A New Criterion for Retransmission in Type I H-ARQ Schemes of LDPC Coded OFDM Systems

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***Abstract*—In this paper, a new criterion for reordering of low-density parity-check (LDPC) coded orthogonal frequency division multiplexing (OFDM) subframes is proposed for type I hybrid automatic repeat request (H-ARQ) systems. It is verified from numerical analysis that a subframe reordering pattern having larger channel capacity shows better bit error rate (BER). Also, it is shown that the subframe reordering pattern achieving equal combined power allocation for each subframe maximizes the channel capacity and outperforms other subframe reordering patterns in terms of BER performance. Simulation results are provided to confirm that for a very slow varying fading channel, the proposed subframe reordering scheme for achieving equal combined power allocation gives better BER performance than the conventional Chase combining scheme without increasing the decoding complexity.**

I. Introduction

Recently, the orthogonal frequency division multiplexing (OFDM) and low-density parity-check (LDPC) codes are considered as attractive techniques for the next generation mobile communication systems. It has been known that the performance of an OFDM system over frequency selective fading channels is better than that of a single carrier modulation system because one tap equalizer can be used [1]. Also, OFDM systems are suitable for achieving high data transmission rate in the wireless communication systems.

The employment of LDPC codes [2] in the wireless communication systems has contributed to achieve the reliability required by high-speed digital communication systems. Especially, the block-type LDPC (B-LDPC) codes [3] enable linear time encoding and efficient decoding to be implemented easily by using a few hardware memories. In practice, the OFDM systems using LDPC codes draw attention as standard platforms for various communication and broadcasting systems. For example, they are included in IEEE 802.16e standard [4] for broadband wireless access systems and adopted as a standard for the wireless local area network (WLAN) and digital video broadcasting (DVB).

The type I hybrid automatic repeat request (H-ARQ) protocol requests the retransmission of the same packet as the previous transmission when the decoding is failed. In the decoding after retransmission, the packet of previous transmission with errors is not used. Chase combining [5] is the simplest combining scheme for the type I H-ARQ. When a received packet fails in decoding, the packet is required to be retransmitted. After the related packet is retransmitted to the receiver, it is added to the previous received packet symbol by symbol and the decoding is performed using the combined values.

In this paper, we divide a packet into several subframes of the same size and investigate the performance gain by adjusting the transmission order of subframes of the packet in the H-ARQ system. Thus, it is shown that the subframe reordering pattern achieving equal combined power allocation for each subframe maximizes the channel capacity and outperforms other subframe reordering patterns in terms of bit error rate (BER) performance. Also, the simulation results are provided to confirm that the performance gain of the proposed subframe reordering scheme with equal combined power allocation can be obtained without increasing the decoding complexity.

This paper is organized as follows. In Section II, after the system modeling for the H-ARQ system is given, the subframe reordering scheme is proposed for the cases with channel state information (CSI) or without CSI. In Section III, it is verified from a new criterion for reordering of subframes that the subframe reordering pattern achieving equal combined power allocation shows the best performance. Finally, in Section IV, it is shown from simulation results that the proposed subframe reordering scheme with equal combined power allocation gives better performance than the conventional Chase combining scheme. The conclusion is given in Section V.

II. System Model and Subframe Reordering Scheme

Two-ray propagation model in [6] is considered as a simple channel model for multipath fading. The channel is also assumed a very slow varying frequency selective channel such as the channel for WLAN in the indoor or pedestrian environments. Each symbol of a packet, an LDPC codeword, is allocated to each subcarrier of an OFDM system and the guard band is not taken into consideration.

If the frequency selective fading is described as two-ray propagation model in [6], the channel coefficient in the frequency domain can be expressed as

$$h(s) = 1 + \exp(j\theta)\exp\left(j\frac{d \times 2\pi s}{N_s}\right) \tag{1}$$

where θ is a uniform random variable in the interval $[0, 2\pi)$, N_s is the number of symbols in a packet, and d is the number of deep fadings in a packet duration.

In this paper, the results for the single deep fading (SDF) case with $d = 1$ are discussed mainly.

Now, the subframe reordering scheme is proposed, which achieves equal combined power allocation for each subframe for the case with or without CSI.

A. Subframe Reordering Scheme with Channel State Information

Let us suppose the OFDM system where CSI can be obtained from channel estimation. In the first transmission, each packet is divided into M subframes with the same size. From the CSI, the power for symbols of each subframe is calculated. Then M subframes are reordered such that the message parts and parity parts of a packet are assigned to the subframes with good and bad channel conditions, respectively, and then they are transmitted in that order.

The receiver carries out decoding for the reordered packet. If the decoding of a packet is failed, the retransmission is requested. In the process of retransmission, in contrast with the first transmission, the message parts and the parity parts of a packet are assigned to the subframes with bad and good channel conditions, respectively, to achieve equal combined power allocation for each subframe. If more retransmissions are required, the reordering patterns of the first and second transmissions are alternately applied.

B. Subframe Reordering Scheme without Channel State Information

If the CSI is not available at the transmitter, the first transmission is carried out without considering any specific ordering of subframes. If the second transmission is required, the subframe reordering scheme is applied as follows. Each packet (frame) is divided into M subframes with the same size in the frequency domain. For the given fading channel, the subframe reordering patterns are determined in advance such that the average power of each combined subframe in the frequency domain becomes as equal as possible. Then, under the H-ARQ system, the subframes in a packet are reordered and retransmitted over the same channel to equalize the accumulated sum of average power of each subframe because we consider a very slow varying fading channel. Here, it is assumed that the channel model is already known. Thus, the subframe reordering pattern can be known to both the transmitter and the receiver.

Through a lot of numerical experiments, it is predicted that the non-equal power allocation in each subframe in LDPC codes degrades the performance of the OFDM system. From the above motivation, the reordering scheme of the retransmission to guarantee the equal power allocation over the combined subframes in a packet after retransmission is proposed and the performance can be expected to be improved because the most of the deep faded subframes can be removed by equal-power combining of subframes. More details are presented in the next section by using the channel capacity.

When the decoder encounters errors in a packet which cannot be corrected in the first transmission, the OFDM system needs a retransmission. The retransmission is performed according to previously determined order of subframes in a packet and the decoding process is performed for the combined received signals. If the third and fourth transmissions are requested for the reliable communication, the subframe reordering scheme based on the equal power allocation of the combined signals can also be applied.

The H-ARQ system with CSI may increase the overhead by using feedback channel and thus it cannot be preferred practically. Therefore, the proposed subframe reordering scheme without CSI can be valuable.

III. A Criterion for Subframe Reordering

In this section, we propose a "channel capacity criterion" for reordering of OFDM subframes, which gives the good performance in H-ARQ systems by an equal combined power allocation.

The mutual information [7] can be computed in the binary input additive white Gaussian noise (BIAWGN) channel with input X and output Y as

$$I(X;Y) = \iint f(x,y) \log \frac{f(x,y)}{f(x)f(y)} dxdy$$

where $f(x)$ and $f(y)$ denote the probability density functions of random variables X and Y, respectively, and $f(x,y)$ stands for the joint probability density function. The channel capacity is the maximum mutual information and the mutual information $I(X;Y)$ is maximum on the condition that the random variable $X \in \{\pm 1\}$ is uniformly distributed.

Therefore, the channel capacity C can be derived as

$$\begin{aligned} C &= \max \{I(X;Y)\} \\ &= 1 - \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(y-\sqrt{E_s}\right)^2}{2\sigma^2}\right) \\ &\quad \times \log\left[1+\exp\left(-\frac{2\sqrt{E_s}}{\sigma^2}y\right)\right] dy \end{aligned} \tag{2}$$

where E_s and σ^2 denote the symbol energy and noise variance, respectively. Also, it is known that $E_s = |h_1(s)|^2 + |h_2(s)|^2$ and $\sigma^2 = \left(|h_1(s)|^2 + |h_2(s)|^2\right)\sigma_n^2$ in H-ARQ systems. Here, h_1 and h_2 are the channel coefficients of the first and second transmissions, respectively.

Now, we pay attention to reordering of subframes in a packet to maximize the channel capacity. For that purpose, the power of each subframe is calculated. But each subframe power cannot be computed directly for the systems without CSI, because θ in (1) is unknown. Thus, in this case, the power is computed in the average sense. First, we can calculate $|h(s)|^2$ using (1) as

$$|h(s)|^2 = 2 + 2\cos\left(\theta + \frac{d \times 2\pi s}{N_s}\right). \tag{3}$$

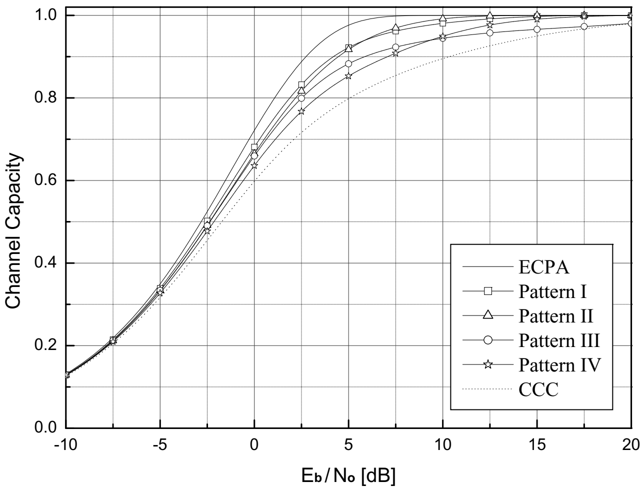

Fig. 1. Channel capacity according to various subframe reordering patterns.

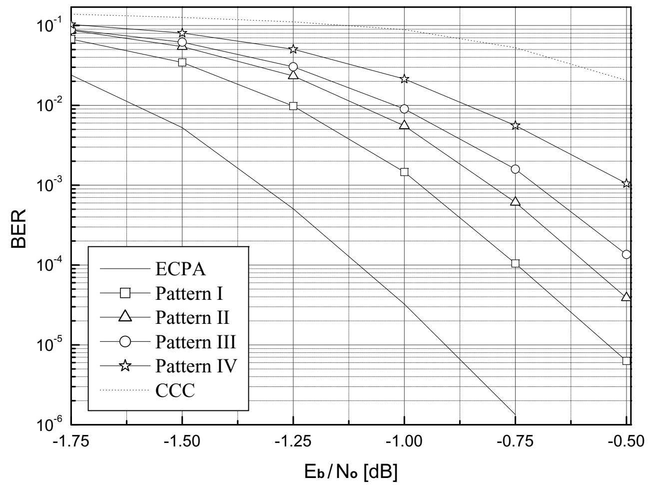

Fig. 2. BER performance according to various subframe reordering patterns.

Therefore, the average power of each subframe can be derived as

$$p_i = \frac{M}{N_s} \int_{\frac{2\pi}{M} i}^{\frac{2\pi}{M}(i+1)} \sum_{s=0}^{\frac{N_s}{M}-1} |h(s)|^2 d\theta \tag{4}$$

where $p_i,\ 0 \le i \le M-1$, stands for the average power of i-th subframe and M denotes the number of subframes in a packet. Also, each subframe includes N_s/M binary phase-shift keying (BPSK) symbols.

Using (3) and (4), the average power of each subframe can be expressed as

$$p_i = \frac{M}{N_s} \int_{\frac{2\pi}{M} i}^{\frac{2\pi}{M}(i+1)} \sum_{s=0}^{\frac{N_s}{M}-1} \left[2 + 2\cos\left(\theta + \frac{d \times 2\pi s}{N_s}\right)\right] d\theta. \tag{5}$$

Using (2) and (5), the channel capacity of the H-ARQ scheme with the subframe reordering patterns can be computed.

Figs 1 and 2 show the channel capacity and BER performance for various subframe reordering patterns in H-ARQ systems, respectively. Here, the ECPA and CCC stand for the patterns of equal combined power allocation and conventional Chase combining, respectively. In the numerical analysis, we consider the SDF channel with $d = 1$ in (1). Also, B-LDPC codes for correcting errors are randomly generated $(3, 6)$ regular codes with the code length $N_s = 2304$ and girth (shortest cycle) 6. It is assumed that a packet with $M = 24$ is transmitted using BPSK modulation. In the first transmission, the subframe reordering is carried out in the sequential order (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24). Thus, it can be confirmed from Fig. 1 that the channel capacity is listed in the order of the equal combined power allocation (13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12), retransmitted pattern I (10, 8, 15, 19, 17, 18, 3, 13, 2, 22, 20, 6, 23, 12, 9, 21, 1, 5, 7, 24, 16, 14, 11, 4), pattern II (13, 14, 2, 9, 5, 19, 18, 1, 3, 21, 8, 17, 15, 7, 16, 24, 10, 4, 11, 12, 6, 20, 23, 22), pattern III (21, 8, 7, 22, 1, 20, 4, 2, 3, 11, 17, 12, 19, 23, 10, 24, 13, 18, 6, 16, 9, 15, 14, 5), pattern IV (16, 5, 24, 18, 21, 7, 6, 4, 12, 8, 13, 15, 9, 10, 1, 17, 2, 3, 23, 19, 22, 11, 20, 14), and conventional Chase combining (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24). As expected, it can be confirmed from Fig. 2 that BER performance depends on channel capacity. In other words, it can be understood that a subframe reordering pattern having larger channel capacity shows better BER. Also, it is verified that the subframe reordering pattern achieving equal combined power allocation for each subframe maximizes the channel capacity and outperforms other subframe reordering patterns in terms of BER performance.

IV. Simulation Results

Fig. 3 compares the frame error rate (FER) performance of the subframe reordering schemes with that of the conventional Chase combining scheme in the H-ARQ scheme. We consider the rate-1/2 LDPC codes in IEEE 802.16e standard [4] with code length 2304 as the error correcting codes and H-ARQ with maximum 4 transmissions. Here, the number of subframes in a packet is selected with $M = 8$. Also, we assume that a packet is transmitted using the 16-QAM as a modulation method and the two-ray propagation model given in (1) with $d = 1$ is employed as a simple channel model.

Fig. 3 shows that the proposed subframe reordering H-ARQ schemes by the channel capacity give better performance than the conventional Chase combining schemes whether to use CSI or not. Also, it is shown that a proposed subframe reordering method using CSI outperforms a retransmission method without reordering ($\{M_1, M_2, M_3, M_4, P_1, P_2, P_3, P_4\}$) in the first transmission. Here, M and P stand for message and parity part in a packet, respectively. Also, for the rate-1/2 LDPC codes, the eight subframes in a packet are split into two parts corresponding to message and parity parts with four subframes, respectively. But, in the second transmission, the FER

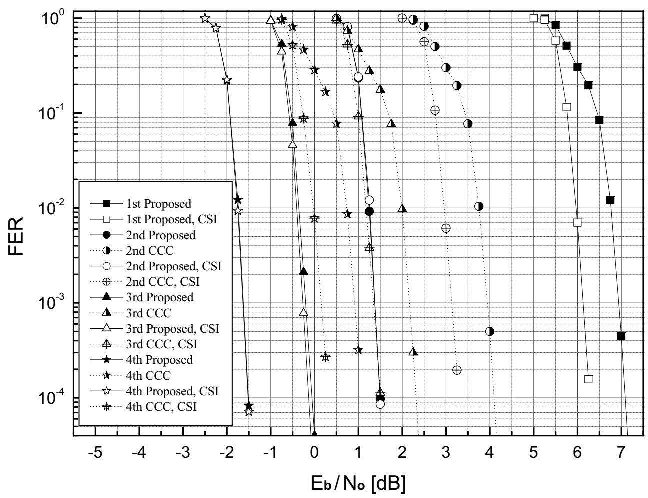

Fig. 3. Performance comparison of subframe reordering schemes.

performance of OFDM systems with the proposed H-ARQ does not depends on whether CSI is used or not. It is shown that the subframe reordering to equalize combined power of each subframe provides the same performance compared with the scheme using CSI.

In this case, we can calculate the average power of each subframe to determine the reordering pattern. Since the CSI is unknown, θ is unknown and thus the power is computed in the average-sense. For SDF case, $|h(s)|^2$ is expressed as

$$|h(s)|^2 = 2 + 2\cos\left(\theta + \frac{2\pi s}{N_s}\right). \tag{6}$$

Therefore, the average power of each subframe can be derived as

$$p_i = \frac{1}{72}\int_{\frac{\pi}{4}i}^{\frac{\pi}{4}(i+1)} \sum_{s=0}^{71} |h(s)|^2 d\theta. \tag{7}$$

From (6) and (7), it can be represented as

$$p_i = \frac{1}{72}\int_{\frac{\pi}{4}i}^{\frac{\pi}{4}(i+1)} \sum_{s=0}^{71} \left[2 + 2\cos\left(\theta + \frac{2\pi s}{N_s}\right)\right] d\theta$$

where $p_i, 0 \leq i \leq 7$, is the average power of i-th subframe and each subframe includes 72 $(= 576/8)$ 16-QAM symbols. Based on the above power computation, in the second transmission, we can determine the subframe reordering pattern to equalize the combining power such as $\{P_1, P_2, P_3, P_4, M_1, M_2, M_3, M_4\}$.

In the third transmission, the similar ordering pattern as the first transmission can be selected and the FER performance is given in Fig. 3. Since the averaging effect on the given channel is completed after the second transmission and thus it is not possible to equalize the power of each subframe in the third transmission, the subframe reordering scheme without CSI degrades the performance compared with the proposed scheme with CSI. However, the gain of channel averaging effect by the equal power allocation is almost saturated with the reordering pattern of the second transmission. Thus, in the third transmission, the difference of performance can be ignored between two schemes. In the last transmission, the FER performance of two schemes is the same because the equal power combining is achieved with the fourth transmission of H-ARQ system.

Therefore, the proposed subframe reordering scheme for achieving equal combined power allocation gives better performance than the conventional Chase combining scheme whether to use CSI or not. Moreover, it need not to increase the decoding complexity. Also, even though CSI is not provided in a system, H-ARQ system with subframe reordering shows nearly the same reliability for both cases of CSI and no CSI without the increase of the complexity.

V. Conclusion

In this paper, we propose a new criterion for reordering schemes of LDPC coded OFDM subframes in type I H-ARQ systems. First, it is verified from numerical analysis that a subframe reordering pattern having larger channel capacity shows better BER performance. Next, assuming the very slow varying fading channel in the OFDM system, the conventional Chase combining is modified to the subframe reordering by using the channel capacity. Thus, it is shown that the subframe reordering pattern achieving equal combined power allocation for each subframe maximizes the channel capacity and outperforms other subframe reordering patterns in terms of BER performance. Also, through numerical results, it is confirmed that the proposed subframe reordering scheme for achieving equal combined power allocation gives better BER performance than the conventional Chase combining scheme without increasing the decoding complexity. Moreover, the proposed scheme has the advantage that it need not transmit the CSI which may increase the overhead. Thus, even though CSI is not provided in the OFDM system, H-ARQ system with subframe reordering shows nearly the same reliability compared to the case with CSI.

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