

# Soft-Decision-and-Forward Protocol for Cooperative Communication Networks Based on Alamouti Code

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**Abstract**—A cooperative protocol called soft-decision-and-forward (SDF) is introduced. SDF protocol exploits the soft decision values of the received signal at the relay node. Alamouti code is used for orthogonal transmission and distributed space-time codes are designed for non-orthogonal transmission. The maximum likelihood decoders with low decoding complexity are proposed. From simulations, it can be seen that SDF protocol outperforms AF protocol.

## I. INTRODUCTION

Recently, there have been much interest in cooperative communication networks because the spectral efficiency and reliability of the wireless communication systems can be improved by the cooperation of relay node. Cooperative communication networks consist of three types of nodes which are source nodes, relay nodes, and destination nodes [1]. In this paper, we express a source node as S, a relay node as R, and a destination node as D. For cooperation, there are several cooperative protocols including amplify-and-forward (AF) and decode-and-forward (DF) protocols [1]–[4]. AF protocol allows R to amplify the received signals from S and forward them to D. DF protocol permits R to decode and re-encode the received signals and then forward them to D.

In this paper, a cooperative protocol called soft-decision-and-forward (SDF) is introduced. Unlike AF and DF protocols, SDF protocol exploits the soft decision values of the received signals at R. Two phase cooperation is assumed. In the first phase, S transmits, and in the second phase, R transmits and S transmits or not. If S does not transmit in the second phase, we call it orthogonal transmission (OT). Otherwise, we call it non-orthogonal transmission (NT). We consider four schemes, i.e., OT-AF, OT-SDF, NT-AF, and NT-SDF, and their ML decoders are derived as simplified forms.

The protocols in [5] and [6] are similar to SDF protocol. In [5], process-and-forward (PF) protocol was proposed, which allows R to perform space-time processing on the received signals. In PF protocol, a relay node with single antenna was considered. However, we consider multiple antennas at all nodes. In [6], decouple-and-forward (DCF) protocol for dual hop cooperative communications was proposed, which allows

R to decouple the received signals. In spite of correlated noise at D, squaring method was used for DCF protocol. However, we assume that D can hear S and maximum likelihood (ML) decoder is used. Further, we show that ML decoder can be simplified in spite of correlated noises at D.

In this paper, the following notations are used: Capital boldface letter denotes a matrix, small boldface letter denotes a vector,  $\Re(\cdot)$  denotes the real part of a complex number,  $(\cdot)^*$  denotes the complex conjugate,  $(\cdot)^T$  denotes the transpose of a matrix,  $(\cdot)^H$  denotes the complex conjugate and transpose of a matrix,  $|\cdot|$  denotes the norm of a complex number,  $\|\cdot\|$  denotes the Frobenius norm of a matrix,  $\mathbf{I}_n$  denotes the  $n \times n$  identity matrix,  $\mathbf{0}$  denotes the all zero matrix.

## II. SYSTEM MODEL

We consider a cooperative communication network consisting three nodes, each of which has two antennas. Let  $\mathbf{G}$  be the fading channel matrix between S and R,  $\mathbf{H}_1$  be the fading channel matrix between S and D, and  $\mathbf{H}_2$  be the fading channel matrix between R and D. Since we assume two antennas, all the fading channel matrices are  $2 \times 2$  matrices whose entries are independent complex Gaussian random variables with zero mean and unit variance. Note that  $g_{ij}$ ,  $h_1^{ij}$ ,  $h_2^{ij}$ , for  $i, j = 1, 2$  are the fading coefficients from the  $i$ th transmit antenna to the  $j$ th receive antenna for the corresponding channels. We assume quasi-static Rayleigh fading for all channels. Further, perfect channel state information is assumed to be known only at the receiver.

Half duplex communication is assumed, that is, all nodes can either transmit or receive. Thus, we assume two phase cooperative communications. For NT, source antenna switching (SAS) [4] is considered such that S utilizes two RF chains, where each RF chain has two transmit antennas. Thus, there are four transmit antennas at S. In the first phase, one transmit antenna at each RF chain is used and then in the second phase the other is used. By using SAS for DF protocol, the additional diversity gain can be obtained [4].

Cooperative protocols are classified by the operation at R. There are two well-known protocols, AF and DF protocols. For AF protocol, R only amplifies the received signals and forwards them to D. Since AF protocol amplifies the received signal, noise component is also amplified and forwarded.

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For DF protocol, R decodes the received signals, re-encodes, and transmits them to D. Since R performs hard decision in DF protocol, the soft information of the received signal is disappeared. Therefore, we propose the SDF protocol which exploits the soft-decision values at R. In the next section, four schemes, i.e., OT-AF, OT-SDF, NT-AF, and NT-SDF are investigated.

### III. AF AND SDF PROTOCOLS

We assume that S transmits Alamouti code [7]. For AF protocol, the received signal at the first (second) receive antenna of R is amplified and forwarded through the first (second) transmit antenna of R. For SDF protocol, R decodes the received signals into the soft decision values. R re-encodes the soft decision values and transmits to D. In this section, four schemes are described, which are OT-AF, OT-SDF, NT-AF, and NT-SDF. To explain each scheme, the following definition will be useful.

**Definition.** *Alamouti operation and complex vectorization:*

- Alamouti operation for two symbols  $a$  and  $b$  is represented as  $\mathbf{A}(a, b) = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$ .
- For  $2 \times 2$  matrices  $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$  and  $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ , let  $\mathbf{B}' = \begin{bmatrix} \mathbf{M} \cdot \mathbf{A}(b_{11}, b_{21}) \\ \mathbf{M} \cdot \mathbf{A}(b_{12}, b_{22}) \end{bmatrix}$ .
- Let  $cv(\cdot)$  denote complex vectorization operation for  $2 \times 2$  matrix, i.e.,  $cv\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [a \ c^* \ b \ d^*]^T$ .

#### A. OT-AF Scheme

In the first phase, S transmits an Alamouti code which consists of two independent symbols  $x_1$  and  $x_2$ , i.e.,  $\mathbf{X} = \mathbf{A}(x_1, x_2)$ . The received signals at R and D in the first phase can be expressed as

$$\mathbf{Y}_R = \sqrt{P_1} \mathbf{X} \mathbf{G} + \mathbf{W}, \quad \mathbf{Y}_{D1} = \sqrt{P_1} \mathbf{X} \mathbf{H}_1 + \mathbf{N}_1 \quad (1)$$

where  $\mathbf{Y}_R$  is the received signal matrix at R,  $\mathbf{Y}_{D1}$  is the received signal at D,  $\mathbf{W}$  and  $\mathbf{N}_1$  are additive white Gaussian noise (AWGN) matrices with entries of zero mean and unit variance complex Gaussian random variables. The average power per antenna at S is assumed to be  $P_1$ .

Considering AF protocol, the transmitted signal at R in the second phase is given as

$$\mathbf{X}_R = \mathbf{Y}_R \boldsymbol{\beta}$$

where  $\mathbf{X}_R$  is the transmitted signal matrix and  $\boldsymbol{\beta} = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}$  is the amplification matrix at R. It can be easily shown that  $\beta_1 = \sqrt{P_2 / (P_1(|g_{11}|^2 + |g_{21}|^2) + 1)}$  and  $\beta_2 = \sqrt{P_2 / (P_1(|g_{12}|^2 + |g_{22}|^2) + 1)}$  to ensure the average power per antenna at R to be  $P_2$ .

Since we consider OT, only R transmits in the second phase. The received signal at D in the second phase is given as

$$\mathbf{Y}_{D2} = \mathbf{Y}_R \boldsymbol{\beta} \mathbf{H}_2 + \mathbf{N}_2 = \sqrt{P_1} \mathbf{X} \mathbf{F} + \mathbf{N} \quad (2)$$

where  $\mathbf{F} = \mathbf{G} \boldsymbol{\beta} \mathbf{H}_2$ ,  $\mathbf{N} = \mathbf{W} \boldsymbol{\beta} \mathbf{H}_2 + \mathbf{N}_2$ , and  $\mathbf{N}_2$  is AWGN matrix with entries of zero mean and unit variance complex Gaussian random variables.

By using (1), (2), and the Alamouti code structure, the equivalent vector model for OT-AF can be expressed as

$$\begin{bmatrix} cv(\mathbf{Y}_{D1}) \\ cv(\mathbf{Y}_{D2}) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1} \mathbf{H}'_1 \\ \sqrt{P_1} \mathbf{F}' \end{bmatrix} \mathbf{x} + \begin{bmatrix} cv(\mathbf{N}_1) \\ cv(\mathbf{N}) \end{bmatrix} \quad (3)$$

where  $\mathbf{x} = [x_1 \ x_2]^T$ .

#### B. OT-SDF Scheme

In the first phase, we have the same situation as in Subsection III-A and thus the received signals at R and D are given in (1).

For SDF protocol, R should decode the received signals into the soft decision values, and re-encode them, and transmit the re-encoded signals. Since S transmits Alamouti code, soft decision values for  $x_1$  and  $x_2$  at R can be obtained as

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \gamma (\mathbf{G}')^H cv(\mathbf{Y}_R) \quad (4)$$

where  $\gamma$  is the power gain at the relay. To guarantee the average power per antenna at R to be  $P_2$ ,  $\gamma$  should be  $\sqrt{P_2 / (\|\mathbf{G}\|^2 (1 + P_1 \|\mathbf{G}\|^2))}$ . Using the soft decision values in (4), R re-encodes them by using Alamouti code and thus we have  $\mathbf{X}_R = \mathbf{A}(\hat{x}_1, \hat{x}_2)$ .

The received signal at D in the second phase is given as

$$\mathbf{Y}_{D2} = \mathbf{X}_R \mathbf{H}_2 + \mathbf{N}_2.$$

Using (4) and after some manipulations, we have the following equivalent model of OT-SDF as

$$\begin{bmatrix} cv(\mathbf{Y}_{D1}) \\ cv(\mathbf{Y}_{D2}) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1} \mathbf{H}'_1 \\ a \mathbf{H}'_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} cv(\mathbf{N}_1) \\ cv(\mathbf{N}) \end{bmatrix} \quad (5)$$

where  $a = \gamma \sqrt{P_1} \|\mathbf{G}\|^2$ ,  $cv(\mathbf{N}) = \gamma \mathbf{H}'_2 (\mathbf{G}')^H cv(\mathbf{W}) + cv(\mathbf{N}_2)$ , and  $\mathbf{x} = [x_1 \ x_2]^T$ .

#### C. NT-AF Scheme

Considering NT, S transmits signals in the second phase. Since S and R transmit signals in the second phase simultaneously, we design a distributed space-time code (DSTC) using a quasi-orthogonal space-time block code (QO-STBC) [8]. For a DSTC in the second phase, we assume that S transmits two Alamouti codes, i.e.,  $\mathbf{X}_1 = \mathbf{A}(x_1, x_2)$  and  $\mathbf{X}_2 = \mathbf{A}(x_3, x_4)$ , in the first phase. Thus, the received signals at R and D in the first phase are given as

$$\begin{aligned} \mathbf{Y}_{R1} &= \sqrt{P_1} \mathbf{X}_1 \mathbf{G} + \mathbf{W}_1, & \mathbf{Y}_{R2} &= \sqrt{P_1} \mathbf{X}_2 \mathbf{G} + \mathbf{W}_2 \\ \mathbf{Y}_{D1,1} &= \sqrt{P_1} \mathbf{X}_1 \mathbf{H}_1 + \mathbf{N}_{1,1}, & \mathbf{Y}_{D1,2} &= \sqrt{P_1} \mathbf{X}_2 \mathbf{H}_1 + \mathbf{N}_{1,2} \end{aligned} \quad (6)$$

where  $\mathbf{Y}_{R1}$  and  $\mathbf{Y}_{R2}$  are  $2 \times 2$  received signal matrices at R,  $\mathbf{Y}_{D1,1}$  and  $\mathbf{Y}_{D1,2}$  are received signal matrices at D, and  $\mathbf{W}_1$ ,  $\mathbf{W}_2$ ,  $\mathbf{N}_{1,1}$ , and  $\mathbf{N}_{1,2}$  are  $2 \times 2$  AWGN matrices.

In the second phase, similarly to Subsection III-A, R transmits  $\mathbf{R}_\theta \mathbf{Y}_{R2} \boldsymbol{\beta}$  at first and then transmits  $\mathbf{Y}_{R1} \boldsymbol{\beta}$ . Note that  $\mathbf{R}_\theta = \mathbf{A}(e^{j\theta}, 0)$  and  $\theta$  is the constellation rotation angle for full diversity in QO-STBC [9]. In this paper,  $\theta$  is assumed to be  $\pi/4$  for quadrature phase shift keying (QPSK) modulation.

S transmits  $\mathbf{X}_1$  at first and then  $\mathbf{R}_\theta \mathbf{X}_2$ . The received signal at D in the second phase can be given as

$$\begin{aligned} \mathbf{Y}_{D2,1} &= \sqrt{P_3} \mathbf{X}_1 \mathbf{H}_1 + \mathbf{R}_\theta \mathbf{Y}_{R2} \boldsymbol{\beta} \mathbf{H}_2 + \mathbf{N}_{2,1} \\ &= \sqrt{P_3} \mathbf{X}_1 \mathbf{H}_1 + \sqrt{P_1} \mathbf{R}_\theta \mathbf{X}_2 \mathbf{F} + \mathbf{N}_{e1} \\ \mathbf{Y}_{D2,2} &= \sqrt{P_3} \mathbf{R}_\theta \mathbf{X}_2 \mathbf{H}_1 + \mathbf{Y}_{R1} \boldsymbol{\beta} \mathbf{H}_2 + \mathbf{N}_{2,2} \\ &= \sqrt{P_3} \mathbf{R}_\theta \mathbf{X}_2 \mathbf{H}_1 + \sqrt{P_1} \mathbf{X}_1 \mathbf{F} + \mathbf{N}_{e2} \end{aligned} \quad (7)$$

where  $P_2$  and  $P_3$  are the average power per antenna at R and S, respectively,  $\mathbf{N}_{2,1}$  and  $\mathbf{N}_{2,2}$  are  $2 \times 2$  AWGN matrices,  $\mathbf{F} = \mathbf{G} \boldsymbol{\beta} \mathbf{H}_2$ ,  $\mathbf{N}_{e1} = \mathbf{R}_\theta \mathbf{W}_2 \boldsymbol{\beta} \mathbf{H}_2 + \mathbf{N}_{2,1}$ , and  $\mathbf{N}_{e2} = \mathbf{W}_1 \boldsymbol{\beta} \mathbf{H}_2 + \mathbf{N}_{2,2}$ . From (7), we can see that a QO-STBC is transmitted to D in the second phase.

For NT-AF, the equivalent vector model can be expressed as

$$\begin{bmatrix} cv(\mathbf{Y}_{D1,1}) \\ cv(\mathbf{Y}_{D1,2}) \\ cv(\mathbf{Y}_{D2,1}) \\ cv(\mathbf{Y}_{D2,2}) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1} \mathbf{H}'_1 & \mathbf{0} \\ \mathbf{0} & \sqrt{P_1} \mathbf{H}'_1 \\ \sqrt{P_3} \mathbf{H}'_1 & \sqrt{P_1} e^{j\theta} \mathbf{F}' \\ \sqrt{P_1} \mathbf{F}' & \sqrt{P_3} e^{j\theta} \mathbf{H}'_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} cv(\mathbf{N}_{1,1}) \\ cv(\mathbf{N}_{1,2}) \\ cv(\mathbf{N}_{e1}) \\ cv(\mathbf{N}_{e2}) \end{bmatrix} \quad (8)$$

where  $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]^T$ .

#### D. NT-SDF Scheme

Similarly to Subsection III-C, for NT-SDF, we assume that S transmits  $\mathbf{X}_1$  and  $\mathbf{X}_2$  in the first phase, and a QO-STBC is used in the second phase. Since there is no difference between NT-AF and NT-SDF in the first phase, the received signals at R and D are given in (6).

Considering SDF protocol, R performs the similar operation in Subsection III-B. Soft decision values at R can be obtained as

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \gamma(\mathbf{G}')^H cv(\mathbf{Y}_{R1}), \quad \begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} = \gamma(\mathbf{G}')^H cv(\mathbf{Y}_{R2})$$

where  $\gamma$  is the same as in (4). Similarly to Subsection III-C, to construct a QO-STBC in the second phase, R transmits  $\mathbf{R}_\theta \mathbf{A}(\hat{x}_3, \hat{x}_4)$  at first and then transmits  $\mathbf{A}(\hat{x}_1, \hat{x}_2)$ . S transmits  $\mathbf{X}_1$  at first and then  $\mathbf{R}_\theta \mathbf{X}_2$  in the second phase.

The equivalent vector model can be expressed as

$$\begin{bmatrix} cv(\mathbf{Y}_{D1,1}) \\ cv(\mathbf{Y}_{D1,2}) \\ cv(\mathbf{Y}_{D2,1}) \\ cv(\mathbf{Y}_{D2,2}) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1} \mathbf{H}'_1 & \mathbf{0} \\ \mathbf{0} & \sqrt{P_1} \mathbf{H}'_1 \\ \sqrt{P_3} \mathbf{H}'_1 & a e^{j\theta} \mathbf{H}'_2 \\ a \mathbf{H}'_2 & \sqrt{P_3} e^{j\theta} \mathbf{H}'_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} cv(\mathbf{N}_{1,1}) \\ cv(\mathbf{N}_{1,2}) \\ cv(\mathbf{N}_{e1}) \\ cv(\mathbf{N}_{e2}) \end{bmatrix} \quad (9)$$

where  $a$  is the same as in (5),  $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]^T$ ,  $cv(\mathbf{N}_{e1}) = \gamma e^{j\theta} \mathbf{H}'_2 (\mathbf{G}')^H cv(\mathbf{W}_2) + cv(\mathbf{N}_{2,1})$ , and  $cv(\mathbf{N}_{e2}) = \gamma \mathbf{H}'_2 (\mathbf{G}')^H cv(\mathbf{W}_1) + cv(\mathbf{N}_{2,2})$ .

#### IV. OPTIMAL ML DECODERS

In this section, we derive the optimal ML decoder for each scheme presented in the previous section. To this end, we have derived the equivalent vector model. The four equivalent vector models (3), (5), (8), and (9) can be commonly expressed as

$$\mathbf{y}_e = \mathbf{H}_e \mathbf{x} + \mathbf{n}_e. \quad (10)$$

Since the noise at R is transmitted to both receive antennas at D, entries of equivalent noise vector  $\mathbf{n}_e$  are correlated. Let  $\mathbf{K}_{\mathbf{n}_e}$  be the covariance matrix of  $\mathbf{n}_e$ . Then, the ML decoder of (10) can be derived as

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} [(\mathbf{y}_e - \mathbf{H}_e \mathbf{x})^H \mathbf{K}_{\mathbf{n}_e}^{-1} (\mathbf{y}_e - \mathbf{H}_e \mathbf{x})]. \quad (11)$$

Thus, we need to derive  $\mathbf{K}_{\mathbf{n}_e}$  and calculate the term inside the bracket in (11) for each scheme.

Since we use Alamouti code as a building block, the following properties for Alamouti operation are useful to derive the ML decoder for each scheme.

**Property.** *Properties of Alamouti operation:*

- 1)  $\mathbf{A}^H(a, b) = \mathbf{A}(a^*, -b)$
- 2)  $\mathbf{A}(a, b) + \mathbf{A}(c, d) = \mathbf{A}(a+c, b+d)$
- 3)  $\mathbf{A}(a, b) \cdot \mathbf{A}(c, d) = \mathbf{A}(ac - bd^*, ad + bc^*)$
- 4)  $\mathbf{A}^H(a, b) \cdot \mathbf{A}(a, b) = \mathbf{A}(a, b) \cdot \mathbf{A}^H(a, b) = (|a|^2 + |b|^2) \mathbf{I}_2$ .

For AF and SDF protocols, we derive the covariance matrix of  $cv(\mathbf{N})$  which corresponds to the equivalent noise at D in the second phase under OT. Considering the AF protocol, the covariance matrix of  $cv(\mathbf{N})$  in (3) is given as

$$\mathbf{K}_{cv(\mathbf{N})} = E(cv(\mathbf{N})(cv(\mathbf{N}))^H) = \begin{bmatrix} p \mathbf{I}_2 & \mathbf{A}(q, 0) \\ \mathbf{A}(q^*, 0) & r \mathbf{I}_2 \end{bmatrix} \quad (12)$$

where  $p = \beta_1^2 |h_2^{11}|^2 + \beta_2^2 |h_2^{21}|^2 + 1$ ,  $q = \beta_1^2 h_2^{11} (h_2^{12})^* + \beta_2^2 h_2^{21} (h_2^{22})^*$ , and  $r = \beta_1^2 |h_2^{12}|^2 + \beta_2^2 |h_2^{22}|^2 + 1$ . For SDF protocol, the covariance matrix of  $cv(\mathbf{N})$  in (5) is given as

$$\begin{aligned} \mathbf{K}_{cv(\mathbf{N})} &= E(cv(\mathbf{N})(cv(\mathbf{N}))^H) = \frac{P_2}{1 + P_1 \|\mathbf{G}\|^2} \mathbf{H}'_2 (\mathbf{H}'_2)^H + \mathbf{I}_4 \\ &= \begin{bmatrix} x \mathbf{I}_2 & \mathbf{A}(u, v) \\ \mathbf{A}(u^*, -v) & y \mathbf{I}_2 \end{bmatrix}. \end{aligned} \quad (13)$$

Let  $\mathbf{y}_e^H \mathbf{K}_{\mathbf{n}_e}^{-1} \mathbf{H}_e = [\eta_1 \ \eta_2]$  for OT and  $\mathbf{y}_e^H \mathbf{K}_{\mathbf{n}_e}^{-1} \mathbf{H}_e = [\eta_1 \ \eta_2 \ \eta_3 \ \eta_4]$  for NT.

Using (3), (12), and the above Property, we can obtain the symbolwise ML decoder for OT-AF as

$$\tilde{x}_i = \arg \min_{x_i} [(P_1 \|\mathbf{H}_1\|^2 + t_a) |x_i|^2 - 2\Re[\eta_i x_i]], \text{ for } i = 1, 2$$

where  $t_a = P_1 s_a / (pr - |q|^2)$  and  $s_a = (|f_{12}|^2 + |f_{22}|^2)p + (|f_{11}|^2 + |f_{21}|^2)r - 2\Re[(f_{11}^* f_{12} + f_{21}^* f_{22})q]$ .

Similarly to OT-AF, the symbolwise ML decoder for OT-SDF can be derived as

$$\tilde{x}_i = \arg \min_{x_i} [(P_1 \|\mathbf{H}_1\|^2 + t_s) |x_i|^2 - 2\Re[\eta_i x_i]], \text{ for } i = 1, 2$$

where  $t_s = a^2 s_s / (xy - |u|^2 - |v|^2)$  and  $s_s = (|h_2^{12}|^2 + |h_2^{22}|^2)x + (|h_2^{11}|^2 + |h_2^{21}|^2)y - 2\Re[(h_2^{11})^* h_2^{12} + (h_2^{21})^* h_2^{22}]u + ((h_2^{11})^* (h_2^{22})^* - (h_2^{12})^* (h_2^{21})^*)v]$ .

For NT-AF, the pairwise ML decoder can be derived as

$$\begin{aligned} (\tilde{x}_i, \tilde{x}_{i+2}) &= \arg \min_{x_i, x_{i+2}} [\alpha_a (|x_i|^2 + |x_{i+2}|^2) \\ &\quad + 2\Re[\beta_a^* x_i x_{i+2}^* - \eta_i x_i - \eta_{i+2} x_{i+2}]], \text{ for } i = 1, 2 \end{aligned}$$

where  $\alpha_a = P_1 \|\mathbf{H}_1\|^2 + (P_3 s_{a1} + P_1 s_{a2}) / (pr - |q|^2)$  and  $\beta_a = \sqrt{P_1 P_3} e^{j\theta} z_a$ . Note that  $s_{a1} = (|h_1^{12}|^2 + |h_1^{22}|^2)p + (|h_1^{11}|^2 + |h_1^{21}|^2)r - 2\Re[(h_1^{11})^* h_1^{12} + (h_1^{21})^* h_1^{22}]q]$ ,  $s_{a2} =$

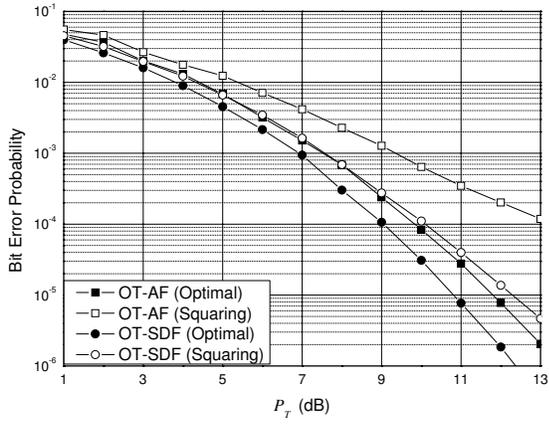


Fig. 1: Performance comparison of various orthogonal protocols.

$(|f_{12}|^2 + |f_{22}|^2)p + (|f_{11}|^2 + |f_{21}|^2)r - 2\Re[(f_{11}^* f_{12} + f_{21}^* f_{22})q]$ , and  $(pr - |q|^2)z_a = 2r\Re[(h_1^{11})^* f_{11} + h_1^{21} f_{21}^*] + 2p\Re[(h_1^{12})^* f_{12} + h_1^{22} f_{22}^*] - 2\Re[(h_1^{11})^* f_{12} + (h_1^{21})^* f_{22} + h_1^{12} f_{11}^* + h_1^{22} f_{21}^*]q$ .

Similarly to NT-AF, the ML decoder can be obtained as

$$(\tilde{x}_i, \tilde{x}_{i+2}) = \arg \min_{x_i, x_{i+2}} [\alpha_s (|x_i|^2 + |x_{i+2}|^2) + 2\Re[\beta_s^* x_i x_{i+2}^* - \eta_i x_i - \eta_{i+2} x_{i+2}]] , \text{ for } i = 1, 2$$

where  $\alpha_s = P_1 \|\mathbf{H}_1\|^2 + (P_3 s_{s1} + a^2 s_{s2}) / (xy - |u|^2 - |v|^2)$  and  $\beta_s = a\sqrt{P_3} e^{j\theta} z_s$ . Note that  $s_{s1} = (|h_1^{11}|^2 + |h_1^{22}|^2)x + (|h_1^{11}|^2 + |h_1^{21}|^2)y - 2\Re[(h_1^{11})^* h_1^{12} + (h_1^{21})^* h_1^{22}]u + ((h_1^{11})^* (h_1^{22})^* - (h_1^{12})^* (h_1^{21})^*)v]$ ,  $s_{s2} = (|h_2^{12}|^2 + |h_2^{22}|^2)x + (|h_2^{11}|^2 + |h_2^{21}|^2)y - 2\Re[(h_2^{11})^* h_2^{12} + (h_2^{21})^* h_2^{22}]u + ((h_2^{11})^* (h_2^{22})^* - (h_2^{12})^* (h_2^{21})^*)v]$ , and  $(xy - |u|^2 - |v|^2)z_s = 2y\Re[(h_1^{11})^* h_2^{11} + h_2^{21} (h_2^{21})^*] + 2x\Re[(h_1^{12})^* h_2^{12} + h_2^{22} (h_2^{22})^*] - 2\Re[(h_1^{11})^* h_2^{12} + (h_1^{21})^* h_2^{22} + h_1^{12} (h_2^{11})^* + h_1^{22} (h_2^{21})^*]u + ((h_1^{11})^* (h_2^{22})^* - (h_1^{21})^* (h_2^{12})^* + (h_2^{22})^* (h_2^{11})^* - (h_2^{12})^* (h_2^{21})^*)v]$ .

## V. NUMERICAL RESULTS

For simulations, all the channels are assumed to be quasi-static Rayleigh fading channels. QPSK is considered and the average power of the transmitted symbol is assumed to be 1. Equal power allocation is assumed. Thus, for OT,  $P_1 = P_2$  and for NT,  $P_1 = P_2 + P_3$  and  $P_2 = P_3$ . Note that  $P_T = P_1 + P_2$  for OT and  $P_T = P_1 + P_2 + P_3$  for NT, where  $P_T$  denotes total power per transmit antenna

Fig. 1 shows the performance of OT-AF and OT-SDF. ‘‘Optimal’’ means that D uses the optimal ML decoder derived in the previous section and ‘‘Squaring’’ means that D uses squaring method as shown in [6]. It is shown that SDF protocol outperforms AF protocol. Also, optimal decoder shows better performance than squaring decoder, although they have the same decoding complexity.

We consider SAS technique to achieve better performance for NT. Although, DF protocol is considered in [4], SAS can be applied to AF and SDF protocols. Fig. 2 shows the performance of NT-AF and NT-SDF. It can be seen that SDF

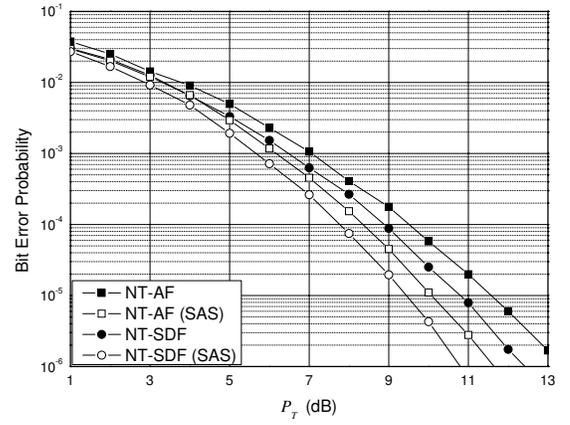


Fig. 2: Performance comparison of various non-orthogonal protocols.

protocol also shows better performance than AF protocol. Further, it can be seen that SAS technique enhances the diversity gain of the cooperative communication networks.

## VI. CONCLUSION

In this paper, we introduce the SDF protocol which decodes the received signals into the soft decision values of the received signals at R. Furthermore, we design DSTCs for the cooperation between S and R. In spite of correlated noises at D, we show that the ML decoders for AF and SDF protocols with both OT and NT can be simplified. It can be seen through simulations that SDF protocol shows better performance than AF protocol.

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