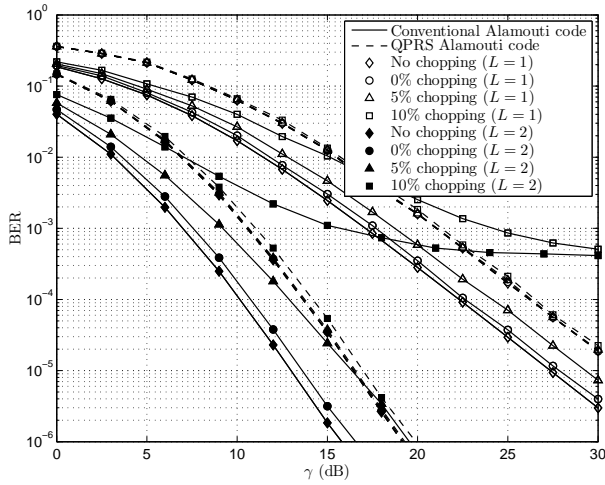

Fig. 5. BER of QPRS Alamouti code and conventional Alamouti code with 4-QAM for one and two receive antennas in the bandlimited Rayleigh fading channel ('solid' line: conventional Alamouti code, 'dashed' line: QPRS Alamouti code, non-filled marker: $L = 1$, filled marker: $L = 2$).

the chopping ratio increases. But the increase of a leads to the performance degradation of BER.

In this simulation, we consider $M = 4$ and use 4096-point fast Fourier transform to implement the chopping filter. For the practical implementation, we use the RC filter with roll-off factors 0.2. In the process of QPRS, the size of constellation increases to 9 for 4-QAM, which causes the 3 dB BER performance degradation in terms of γ.

Fig. 5 shows BERs of QPRS Alamouti code ('dashed line') and conventional Alamouti code with M-QAM ('solid line') for various chopping ratios and for one or two receive antennas. From this result, we can observe the following: First, without chopping, QPRS Alamouti code with M-QAM has poorer performance than the conventional Alamouti code does. This is caused by the increase of the alphabet size and the number of decision boundaries. Second, as chopping ratio increases, the QPRS Alamouti code with M-QAM shows a better BER performance than the conventional Alamouti code with M-QAM in high SNR region. Third, as the chopping ratio becomes larger and larger, the diversity effect in the conventional Alamouti code disappears while it is almost preserved in the proposed scheme.

V. CONCLUSIONS

We proposed Alamouti code combined with QPRS for the bandlimited wireless communication systems and derived the closed-form expression for the BER of the proposed scheme with 4-QAM under MAP symbol decoding. We compared the performance of the proposed QPRS Alamouti code with M-QAM with the conventional Alamouti code with M-QAM. From the numerical results, we conclude that QPRS Alamouti code with M-QAM outperforms the conventional Alamouti code with M-QAM, especially, when the bandwidth smaller than the Nyquist rate is used.

ACKLEDGEMENT

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