Partial Bit Inverted SLM Scheme for PAPR Reduction in QAM Modulated OFDM Signals

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***Abstract*—In this paper, we propose a new selected mapping (SLM) scheme for reducing peak to average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals modulated with quadrature amplitude modulation (QAM), called partial bit inverted SLM (PBISLM). The proposed scheme changes the magnitudes as well as the phases of QAM symbols by applying binary phase sequences to the binary data sequence before mapped to QAM symbols. Simulation results show that the proposed scheme has better PAPR reduction performance than the conventional SLM scheme for the QAM modulated OFDM signals, especially for the small number of subcarriers.**

I. Introduction

Since orthogonal frequency division multiplexing (OFDM) can support high data rate and provides high reliability to voice, data, and multimedia communications, it has been adopted as a standard in many wireless communication systems. One of main advantages of OFDM is the robustness against frequency selective fading or narrowband interference. However, the OFDM system suffers from high peak to average power ratio (PAPR) of time domain signal obtained by inverse fast Fourier transform (IFFT). If an OFDM signal has high PAPR, it causes significant signal distortion such as in-band distortion and out-of-band radiation in a nonlinear high power amplifier (HPA) [1]. There are many techniques to reduce PAPR such as clipping and filtering [2], tone reservation (TR) [3], partial transmit sequence (PTS) [4], and selected mapping (SLM) [5]. Selected mapping (SLM) selects the signal with the minimum PAPR among several candidate signals generated by multiplying phase sequences to the data sequence before IFFT.

In this paper, we review the conditions of phase sequences for good SLM scheme and analyze the relation between the independency of alternative signal sequences and the covariance of the average symbol powers of them. Based on these results, we propose a new SLM called partial bit inverted SLM (PBISLM) which generates the alternative symbol sequences by inverting data bits at predetermined bit positions or not according to the binary phase sequences. PAPR reduction performance of the PBISLM scheme is better than that of the conventional SLM scheme for the QAM modulated OFDM signals, especially for the small number of subcarriers.

II. Conventional SLM Scheme

An OFDM signal sequence $\mathbf{a} = [a_0\ a_1\ \cdots\ a_{N-1}]$ using N subcarriers can be expressed as

$$a_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j2\pi \frac{k}{N} n}, \quad 0 \le n < N \tag{1}$$

where $\mathbf{A} = [A_0\ A_1\ \cdots\ A_{N-1}]$ is an input symbol sequence usually modulated by using phase shift keying (PSK) or QAM and n stands for a discrete time index. The PAPR of the OFDM signal sequence $\mathbf{a}$ in the discrete time domain can be defined as

$$\mathrm{PAPR}(\mathbf{a}) \doteq \frac{\max\limits_{0\le n<N} |a_n|^2}{\mathrm{E}\left[|a_n|^2\right]} \tag{2}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator.

In the conventional SLM scheme, a transmitter generates U distinct alternative symbol sequences, all representing the same input symbol sequence, and selects the one with the minimum PAPR for transmission. To generate U alternative symbol sequences, an input symbol sequence is multiplied by U different phase sequences of length N, $\mathbf{P}^{(u)} = [P_0^{(u)}\ \ P_1^{(u)}\ \ \cdots\ \ P_{N-1}^{(u)}], 0 \le u < U$, where $P_i^{(u)} = e^{j\phi_i^{(u)}}$. Then, the alternative symbol sequences $\mathbf{A}^{(u)}$ = $[A_0^{(u)}\ \ A_1^{(u)}\ \ \cdots\ \ A_{N-1}^{(u)}], 0 \le u < U$, are generated by $A_i^{(u)} = A_i P_i^{(u)}$. After U alternative symbol sequences are transformed by IFFT, the alternative OFDM signal sequence $\mathbf{a}^{(\tilde{u})} = \mathrm{IFFT}(\mathbf{A}^{(\tilde{u})})$ with the smallest PAPR is selected for transmission.

If we assume that the alternative OFDM signal sequences $\mathbf{a}^{(u)}$ = $[a_0^{(u)}\ a_1^{(u)}\ \cdots\ a_{N-1}^{(u)}], 0 \le u < U$, are mutually independent, the complementary cumulative distribution function (CCDF) for the SLM scheme can be given [4] as

$$\Pr(\mathrm{PAPR}(\mathbf{a}^{(\tilde{u})}) > \mathrm{PAPR}_0) = (1-(1-e^{-\mathrm{PAPR}_0})^N)^U. \tag{3}$$

III. Conditions for Mutually Independent OFDM Signals

OFDM signal sequences obtained after IFFT can be assumed to be complex Gaussian distributed for large N by the

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cental limit theorem. Thus zero covariance of two alternative OFDM signals implies that they are mutually independent [6]. However, this assumption does not hold for small N because OFDM signal sequence for small N may not be approximated as complex Gaussian random vector. In this case, instead of covariance, we consider the property of joint cumulants of alternative OFDM signals such that two alternative OFDM signal sequences are mutually independent if the joint cumulants of all orders are equal to zero [7]. Since it is not easy to calculate high order joint cumulants, we will only consider joint cumulants up to the fourth order to investigate the independency of alternative OFDM signal sequences. Also, we will propose a new SLM scheme which makes the second and fourth order joint cumulants of any pair of alternative OFDM signals close to zero. Through numerical analysis, it will be shown that the PAPR reduction performance improves as the fourth order joint cumulant between alternative OFDM signal sequences decreases.

In general, the kth symbol of the uth alternative symbol sequence $\mathbf{X}^{(u)}$ can be expressed as

$$X_k^{(u)} = A_k \Lambda_k^{(u)} e^{j\phi_k^{(u)}}, \quad 0 \le k < N \quad \text{and} \quad 0 \le u < U \tag{4}$$

where $\Lambda_k^{(u)}$ and $\phi_k^{(u)}$ are the amplitude gain and phase rotation of the kth symbol in the uth alternative symbol sequence, respectively. Note that in the conventional SLM, $\Lambda_k^{(u)} = 1$ for all k and u. Then the uth alternative OFDM signal sequence is given as

$$x_n^{(u)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k \Lambda_k^{(u)} e^{j\phi_k^{(u)}} e^{j2\pi \frac{k}{N} n}, \quad 0 \le n < N. \tag{5}$$

The second order joint cumulant of two alternative OFDM signals, which is also the covariance of them is zero regardless of Λ_k if $\phi_k^{(u)}$ is i.i.d. with $E[e^{j\phi_k^{(u)}}] = 0$ for $u = 0, 1, ..., U-1$ [6]. If the phase sequences satisfy these i.i.d. and mean zero conditions, the fourth order joint cumulant of two alternative OFDM signal sequences can be given as

$$\begin{aligned}
&\text{cum}\left(x_k^{(l)}, x_k^{(l)*}, x_k^{(m)}, x_k^{(m)*}\right) \\
&= E\left[x_k^{(l)} x_k^{(l)*} x_k^{(m)} x_k^{(m)*}\right] - E\left[x_k^{(l)} x_k^{(l)*}\right] E\left[x_k^{(m)} x_k^{(m)*}\right] \\
&= \frac{1}{N^2}\left\{ E\left[\left(\sum_{k=0}^{N-1} |A_k \Lambda_k^{(l)}|^2\right)\left(\sum_{k=0}^{N-1} |A_k \Lambda_k^{(m)}|^2\right)\right] \right. \\
&\quad \left. - E\left[\sum_{k=0}^{N-1} |A_k \Lambda_k^{(l)}|^2\right] E\left[\sum_{k=0}^{N-1} |A_k \Lambda_k^{(m)}|^2\right] \right\}.
\end{aligned} \tag{6}$$

It can be easily shown that the fourth order joint cumulant in (6) is equivalent to the covariance of average symbol powers of alternative symbol sequences given by

$$\begin{aligned}
\text{cov}\left(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)}\right) &= E\left[\left(\bar{\mathrm{P}}^{(l)} - E\left[\bar{\mathrm{P}}^{(l)}\right]\right)\left(\bar{\mathrm{P}}^{(m)} - E\left[\bar{\mathrm{P}}^{(m)}\right]\right)\right] \\
&= E\left[\bar{\mathrm{P}}^{(l)}\bar{\mathrm{P}}^{(m)}\right] - E\left[\bar{\mathrm{P}}^{(l)}\right] E\left[\bar{\mathrm{P}}^{(m)}\right]
\end{aligned} \tag{7}$$

where

$$\bar{\mathrm{P}}^{(u)} = \frac{1}{N} \sum_{k=0}^{N-1} |A_k \Lambda_k^{(u)}|^2. \tag{8}$$

If $E[\bar{\mathrm{P}}^{(u)}]$ is normalized to one, we have

$$\begin{aligned}
&\text{cov}\left(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)}\right) = E\left[\bar{\mathrm{P}}^{(l)}\bar{\mathrm{P}}^{(m)}\right] - E\left[\bar{\mathrm{P}}^{(l)}\right] E\left[\bar{\mathrm{P}}^{(m)}\right] \\
&= \frac{1}{N^2}\left\{ E\left[N|A_k|^4 |\Lambda_k^{(l)}|^2 |\Lambda_k^{(m)}|^2\right] \right. \\
&\quad \left. + E\left[N(N-1)|A_i \Lambda_i^{(l)}|^2 |A_k \Lambda_k^{(m)}|^2\right] - N^2 \right\} \\
&= \frac{1}{N^2}\left(N E[|A_k|^4 |\Lambda_k^{(l)}|^2 |\Lambda_k^{(m)}|^2] + N(N-1) - N^2\right) \\
&= \frac{1}{N}\left(E[|A_k|^4 |\Lambda_k^{(l)}|^2 |\Lambda_k^{(m)}|^2] - 1\right).
\end{aligned} \tag{9}$$

For simplicity, we assume that the average symbol power of OFDM symbol sequences modulated with M-QAM is normalized to one hereafter.

If $\mathbf{X}^{(l)}$ and $\mathbf{X}^{(m)}$ are mutually independent, that is, two alternative symbol sequences are generated independently for a given modulation, the covariance of average symbol powers of them is zero. However, the covariance is not zero in the conventional SLM even if the phase sequences satisfy the optimality conditions for phases because $E[|A_k|^4|]$ in (9) is not one, which means that mutually independent alternative OFDM signal sequences cannot be generated by using the conventional SLM scheme for QAM modulation. Therefore, we have to change the amplitude gain $\Lambda_k^{(u)}$ to make $E[|A_k|^4 |\Lambda_k^{(l)}|^2 |\Lambda_k^{(m)}|^2]$ close to one.

IV. Partial Bit Inverted SLM Scheme

In this section, we propose a new SLM scheme called PBISLM. Let $\mathbf{S} = \{i_0, i_1, ..., i_{W-1}\}$ denote a subset of bit indices $\mathbf{L} = \{0, 1, ..., \log_2 M - 1\}$ for M-QAM symbol and $\mathbf{S}^{\mathrm{C}}$ be the complement set of $\mathbf{S}$ in $\mathbf{L}$. The lth bit $X_{k,l}^{(u)}$ of the kth symbol in binary form of the uth alternative symbol sequence can be written as

$$X_{k,l}^{(u)} = \begin{cases} A_{k,l} P_k^{(u)}, & l \in \mathbf{S} \\ A_{k,l}, & l \in \mathbf{S}^{\mathrm{C}} \end{cases} \tag{10}$$

where $P_k^{(u)} = \{\pm 1\}$. If $P_k^{(u)}$ is -1, the bits of A_k corresponding to $\mathbf{S}$ are inverted and $A_k^{(u)}$ is mapped to other M-QAM symbol $X_k^{(u)}$. After the alternative symbol sequences $\mathbf{X}^{(u)}$ are IFFTed, the OFDM signal sequence $\mathbf{x}^{(\tilde{u})} = \text{IFFT}(\mathbf{X}^{(\tilde{u})})$ with the lowest PAPR is selected for transmission.

In the PBISLM scheme, the average power $\bar{\mathrm{P}}^{(u)}$ of $\mathbf{X}^{(u)}$ is different from the average power of $\mathbf{A}$ for the OFDM signal with M-QAM and depends on the selection of the set $\mathbf{S}$ for the given constellation mapping. From (9), we have to make $E[|X_k^{(l)}|^2 |X_k^{(m)}|^2]$ as close to one as possible to have the covariance $\text{cov}(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)})$ close to zero. For some M-QAM symbol mappings, we analyze the covariance of average symbol powers of alternative symbol sequences

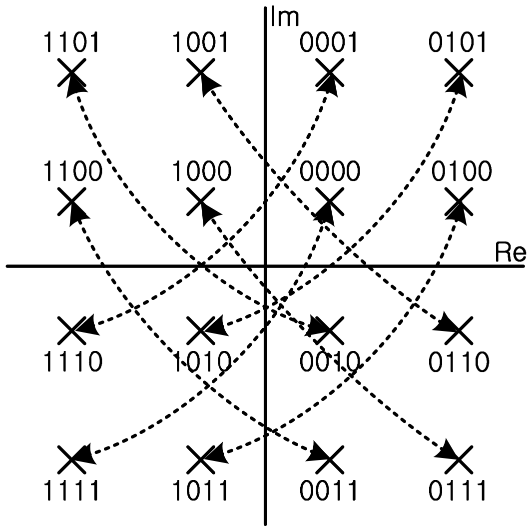

Fig. 1. An example of constellation mapping of 16-QAM with Gray mapping in PBISLM.

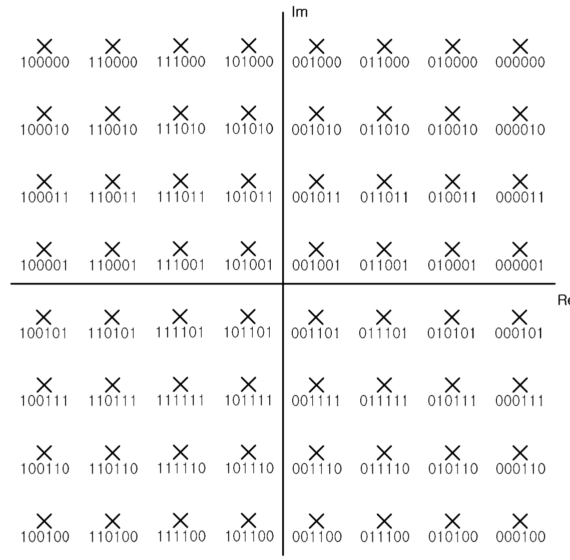

Fig. 2. An example of 64-QAM constellation with Gray mapping.

in PBISLM as follows. Fig. 1 shows an example of 16-QAM constellation mapping with Gray mapping in PBISLM. If we use $\mathbf{S} = \{0,1,2,3\}$ and $\mathbf{S}^{\mathrm{C}} = \emptyset$, all bits for the kth input symbol are inverted when $P_k^{(u)} = -1$. Assuming that $E[|A_k|^2] = 1$, the input symbols A_k are classified into three groups E_1, E_2, and E_3 according to their powers such that $E_1 = \{0000, 1000, 1010, 0010\}$ with $P_1 = 0.2$, $E_2 = \{0100, 0001, 1001, 1100, 1110, 1011, 0011, 0110\}$ with $P_2 = 1.0$, and $E_3 = \{0101, 1101, 1111, 0111\}$ with $P_3 = 1.8$. Then the amplitude gain of the symbol in alternative symbol sequence generated by PBISLM is

$$\Lambda_k^{(u)} = \begin{cases} 1, & A_k \in E_2 \text{ or } P_k^{(u)} = 1 \\ \sqrt{P_1/P_3}, & A_k \in E_3 \text{ and } P_k^{(u)} = -1 \\ \sqrt{P_3/P_1}, & A_k \in E_1 \text{ and } P_k^{(u)} = -1. \end{cases} \tag{11}$$

If the phase sequences $\mathbf{P}^{(u)}$ are randomly generated and they are balanced in terms of the number of $+1$'s and -1's, $E[e^{j\phi_k^{(u)}}]$ is also zero. Then the covariance of average symbol powers of two alternative symbol sequences in the PBISLM is calculated as

$$\begin{aligned} \mathrm{cov}\left(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)}\right) &= \frac{1}{N}\left(E\left[|A_k|^4 |\Lambda_k^{(l)}|^2 |\Lambda_k^{(m)}|^2\right] - 1\right) \\ &= \frac{1}{N}\left\{\frac{1}{2}\left(\frac{1}{4}(P_1)^2 \frac{P_3}{P_1} + \frac{1}{2} + \frac{1}{4}(P_3)^2 \frac{P_1}{P_3}\right)\right. \\ &\quad \left. + \frac{1}{2}\left(\frac{1}{4}(P_1)^2 + \frac{1}{2} + \frac{1}{4}(P_3)^2\right) - 1\right\} \\ &= 0. \end{aligned} \tag{12}$$

Therefore, the average symbol power of alternative symbol sequences in the PBISLM is uncorrelated as the case of independently generated symbol sequences.

Now, we consider PBISLM for 64-QAM and Fig. 2 shows a 64-QAM constellation with Gray mapping. In order to compare the covariance of average symbol powers of two alternative symbol sequences, we introduce two types of PBISLM according to the selection of $\mathbf{S}$. Let Type-I be $\mathbf{S} = \{0,2,3,5\}$ and $\mathbf{S}^{\mathrm{C}} = \{1,4\}$, and Type-II be $\mathbf{S} = \{0,1,2,3,4,5\}$ and $\mathbf{S}^{\mathrm{C}} = \emptyset$. The covariance of average symbol powers of two alternative symbol sequences in Type-I is smaller than that in Type-II, that is, 2.83×10^{-4} for Type-I and 1.13×10^{-3} for Type-II for $N = 64$, which are calculated similarly to (12). The PAPR reduction performance of these two types of PBISLM are compared through numerical analysis in the next section.

It is clear that PBISLM preserves the distance property of Gray mapping after removing the binary phase sequence $\mathbf{P}^{(\tilde{u})}$ at the receiver. Therefore, the BER performance is not deteriorated by using the proposed scheme. Also, PBISLM does not increase the computational complexity and the size of side information compared to the conventional SLM.

V. Simulation Results

In this section, we compare the covariance of average symbol powers of two alternative symbol sequences for the conventional SLM and PBISLM and also compare the PAPR reduction performance of them for $U = 8$. The rows of cyclic Hadamard matrix are used for phase sequences [8] and the all-1 sequence is used for $u = 0$ to include the original input symbol sequence among the alternative symbol sequences.

Fig. 3 compares the covariance of average symbol powers of two alternative symbol sequences for $N = 64$, 128, 256, and 512 when 16-QAM and 64-QAM are used. The conventional SLM scheme has the largest covariance and the covariance for 64-QAM is larger than that for 16-QAM. In case of PBISLM for 64-QAM, covariance in Type-I is smaller than that in Type-II. It is clear that the covariance in PBISLM with Type-I is close to zero for any N and any M.

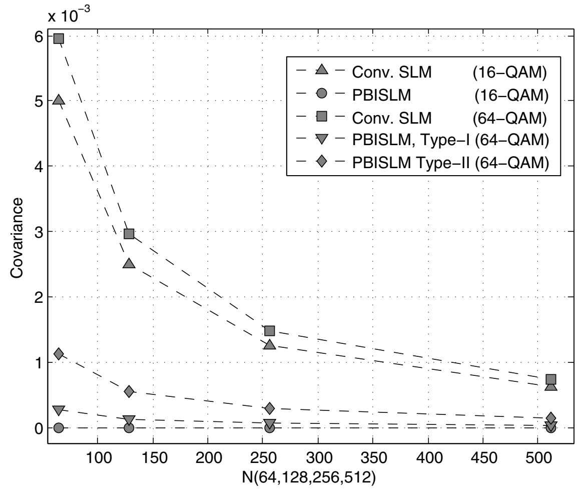

Fig. 3. Comparison of the covariance of average symbol powers of two alternative symbol sequences for the conventional SLM and PBISLM.

Fig. 4 compares the PAPR reduction performance of various SLM schemes for $N = 64$ and 256. Note that oversampling is not performed when PAPR is obtained in order to compare the results with the theoretical CCDF in (3). The conventional SLM scheme shows the worst performance, especially for 64-QAM, but the performance gap with other schemes decreases as N increases. The PAPR reduction performance of PBISLM with Type-II is slightly worse than that of PBISLM with Type-I for $N = 64$. The PAPR reduction performance of PBISLM with Type-I is almost identical to the theoretical CCDF curves in (3). Note that the PAPR reduction performance shows similar trend as the covariance given in Fig. 3.

VI. Conclusion

We proposed a new SLM scheme for PAPR reduction in QAM modulated OFDM signals called PBISLM scheme. The proposed scheme modifies the magnitude as well as the phase by applying binary phase sequence to the input symbol sequence in the binary form to make alternative OFDM signal sequences more independent. The proposed scheme does not increase the computational complexity and the amount of side information compared to the conventional SLM. Simulation results show that the proposed scheme has better PAPR reduction performance than the conventional SLM, which is close to the theoretical performance. The improvement in the PAPR reduction performance of the proposed SLM scheme increases as U increases and N decreases for QAM modulated OFDM signals. Furthermore, the proposed scheme achieves shaping gain which can improve BER performance.

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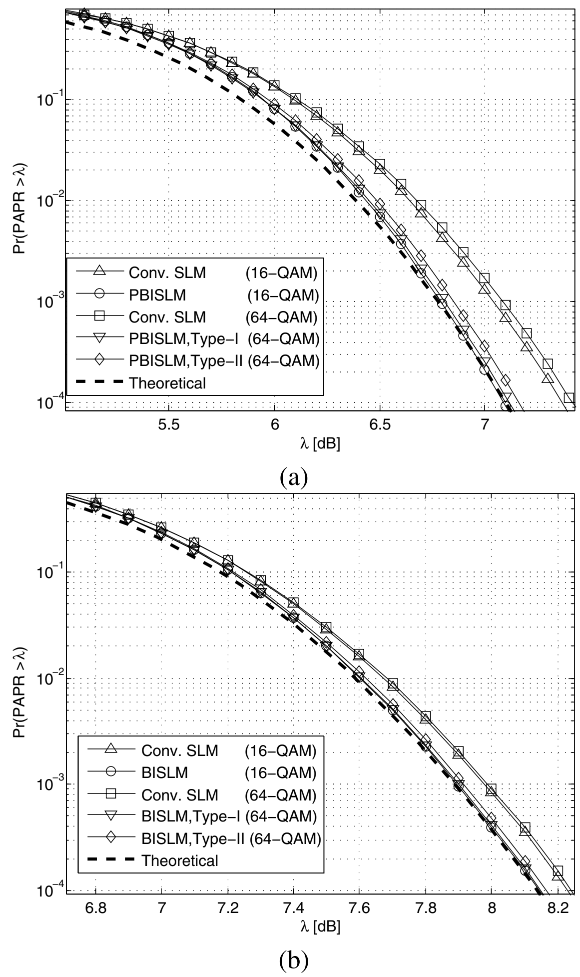

Fig. 4. Comparison of PAPR reduction performance of the conventional SLM, PBISLM, and theoretical CCDF of (3) for $U = 8$. (a) $N = 64$. (b) $N = 256$.

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