Peak to Average Power Ratio Reduction for OFDM Systems With Low Complexity

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) has gained popularity as a standard for various high data rate wireless communication systems due to the spectral bandwidth efficiency, robustness to frequency selective fading channels, etc. However, its high peak-to-average power ratio (PAPR) causes significant in-band distortion and high out-of-band radiation when it passes through a nonlinear device such as a high power amplifier (HPA). This paper reviews the main PAPR reduction schemes and introduces their modifications for achieving low-complexity required for practical implementation in wireless communication systems.**

I. Introduction

Recently, orthogonal frequency division multiplexing (OFDM) has been regarded as one of the core technologies for various communication systems. Especially, OFDM has been adopted as a standard for various wireless communication systems such as wireless local area networks, wireless metropolitan area networks, digital audio broadcasting, and digital video broadcasting. It is widely known that OFDM is an attractive technique for achieving high data transmission rate in wireless communication systems and it is robust to the frequency selective fading channels. However, an OFDM signal has a high peak-to-average power ratio (PAPR) at the transmitter, which causes signal distortion such as in-band distortion and out-of-band radiation due to the nonlinearity of the high power amplifier (HPA) [1]. In general, HPA requires a large backoff from the peak power to reduce the distortion caused by the nonlinearity of HPA and this gives rise to a low power efficiency, which is a significant burden, especially in mobile terminals. The large PAPR also requires a large amount of dynamic range of data which results in the increased complexity of the transmitter and the receiver. Thus, PAPR reduction is one of the most important research areas in OFDM systems.

Several techniques have been proposed to mitigate the PAPR problem in OFDM signals. Clipping and filtering (CAF), tone reservation (TR), and active constellation extension (ACE) are the iterative schemes which reduce PAPR in time domain and apply some constraints in frequency domain. Selected mapping (SLM) and partial transmit sequence (PTS) are probabilistic scheme that generate several candidate signals and select one with the minimum PAPR for transmission.

The aim of this paper is to review the conventional PAPR reduction schemes and the various modifications of the conventional PAPR reduction schemes for achieving a low computational complexity.

This paper is organized as follows: Section II defines PAPR and analyzes the characteristics of PAPR. Then, we present the iterative PAPR reduction schemes and review their low-complexity implementation in Section III. The probabilistic PAPR reduction schemes and their modifications for low-complexity are discussed in Section IV. Finally, the concluding remarks are given in Section V.

II. OFDM System and PAPR

The OFDM signal sequence $\mathbf{a} = [a_0, a_1, \cdots, a_{N-1}]$ in discrete time domain can be expressed as

$$a_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j2\pi \frac{k}{N} n}, \quad 0 \le n < N \tag{1}$$

where $\mathbf{A} = [A_0, A_1, \cdots, A_{N-1}]^T$ is an input symbol sequence usually modulated by phase shift keying (PSK) or quadrature amplitude modulation (QAM) and n stands for a discrete time index.

OFDM signal is usually oversampled to represent analog OFDM signal as closely as possible because nonlinear processing such as HPA is done for the analog OFDM signal. Let J be an oversampling factor. Then the oversampled OFDM signal can be generated by inverse fast Fourier transform (IFFT) of the input symbol sequence padded with $(J-1)N$ zeros at the end of the sequence, that is, $\mathbf{A} = [A_0, A_1, \cdots, A_{N-1}, \underbrace{0, \cdots, 0}_{(J-1)N}]^T$. Then, the oversampled OFDM signal is expressed as

$$a_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j2\pi \frac{k}{JN} n}, \quad 0 \le n < JN. \tag{2}$$

Then, PAPR of the oversampled OFDM signal is defined as

$$\mathrm{PAPR}(\mathbf{a}) \doteq \frac{\max\limits_{0 \le n < JN} |a_n|^2}{\mathrm{E}\left[|a_n|^2\right]} \tag{3}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator.

III. Iterative PAPR Reduction Scheme With Low-Complexity

A. Clipping and Filtering

Clipping the OFDM signal is usually realized by soft envelope limiter as

$$\bar{a}_n = g(a_n) = \begin{cases} a_n, & |a_n| \leq A_{th} \\ A_{th} e^{j\theta_n}, & |a_n| > A_{th} \end{cases} \tag{4}$$

where $a_n = |a_n| e^{j\theta_n}$ and positive value A_{th} is the threshold level. Clipping method guarantees to reduce PAPR to the threshold level. However, it introduces in-band distortion and out-of-band (OOB) radiation by distorting amplitude nonlinearly. The former increases bit error rate (BER) and the latter causes interference to the signals in the neighboring channel.

CAF is the iterative scheme such that the clipped signal $\bar{a}_n$ is transformed to the frequency domain symbol $\bar{A}_k$ by FFTing $\bar{a}_n$ and $\bar{A}_n$'s in OOB, that is, $N \leq n < JN$ are set to zero, which is the operation of OOB radiation removal. Since removing OOB radiation causes peak regrowth again, this clipping and filtering process is iterated until PAPR meets the threshold level or the number of iterations reaches the predetermined maximum number. CAF is very simple but many fast Fourier transform (FFT) and IFFT operations are required to achieve the desired PAPR reduction performance.

B. Tone Reservation

The TR scheme reserves some tones for generating a PAPR reduction signal instead of data transmission [3]. Let $\mathcal{R} = \{i_1, i_2, \cdots, i_W\}$ denote the ordered set of the positions of the reserved tones and $\mathcal{R}^c$ denote the complement set of $\mathcal{R}$ in $\mathcal{N} = \{0, 1, \cdots, N-1\}$, where N and W are numbers of subcarriers and reserved tones, respectively. The input symbol A_k is expressed as

$$A_k = X_k + C_k = \begin{cases} C_k, & k \in \mathcal{R} \\ X_k, & k \in \mathcal{R}^c \end{cases}$$

where X_k is the data symbol with 0 in the set $\mathcal{R}$ and C_k is the PAPR reduction symbol with 0 in the set $\mathcal{R}^c$. Let $a_n,\ x_n$, and c_n be the time domain signals obtained by IFFTing $A_k,\ X_k$, and C_k, respectively. Since IFFT is a linear operation, the OFDM signal a_n corresponds to the summation of the data signal x_n and the PAPR reduction signal c_n, i.e., $a_n = x_n + c_n$.

Next, we consider the method of generation of peak reduction signals. Peak reduction signals are iteratively generated as follows. Let $\mathbf{p} = [p_0\ p_1\ \cdots\ p_{N-1}]^T$ be the time domain kernel defined by

$$p_n = \frac{1}{\sqrt{N}} \sum_{k \in \mathcal{R}} P_k e^{j2\pi \frac{k}{N} t}$$

where $\mathbf{P} = [P_0\ P_1\ \cdots\ P_{N-1}]^T$ is called peak reduction tone (PRT) with $P_k = 0$ for $k \in \mathcal{R}^c$. The time domain kernel $\mathbf{p}$ is used to compute the PAPR reduction signal sequence $\mathbf{c}$ iteratively [3]. That is, the PAPR reduction signal sequence $\mathbf{c}^l$ at the lth iteration is obtained as

$$\mathbf{c}^l = \sum_{i=1}^{l} \alpha_i \mathbf{p}_{((\tau_i))} \tag{5}$$

where $\mathbf{p}_{((\tau_i))}$ denotes a circular shift of $\mathbf{p}$ by τ_i and α_i is a complex scaling factor computed according to the threshold level and the maximum peak value at the ith iteration. The circular shift τ_i is determined as

$$\tau_i = \underset{0 \leq n \leq N-1}{\operatorname{argmax}} |x_n + c_n^{i-1}|.$$

Then, the OFDM signal in the TR scheme can be represented as

$$\mathbf{a} = \mathbf{x} + \mathbf{c}^l. \tag{6}$$

If the maximum number of iterations is reached or the desired peak power is obtained, iteration stops. For simplicity, we assume that only one maximum peak of the OFDM signal is reduced per iteration in (6).

A multi-stage TR scheme was proposed in order to achieve a low PAPR that has a reduced data rate loss [7]. The multi-stage TR scheme adaptively selects one of several PRT sets according to the PAPR of the OFDM signal while the PRT set is fixed for the conventional TR scheme. In fact, the multi-stage TR scheme utilizes the conventional TR schemes in a sequential manner.

Assume a two-stage TR scheme where the first TR block TR_1 is the conventional TR scheme using $\mathcal{R}_1$ and γ_1 as its PRT set and threshold level, respectively, while the second TR block TR_2 uses $\mathcal{R}_2$ and γ_2. The two PRT sets must be designed to satisfy the condition, $\mathcal{R}_1 \subset \mathcal{R}_2$. The peak of an OFDM signal is initially reduced by TR_1 using the threshold level γ_1. After processing by TR_1, the OFDM signal is transmitted, if the PAPR of the processed OFDM signal is lower than the target PAPR threshold level γ_2. Otherwise, the OFDM signal must be processed by TR_2 for further reduction of PAPR. For the two-stage TR scheme, additional side information of 1-bit must be transmitted to indicate which TR block was used.

The average tone reserved ratio (TRR) of the two-stage TR scheme is defined as

$$\rho_{av} = \rho_1 \Pr(\text{PAPR}_{\mathbf{x}_1} < \gamma_2) + \rho_2 \{1 - \Pr(\text{PAPR}_{\mathbf{x}_1} < \gamma_2)\} \tag{7}$$

where ρ_1 and ρ_2 are the TRR values of TR_1 and TR_2, respectively, and $\text{PAPR}_{\mathbf{x}_1}$ is the PAPR of the OFDM signal $\mathbf{x}_1$ after TR_1 is applied.

Since $\rho_2 > \rho_1$, to minimize ρ_{av}, it is desirable to select the threshold level γ_1 such that $\Pr(\text{PAPR}_{\mathbf{x}_1} < \gamma_2)$ is quite high (≥ 0.9). Then, the two-stage TR scheme can reduce the PAPR level of OFDM signals below the target threshold level γ_2 while achieving an average TRR close to ρ_1. In other words, it is possible to obtain the desired peak power reduction with the smaller number of iteration than that of the conventional TR for the same data rate and as a consequence, the proposed scheme reduces the computational complexity considerably.

C. Active Constellatio Extension

ACE reduces the amplitude of peak signals by extending subcarriers located at exterior constellations to outer region to keep minimum distance between symbols [4]. It is aimed to reduce PAPR by clipping without BER performance degradation and OOB radiation. However, there are some transmit power increase and additional complexity with this scheme.

Let $\mathbf{C}_{PRD} = \{C_0, C_1, ..., C_{JN-1}\}$ be the set of peak reduction data in frequency domain and $\mathcal{C}_{ext}$ represent allowable space of peak reduction data satisfying ACE constraint. Then ACE minmax problem can be formulated as [4]

$$\min_{\mathbf{C}\in\mathcal{C}_{\mathbf{ext}}} \max |\hat{a}_n| \tag{8}$$

where

$$\hat{a}_n = a_n + c_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{JN-1} (A_k + C_k) e^{j2\pi nk/JN}. \tag{9}$$

For example, if we use soft envelope limiter in (4) to reduce signals of which amplitude is larger than A_{th}, $\mathbf{C}_{PRD}$ is the set of data given by IFFTing the clipping noise c_n and it is usually non-zero value for $C_k, 0 \leq k \leq JN-1$. After applying ACE constraint, peak reduction data located at outward of exterior constellation are projected to the positon where the resulting OFDM symbols keep their minimum distance and the others including peak reduction data in OOB are set to zero. Consequently, $\mathbf{C}_{PRD}$ becomes the subset of $\mathcal{C}_{ext}$.

Some practical methods to solve this optimization problem are proposed in [4]. Projection onto convex set (POCS) method achieves optimal solution but converges very slowly. Smart gradient project (SGP) method which uses gradient step size to improve convergence speed is sub-optimal solution but faster than POCS.

Since ACE method uses degree of freedom of data located at exterior constellation to reduce PAPR, it does not need any additional process at receiver which means that it can be applied to the existing systems with minor modification at transmitter. Moreover, there is no loss in data rate because it does not need any side information like SLM and PTS or extra tones like TR. Drawbacks of ACE are the increased signal power and the large computational complexity due to iterative process. If it is possible to reduce the amount of peak regrowth due to ACE constraint, ACE can achieve the same PAPR reduction performance as that of the conventional ACE with the smaller number of iteration. One of the possible scheme is to use peak cancelling instead of clipping. There will be no peak regrowth if PRT is constructed such that the peak reduction data after peak cancelling are in a subset of $\mathcal{C}_{ext}$. However, it is impossible to find PRT satisfying this condition before peak cancelling. Instead, it can be suboptimal to allow PRT to have valid value only at the same indices as symbols located at exterior constellation.

IV. Probabilistic PAPR Reduction Scheme With Low-Complexity

A. Partial Transmit Sequence

The main principle of the PTS scheme is that an input symbol vector $\mathbf{A}$ is partitioned into V disjoint symbol subvectors $\mathbf{A}_v = [A_{v,0}\ A_{v,1}\ \cdots\ A_{v,N-1}]^T,\ \ v = 1,\ 2, \cdots,\ V$ as follows [6]

$$\mathbf{A} = \sum_{v=1}^{V} \mathbf{A}_v. \tag{10}$$

Here, disjoint implies that for each k, $0 \leq k \leq N-1, A_{v,k} = 0$ except for a single v. In other words, the support sets of $\mathbf{A}_v$'s are disjoint. The signal subvector $\mathbf{a}_v = [a_{v,0}\ a_{v,1}\ \cdots\ a_{v,N-1}]^T$ is generated by applying IFFT to each symbol subvector $\mathbf{A}_v$, also known as a subblock. Each signal subvector $\mathbf{a}_v$ is then multiplied by a unit magnitude constant r_v^w chosen from a given alphabet $\mathcal{Z}$, which is usually $\mathcal{Z} = \{\pm 1\}$ or $\mathcal{Z} = \{\pm 1, \pm j\}$. Then, they are summed, which results in a PTS OFDM signal vector $\mathbf{a}^w = [a_0^w\ a_1^w\ \cdots\ a_{N-1}^w]^T$ given as

$$\mathbf{a}^w = \sum_{v=1}^{V} r_v^w \mathbf{a}_v$$

where $\mathbf{r}^w = [r_1^w\ r_2^w\ \cdots\ r_V^w],\ 1 \leq w \leq W,\ W = |\mathcal{Z}|^{V-1}$, is called a rotating vector. The PAPR of $\mathbf{a}^w$ is computed for W rotating vectors and compared and finally one with the minimum PAPR is selected for transmission.

In [8], a new PTS scheme was proposed for reducing the computational complexity of IFFTs. Unlike the conventional PTS scheme, where input symbol vectors are partitioned at the initial stage, the proposed PTS scheme performs block partitioning after the first l stages of IFFT. In this scheme, the 2^n-point IFFT based on the decimation-in-time algorithm is divided into two parts. The first part is the first l stages of IFFT. The second part is the remaining $n-l$ stages. In the first l stages of IFFT, the input symbol vector $\mathbf{A}$ is partially IFFT-ed to form an intermediate signal vector $\hat{\mathbf{a}}$. This intermediate signal vector is partitioned into V intermediate signal subvectors and then, the remaining $n-l$ stages of IFFT are applied to each of the intermediate signal subvectors.

Let $\mathbf{T}_i$ be an $N \times N$ symmetric matrix representing the ith stage of IFFT and $\mathbf{Q}_i^j = \mathbf{T}_j \mathbf{T}_{j-1} \cdots \mathbf{T}_{i+1} \mathbf{T}_i,\ j \geq i$. Then, the $N = 2^n$ point IFFT matrix $\mathbf{Q}$ can be expressed as $\mathbf{Q} = \mathbf{Q}_0^{n-1}$.

Compared to the conventional PTS scheme, the computational complexity of the proposed PTS scheme is lower, because there is a common intermediate signal vector $\hat{\mathbf{a}} = \mathbf{Q}_0^{l-1}\mathbf{A}$ for IFFT of V symbol subvectors. When the number of subcarriers is $N = 2^n$, the numbers of complex multiplications n_{mul} and complex additions n_{add} of the conventional PTS scheme are given by $n_{\mathrm{mul}} = 2^{n-1}nV$ and $n_{\mathrm{add}} = 2^n nV$, respectively, where V is the number of subblocks. If the intermediate signal is partitioned after the lth stage of IFFT, it is clear that the numbers of complex computations of the proposed PTS scheme are given as $n_{\rm mul}=2^{n-1}n+2^{n-l}(n-l)(V-1)$ and $n_{\rm add}=2^n n+2^n(n-l)(V-1)$.

B. Selected Mapping

In the SLM scheme, U alternative input symbol vectors $\mathbf{A}^u=[A_0^u\ A_1^u\ \cdots\ A_{N-1}^u]^T,\ 1\le u\le U$ are generated via componentwise vector multiplication of the input symbol vector $\mathbf{A}$ and U phase sequences $\mathbf{P}^u=[P_0^u\ P_1^u\ \cdots\ P_{N-1}^u]^T$ [5]. We use the notation $\mathbf{A}^u=\mathbf{A}\otimes\mathbf{P}^u$ to represent componentwise multiplication, i.e., $A_k^u=A_kP_k^u,\ 0\le k\le N-1$.

The phase sequence $\mathbf{P}^u$ is generated by using the unit-magnitude complex number, that is, $P_k^u=e^{j\phi_k^u}$, where $\phi_k^u\in[0,\ 2\pi)$. In general, binary or quaternary elements are used for P_k^u, that is, $\{\pm1\}$ or $\{\pm1,\pm j\}$.

IFFT is performed for each of U alternative input symbol vectors to generate U alternative OFDM signal vectors as

$$\mathbf{a}^u=\mathbf{Q}\mathbf{A}^u=\mathbf{Q}(\mathbf{A}\otimes\mathbf{P}^u),\qquad 1\le u\le U. \tag{11}$$

Then, the OFDM signal with the minimum PAPR among U alternative OFDM signal vectors $\mathbf{a}^u$ is selected and transmitted. In order to recover the original input symbol vector in the receiver, the transmitter must send the side information as PTS.

A new SLM scheme with low computational complexity is proposed in [9]. This is a method for applying the SLM scheme to the intermediate stage of IFFT rather than the first stage as in the previous subsection. In this scheme, the N point IFFT based on decimation-in-time algorithm is partitioned into two parts, i.e., the first l stages and the remaining $n-l$ stages. To make alternative OFDM signals, we multiply the different U phase sequences, $\mathbf{P}^u,\ 1\le u\le U$, using the signal in the intermediate lth stage of IFFT. Based on the proposed SLM scheme, the computational complexity is reduced compared to the conventional SLM scheme, because it uses a common IFFT upto l stages and then the SLM scheme is applied to the intermediate stage IFFTed signals.

Since the proposed SLM scheme is performed using a stage-by-stage IFFT approach, its computational complexity can be reduced compared to the common IFFT operation $\mathbf{Q}_0^{l-1}=\mathbf{T}_{l-1}\mathbf{T}_{l-1}\cdots\mathbf{T}_0$. The output signal corresponding to the phase sequence in the proposed SLM scheme $\tilde{\mathbf{a}}$ can be expressed as

$$\tilde{\mathbf{a}}=\mathbf{T}_n\cdots\mathbf{T}_{k+1}\tilde{\mathbf{P}}\mathbf{T}_k\cdots\mathbf{T}_1\mathbf{A} \tag{12}$$

where $\tilde{\mathbf{P}}$ is a $2^{n-l}\times2^{n-l}$ diagonal block matrix, i.e., each $2^l\times2^l$ subblock of $\tilde{\mathbf{P}}$ is either $\pm\mathbf{I}_{2^l}$. Here, $\mathbf{I}_{2^l}$ is the $2^l\times2^l$ identity matrix.

When the number of subcarriers is $N=2^n$, the numbers of complex multiplications $n_{\rm mul}$ and complex additions $n_{\rm add}$ of the conventional SLM scheme are given by $n_{\rm mul}=2^{n-1}nU$ and $n_{\rm add}=2^n nU$, respectively, where U is the total number of phase sequences. If the phase sequences are multiplied after the lth stage of IFFT, the numbers of complex computations of the proposed SLM scheme are given by $n_{\rm mul}=2^{n-1}n+2^{n-1}(n-l)(U-1)$ and $n_{\rm add}=2^n n+2^n(n-l)(U-1)$.

The proposed SLM scheme has almost the same PAPR reduction performance as that of the conventional SLM scheme for $n-l=5$ and 16-QAM constellation. In the case of $n-l=5$, the proposed SLM OFDM system reduces the computational complexity by $41\sim51\%$ as U increases from 4 to 16.

The modified SLM scheme in [10] generates some of alternative OFDM signal vectors using the other ones in the time domain. Let $\mathbf{a}^i$ and $\mathbf{a}^k$ be the alternative OFDM signal vectors, generated by the conventional SLM scheme as in (11). Based on linear property of the Fourier transform, the linear combination of these two vectors can be given as

$$\begin{aligned}\mathbf{a}^{i,k}&=c_i\mathbf{a}^i+c_k\mathbf{a}^k\\&=c_i\mathbf{Q}(\mathbf{A}\otimes\mathbf{P}^i)+c_k\mathbf{Q}(\mathbf{A}\otimes\mathbf{P}^k)\\&=\mathbf{Q}(\mathbf{A}\otimes(c_i\mathbf{P}^i+c_k\mathbf{P}^k))\end{aligned} \tag{13}$$

where c_i and c_k are complex numbers. If each element of the vector $c_i\mathbf{P}^i+c_k\mathbf{P}^k$ in (13) has a unit magnitude, $c_i\mathbf{P}^i+c_k\mathbf{P}^k$ can be also used as a phase sequence for the SLM scheme and $\mathbf{a}^{i,k}$ can be considered as the corresponding alternative OFDM signal vector. Therefore, if we have alternative OFDM signal vectors $\mathbf{a}^i$ and $\mathbf{a}^k$, another alternative OFDM signal vector $\mathbf{a}^{i,k}$ can be obtained, which avoids the need for IFFT. Note that the phase sequence $c_i\mathbf{P}^i+c_k\mathbf{P}^k$ is not statistically independent of $\mathbf{P}^i$ and $\mathbf{P}^k$. Here, we investigate how to make each element of $c_i\mathbf{P}^i+c_k\mathbf{P}^k$ a unit magnitude, in the condition that each element of the phase sequences $\mathbf{P}^i$ and $\mathbf{P}^k$ has a unit magnitude. Clearly, the elements of the vector $c_i\mathbf{P}^i+c_k\mathbf{P}^k$ have a unit magnitude if the following conditions are satisfied:

i) Each element of $\mathbf{P}^i$ and $\mathbf{P}^k$ has a value in $\{+1,-1\}$;
ii) $c_i=\pm1/\sqrt{2}$ and $c_k=\pm j/\sqrt{2}$.

Since the two alternative OFDM signal vectors generated from the phase sequences $\pm(c_i\mathbf{P}^i+c_k\mathbf{P}^k)$ have the same PAPR, we only consider the case of $c_i=1/\sqrt{2}$ and $c_k=\pm j/\sqrt{2}$. Since $|c_i|^2=|c_k|^2=1/2$, the average power of $\mathbf{a}^{i,k}$ is equal to half the sum of the average power of $\mathbf{a}^i$ and $\mathbf{a}^k$. Using U binary phase sequences, $2\binom{U}{2}$ additional phase sequences are obtained, where $\binom{U}{2}=U(U-1)/2$. Thus, the total U^2 phase sequences are obtained as

$$\{\mathbf{P}^1,\mathbf{P}^2,\cdots,\mathbf{P}^U,\frac{1}{\sqrt{2}}(\mathbf{P}^1\pm j\mathbf{P}^2),\\ \frac{1}{\sqrt{2}}(\mathbf{P}^1\pm j\mathbf{P}^3),\cdots,\frac{1}{\sqrt{2}}(\mathbf{P}^{U-1}\pm j\mathbf{P}^U)\}.$$

By combining each pair among U alternative OFDM signal vectors $\mathbf{a}^u$ obtained by using U binary phase sequences as above, a set $\mathcal{S}$ of U^2 alternative OFDM signal vectors is generated as

$$\begin{aligned}\mathcal{S}=&\{\mathbf{a}^u|\ 1\le u\le U^2\}\\=&\{\mathbf{a}^u|\ 1\le u\le U\}\\&\cup\left\{\frac{1}{\sqrt{2}}(\mathbf{a}^i+j\mathbf{a}^k),\frac{1}{\sqrt{2}}(\mathbf{a}^i-j\mathbf{a}^k)\ |\ 1\le i<k\le U\right\}\end{aligned} \tag{14}$$

where only U IFFTs and the additional summations of U^2-U pairs of OFDM signal vectors are needed. However, the computational complexity for the summations of OFDM signal vectors is negligible compared with that of IFFT.

The modified SLM scheme with U binary phase sequences can be compared with the conventional SLM scheme with U^2 binary phase sequences. These two schemes show a similar PAPR reduction performance for a small U. However, as U increases, the PAPR reduction performance of the modified scheme becomes worse than that of the conventional SLM scheme with U^2 binary phase sequences, because U^2 phase sequences of the modified scheme are statistically correlated.

It is possible to generate the alternative OFDM signal vectors of SLM scheme by additive representation in time domain. An input symbol A_k modulated with M-ary QAM can be expressed as a complex symbol comprised of in-phase and quadrature components, that is, $A_k = A_{I,k} + jA_{Q,k}$. For the phase sequence $P_k^{(u)} \in \{\pm 1\}$, the kth additive mapping sequence $\mathbf{D}^{(u)} = [D_0^{(u)} \ D_1^{(u)} \ \cdots \ D_{N-1}^{(u)}], 0 \le u < U$ is defined as

$$D_k^{(u)} = \begin{cases} d(-1-j)\sqrt{M}/2, & A_k \in \mathbf{Q}^{(1)} \text{ and } P_k^{(u)} = -1 \\ d(+1-j)\sqrt{M}/2, & A_k \in \mathbf{Q}^{(2)} \text{ and } P_k^{(u)} = -1 \\ d(+1+j)\sqrt{M}/2, & A_k \in \mathbf{Q}^{(3)} \text{ and } P_k^{(u)} = -1 \\ d(-1+j)\sqrt{M}/2, & A_k \in \mathbf{Q}^{(4)} \text{ and } P_k^{(u)} = -1 \\ 0, & P_k^{(u)} = 1 \end{cases} \tag{15}$$

where d is the smallest distance between symbols and $\mathbf{Q}^{(i)}$ is the set of symbols belonging to the ith quadrant of 2-dimensional signal space. Let $\mathbf{P}^{(-1)}$ be the phase sequence whose elements are all -1 and $\mathbf{D}^{(-1)} = \mathbf{D}_I^{(-1)} + j\mathbf{D}_Q^{(-1)}$ be the corresponding additive mapping sequence where $\mathbf{D}_I^{(-1)}$ and $\mathbf{D}_Q^{(-1)}$ are real in-phase and quadrature components. Note that $\mathbf{D}^{(-1)}$ is determined also by using the input symbol sequence $\mathbf{A}$ as given in (15). Then the additive mapping signal sequence $\mathbf{d}^{(-1)}$ is generated as

$$\mathbf{d}^{(-1)} = \text{IFFT}(\mathbf{D}^{(-1)}) = \text{IFFT}(\mathbf{D}_I^{(-1)}) + j\text{IFFT}(\mathbf{D}_Q^{(-1)}). \tag{16}$$

Let $\mathbf{d}_I^{(-1)} = \text{IFFT}(\mathbf{D}_I^{(-1)})$ and $\mathbf{d}_Q^{(-1)} = \text{IFFT}(\mathbf{D}_Q^{(-1)})$. Then $\mathbf{d}_I^{(-1)}$ and $\mathbf{d}_Q^{(-1)}$ have the property of complex conjugate symmetry because $\mathbf{D}_I^{(-1)}$ and $\mathbf{D}_Q^{(-1)}$ are real. Similarly, the lth additive mapping signal sequence for $\mathbf{P}_k^{(l)}$ is given as

$$\mathbf{d}^{(l)} = \text{IFFT}(\mathbf{D}^{(l)}) = \text{IFFT}(\mathbf{D}_I^{(l)}) + j\text{IFFT}(\mathbf{D}_Q^{(l)}), \ 1 \le l \le V \tag{17}$$

where $\mathbf{d}_I^{(l)} = \text{IFFT}(\mathbf{D}_I^{(l)})$ and $\mathbf{d}_Q^{(l)} = \text{IFFT}(\mathbf{D}_Q^{(l)})$ also have the property of complex conjugate symmetry because $\mathbf{D}_I^{(l)}$ and $\mathbf{D}_Q^{(l)}$ are real.

Using linear combinations of these two additive mapping signal sequences $\mathbf{d}^{(-1)}$ and $\mathbf{d}^{(l)}$, we can make 16 alternative signal sequences $\mathbf{x}^{(u)} = \mathbf{a} + \mathbf{m}^{(u)}$ where $\mathbf{m}^{(u)}$ is linear combination of the additive mapping signal sequences denoted as $+$ or $-$.

$\mathbf{m}^{(u)}$'s for the first 4 alternative signal sequences are generated by linear combination of $\mathbf{d}_I^{(-1)}$ and $\mathbf{d}_Q^{(-1)}$ and $\mathbf{m}^{(u)}$'s for the remaining 12 alternative signal sequences are obtained by linear combination of $\mathbf{d}_I^{(-1)}$, $\mathbf{d}_Q^{(-1)}$, $\mathbf{d}_I^{(l)}$ and $\mathbf{d}_Q^{(l)}$. Therefore, Another 12 alternative signal sequences can be generated by introducing additional $\mathbf{P}^{(l)}$. Thus, the total number of alternative signal sequences generated from V phase sequences is $12V + 4$.

V. Conclusion

In this paper, we reviewed some of the main PAPR reduction algorithms for OFDM systems and provided their low-complexity schemes. In the iterative schemes such as CAF, TR, and ACE, it is important to reduce the number of iteration. The probabilistic schemes such as SLM and PTS should be implemented with the reduced number of FFT operation. Performances such as PAPR reduction performance, BER, or data rate should be also considered when a low-complexity PAPR reduction scheme is developed. Future work on PAPR reduction is to compare these overall performances and find the optimal PAPR reduction scheme to be adopted as a standard for wireless communication systems.

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