Diversity Analysis of the Best Relay Selection for Soft-Decision-and-Forward Cooperative Network

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Abstract—In this paper, we propose the best relay selection scheme for the soft-decision-and-forward cooperative network with multiple relays. The term ‘best relay selection’ implies that the relay having the largest end-to-end signal-to-noise ratio is selected to transmit in the second phase transmission. The approximate performances are analyzed in terms of pairwise error probability. Using the Fox’s $H$-function, it is shown that the proposed scheme has full diversity order.

Index Terms—Cooperative diversity, Fox’s $H$-function, maximum distribution, relay selection, soft-decision-and-forward (SDF).

I. INTRODUCTION

Higher spectral efficiency and high data rate might be achieved by multiple-antenna technique [1] which increases the channel capacity. However, due to the limitation on implementation, the virtual multiple-antenna technique in a distributed sense is recommended. In [2], Ikki and Ahmed proposed and analyzed the best relay selection scheme based on amplify-and-forward protocol equipped with one antenna in each node. But their performance analysis left a room for an improvement since it was based on a rough approximation. Bletasas et al. [3] described the forward channel estimation for the opportunistic relay selection case in only one of the multiple relays.

Yang, Song, No, and Shin [4] proposed the new cooperation protocol called soft-decision-and-forward (SDF) using Alamouti code [5]. And in [7], Song, No, and Chung analyzed the bit error rate (BER) and suboptimal power allocation of the single-relay SDF cooperative network. In this paper, a best relay selection scheme using Alamouti code for the SDF cooperative network with multiple relays is proposed. For the proposed scheme, we express the end-to-end signal-to-noise ratios (SNRs) and derive the PEP under the ML decoder. The total transmit power $P_t$ at $S$ and the total transmit power $P_r$ at the relays. The channel gains of each link $S \to D$, $S \to R_m$, and $R_m \to D$ are assumed to be Rayleigh-faded, i.e., $f_{ij}^{(S)} \sim \mathcal{CN}(0, \sigma_{SD}^2)$, $f_{ij}^{(R)} \sim \mathcal{CN}(0, \sigma_{SR}^2)$, and $g_{ij}^{(R)} \sim \mathcal{CN}(0, \sigma_{RD}^2)$, where $f_{ij}^{(S)}$, $f_{ij}^{(R)}$, and $g_{ij}^{(R)}$, $i, j = 1, 2$, $m = 1, \cdots, M$, denote the path gain from the $i$th transmit antenna at $S$ to the $j$th receive antenna at $D$, from the $i$th transmit antenna at $S$ to the $j$th receive antenna at $R_m$, and from the $i$th transmit antenna at $R_m$ to the $j$th receive antenna at $D$, respectively. These path gains are represented as the channel matrices $F_0 = [f_{ij}^{(S)}]$, $F_m = [f_{ij}^{(R)}]$, and $G_m = [g_{ij}^{(R)}]$.

The signal transmission in the cooperative networks is composed of two phases. In the first phase, $S$ transmits the
signals using Alamouti code to $R_m$, $m = 1, \cdots, M$ and $D$. The received signals at $R_m$ and $D$ are represented, respectively, as
\[
Y_{R_m} = \sqrt{\frac{P_1}{2}} X F_m + N_{R_m}, \quad m = 1, \cdots, M
\]
\[
Y_{D1} = \sqrt{\frac{P_1}{2}} X F_0 + N_{D1}
\]
(1)
where $X = A(x_1, x_2)$ is the transmit codeword for the message vector $x = [x_1, x_2]^T$ at $S$ in the first phase, $F_0$ and $F_m$ denote the channel matrices of $S \rightarrow D$ and $S \rightarrow R_m$, respectively, and $N_{R_m}$ and $N_{D1}$ are the $2 \times 2$ additive white Gaussian noise (AWGN) matrices with zero-mean and unit-variance entries. The equation (1) can be also rewritten as the vector-form
\[
\begin{bmatrix}
\text{cv}(Y_{R_m}) \\
\text{cv}(Y_{D1})
\end{bmatrix} = \sqrt{\frac{P_1}{2}} X \begin{bmatrix}
F'_m \\
F'_0
\end{bmatrix} x + \begin{bmatrix}
\text{cv}(N_{R_m}) \\
\text{cv}(N_{D1})
\end{bmatrix}
\]
(4)
where $\text{cv}(N_D)$ means the equivalent noise at $D$ in the vector form given by
\[
\text{cv}(N_D) = \sqrt{\frac{P_2}{2} \lambda_m \|F_m\|^2 G_m' I_n} + \text{cv}(N_{D2})
\]
The received signal at $D$ in both phases can be rewritten as an equivalent vector model
Then, the selected $\hat{m}$th relay $R_{\hat{m}}$ transmits the following codeword to $D$
\[
X_{R_{\hat{m}}} = A(\hat{x}_{\hat{m},1}, \hat{x}_{\hat{m},2}) = \begin{bmatrix}
\hat{x}_{\hat{m},1} \\
-\hat{x}_{\hat{m},2}
\end{bmatrix}
\]
In the second phase, the destination $D$ receives the signal from the $\hat{m}$th relay as
\[
Y_{D2} = \sqrt{\frac{P_2}{2}} X_{R_{\hat{m}}} G_{\hat{m}} + N_{D2}
\]
(2)
\[
\gamma_m = P \|F_0\|^2 / 4, \gamma_{\hat{m},1} = P \|F_{\hat{m}}\|^2 / 4, \gamma_{\hat{m},2} = P \|G_{\hat{m}}\|^2 / 4.
\]
Then, the instantaneous end-to-end SNR for best relay selection can be expressed as
\[
\gamma_{eq} = \gamma_0 + \max_m \frac{\gamma_{m,1} \gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1}
\]
(5)
where \( \gamma_0 \sim G\left(4, \sigma_{SDP}^2/4\right) \), \( \gamma_{m,1} \sim G\left(4, \sigma_{SR_m}^2/4\right) \), and \( \gamma_{m,2} \sim G\left(4, \sigma_{R_mDP}^2/4\right) \), respectively.

For two positive numbers \( x \) and \( y \), if we consider the constant \( k > (x + y + 1)/(x + y) \), the following inequality holds:

\[
\frac{x}{k(x+y)} < \frac{x}{x+y+1} < \frac{x}{x+y}.
\]

Thus, (5) can be rewritten as

\[
\gamma_0 + \frac{\mu_H(\gamma_{m,1}, \gamma_{m,2})}{2k} < \gamma_{eq} < \gamma_0 + \frac{\mu_H(\gamma_{m,1}, \gamma_{m,2})}{2}
\]

where \( \mu_H(a, b) = \frac{2ab}{a+b} \) and \( k > 1 + (\gamma_{m,1} + \gamma_{m,2})^{-1} \).

The conditional PEP \( \Pr(\gamma \rightarrow \bar{\gamma}|\mathbf{H}) \) can be obtained by averaging the conditional PEP in the above equation over \( \mathbf{H} \). Furthermore, \( Q \)-function is upperly and lowerly bounded by

\[
\sum_{n=1}^{N} a_n \exp\left(-b_n-1 x^2\right) \leq Q(x) \leq \sum_{n=1}^{N} a_n \exp\left(-b_n x^2\right)
\]

where \( a_n = (\theta_n - \theta_{n-1})/\pi \) and \( b_n = 1/(2\sin^2 \theta_n) \) for \( n = 1, \ldots, N \) with \( \theta_0 = 0 \). And then, the PEP can be upperly and lowerly bounded by

\[
\Pr(\gamma \rightarrow \bar{\gamma}) \leq \sum_{n=1}^{N} a_n \mathcal{M}_\gamma(b_n \delta_x^2) \mathcal{M}_{\gamma_{\max}}(b_n \delta_x^2)
\]

\[
\Pr(\gamma \rightarrow \bar{\gamma}) \geq \sum_{n=1}^{N} a_n \mathcal{M}_\gamma(b_n-1 \delta_x^2) \mathcal{M}_{\gamma_{\max}}(b_n-1 \delta_x^2)
\]

where \( \mathcal{M}_X(s) \) is the MGF of random variable \( X \), i.e., the Laplace transform of \( X \), and \( \gamma_{\max} = \mu_H(\gamma_{m,1}, \gamma_{m,2}) \).

Since the lower bound can have the same form as the upper bound except the index of \( b_n \), it is enough to show the diversity order for the upper or lower bound of the PEP.

However, it is difficult to derive the maximum distribution of the harmonic mean between two gamma random variables. For derivation of the diversity order, we will use the following relation:

\[
\Pr(\gamma_{\max} \leq \gamma) = \Pr(\gamma_1 \leq \gamma, \ldots, \gamma_M \leq \gamma)
\]

\[
= \Pr(\lim_{r \to \infty} \left[ \gamma_{1}^r + \cdots + \gamma_{M}^r \leq \gamma^r \right]).
\]

Since \( \gamma_m \)'s are independent, the MGF of \( \gamma_{\max} \) equals to the \( M \)th power of the MGF of \( \gamma_m \), i.e.,

\[
\mathcal{M}_{\gamma_{\max}}(s) = \left[ \mathcal{M}_{\gamma_m}(s) \right]_M
\]

as \( r \to \infty \). As \( \gamma_m \) has \( H \)-function distribution [9], so does \( \gamma_m^r \).

Since the MGF of \( \gamma_m^r \) is expressed in terms of \( H \)-function, the MGF of \( \gamma_{\max}^r \) can be written as \( H \)-function as in the following theorem.

**Theorem 1:** The asymptotic MGF of \( \gamma_{\max}^r \) is represented as

\[
\mathcal{M}_{\gamma_{\max}^r}(s) \approx \left\{ \frac{\Omega}{2} \right\}^{-4} \left[ 3 + \frac{95}{27} \left( \frac{\Omega}{2} \right)^{-4} \log \left( \frac{\Omega}{2} \right) \right]\]

**Proof:** Let \( Z_m = Y_m^r \) where \( Y_m = \mu_H(\gamma_{m,1}, \gamma_{m,2}) \) for \( \gamma_{m,1}, \gamma_{m,2} \sim G(K, \Omega) \). By the property of \( H \)-function distribution [9], the PDF of \( Z_m \) can be expressed by using \( H \)-function given as

\[
f_{Z_m}(z) = \frac{\sqrt{\pi}}{2^{2K-2} K \Omega} \left[ \frac{2}{(K-r, 2K-r)} \right] \left( \frac{2}{\Omega} \right)^{z} \left( \frac{K-r}{2K-r} \right)
\]

where \( \Omega = \sigma^2 P/4 \) and \( 3_{m,n} \) is the Fox’s \( H \)-function defined in [10]. Taking the Laplace transform into the above PDF, the MGF of \( Z_m \) can be expressed as

\[
\mathcal{M}_{Z_m}(s) = \frac{\sqrt{\pi}}{2^{2K-1} \Gamma(K)} \mathcal{G}_{2,2}(1, 1, \Omega, 1-K, (1/2-K, r))
\]

where \( c = (\frac{1}{2})^r \). In this case, \( K = 4 \) is considered.

Using the result in [8], the following relation holds for for \( s \) in infinity :

\[
\mathcal{G}_{2,2}(1, 1, \Omega, 1-K, (1/2-K, r)) \equiv \sum_{i} \bar{H}_{1,i}(cs)^{a_i-1} + \sum_{i} \bar{H}_{2,i}(cs)^{a_i-1} \log(cs)
\]

\[
= h_1(cs)^{-\frac{1}{r}} + \left( H_{1,4}(cs)^{-\frac{1}{r}} + H_{2,0}(cs)^{-\frac{1}{r}} \right) \log(cs).
\]

The constants \( h_1, H_{1,k_1}, \) and \( H_{2,k_2} \) are expressed as

\[
h_1 = \frac{1}{r} \Gamma(\frac{1}{2}) \Gamma(4) \Gamma(\frac{1}{2}) \Gamma(4)
\]

\[
H_{1,k_1} = \frac{1}{2r^2 \Gamma(\frac{1}{2} - k_1)}
\]

\[
H_{2,k_2} = \frac{1}{2r^2 \Gamma(\frac{1}{2} - k_2)}
\]

Thus, using the relation

\[
\lim_{r \to \infty} \frac{\Gamma(\frac{1}{2})}{r} = \frac{1}{x} \text{ and } \lim_{r \to \infty} s^{-\frac{1}{r}} = 1,
\]

the limits of each term in (9) are calculated as

\[
\lim_{r \to \infty} \left[ h_1(cs)^{-\frac{1}{r}} \right] = \left[ \frac{1}{2r^2 \Gamma(\frac{1}{2})} \right] \left( \frac{1}{2} \right)^{-4} s^{-\frac{1}{r}}
\]

\[
= 3 \left[ \frac{2}{(\frac{1}{2})^4} \right] \left( \frac{\Omega}{2} \right)^{-4} s^{-\frac{1}{r}}
\]  

(10)
\[
\lim_{r \to \infty} \left[ H_{1,4}(cs)^{-\frac{3}{2}} \log(cs) \right] = \lim_{r \to \infty} \left[ \frac{1}{2r^2} \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \left( \frac{\Omega}{2} \right)^{-3} s^{-\frac{3}{2}} \log \left( \frac{\Omega}{2} \right)^r \right] = \frac{7 \cdot 5 \cdot 3}{2^8 \sqrt{\pi}} \left( \frac{\Omega}{2} \right)^{-3} \log \left( \frac{\Omega}{2} \right) \tag{11}
\]
\[
\lim_{r \to \infty} \left[ H_{2,0}(cs)^{-\frac{3}{2}} \log(cs) \right] = \lim_{r \to \infty} \left[ \frac{1}{2r^2} \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \left( \frac{\Omega}{2} \right)^{-3} s^{-\frac{3}{2}} \log \left( \frac{\Omega}{2} \right)^r \right] = -\frac{1}{2^8 \sqrt{\pi}} \left( \frac{\Omega}{2} \right)^{-3} \log \left( \frac{\Omega}{2} \right). \tag{12}
\]

Summing up the above results, (8) is approximated as
\[
\lim_{r \to \infty} M_{Z_m}(s) \approx \frac{1}{21032} \left( \frac{\Omega}{2} \right)^{-4} \left[ 3 + \frac{95}{27} \left( \frac{\Omega}{2} \right)^{-4} \log \left( \frac{\Omega}{2} \right) \right].
\]

Let \( \gamma_{\text{max}} = \sum_{m=1}^{M} Z_m \) where \( Z_m \)'s are i.i.d. random variables for \( m = 1, \ldots, M \). Then, the MGF for \( \gamma_{\text{max}} \) can be obtained as
\[
M_{\gamma_{\text{max}}}(s) \approx \left( \frac{1}{21032} \left( \frac{\Omega}{2} \right)^{-4} \left[ 3 + \frac{95}{27} \left( \frac{\Omega}{2} \right)^{-4} \log \left( \frac{\Omega}{2} \right) \right] \right)^M
\]
as \( r \to \infty \).

From the above result, we can conclude that the diversity order of \( S \to R_{\text{th}} \to D \) is \( 4M \). Since the diversity order of \( S \to D \) is four, the diversity order of the cooperative network with the proposed best relay selection under ML decoder is \( 4(M + 1) \).

Fig. 2 shows the analytical and numerical results for the cooperative network with the proposed best relay selection when the uniform power allocation between \( S \) and \( R_{\text{th}} \) is used, i.e., \( P_1 = P_2 = P/2 \). The ‘Pade approximation’ results are obtained using Padé approximation technique [11]. As the number of the relays increases, the BER performance of the proposed scheme is always enhanced. And also, diversity orders from PEP are plotted. From this result, we can confirm that the proposed scheme has full diversity order.

IV. CONCLUSION

In this paper, the performance of best relay selection scheme has been shown. The PEP and diversity of the proposed relay selection scheme has been derived. And it has been shown that it has full diversity. From the numerical results, it has been shown that the best relay selection scheme has full diversity order.

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