Diversity Analysis of the Best Relay Selection for Soft-Decision-and-Forward Cooperative Network

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***Abstract*—In this paper, we propose the best relay selection scheme for the soft-decision-and-forward cooperative network with multiple relays. The term 'best relay selection' implies that the relay having the largest end-to-end signal-to-noise ratio is selected to transmit in the second phase transmission. The approximate performances are analyzed in terms of pairwise error probability. Using the Fox's H-function, it is shown that the proposed scheme has full diversity order.**

***Index Terms*—Cooperative diversity, Fox's H-function, maximum distribution, relay selection, soft-decision-and-forward (SDF).**

I. Introduction

Higher spectral efficiency and high data rate might be achieved by multiple-antenna technique [1] which increases the channel capacity. However, due to the limitation on implementation, the virtual multiple-antenna technique in a distributed sense is recommended. In [2], Ikki and Ahmed proposed and analyzed the best relay selection scheme based on amplify-and-forward protocol equipped with one antenna in each node. But their performance analysis left a room for an improvement since it was based on a rough approximation. Bletsas *et al.* [3] described the forward channel estimation for the opportunistic relay selection in case of the multiple relays.

Yang, Song, No, and Shin [4] proposed the new cooperation protocol called soft-decision-and-forward (SDF) using Alamouti code [5]. And in [7], Song, No, and Chung analyzed the bit error rate (BER) and suboptimal power allocation of the single-relay SDF cooperative network. In this paper, a best relay selection scheme using Alamouti code for the SDF cooperative network with multiple relays is proposed. For the proposed scheme, we express the end-to-end signal-to-noise ratios (SNRs) and derive the PEP under the ML decoder. From the derived PEP for the SDF cooperative networks with multiple relays, its diversity order is obtained using the property of H-function distribution [9]. It is shown that the proposed best relay selection has full diversity order.

This paper is organized as follows. Section II describes the system model for the proposed 'best relay selection' schemes. In Section III, the PEP and the diversity order for the best relay selection scheme are derived. Finally, the concluding remarks are given in Section IV.

Notations: $\mathbb{E}[\cdot]$ denotes the expectation of a random variable. $X \sim \mathcal{CN}(0, \sigma^2)$ means that X is a complex normal random variable with zero mean and variance $\sigma^2/2$ in both real and imaginary parts, respectively. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\dagger}$, and $\|\cdot\|$ denote the transpose of a matrix, the conjugate transpose of a matrix, and the Frobenius norm of a matrix or a vector, respectively. Bold-face uppercase and lowercase letters denote matrices and vectors, respectively.

II. System Model

In this section, the system model of SDF protocol [4] with multiple relays equipped with two antennas in each node as shown in Fig. 1 is described. This cooperative communication system is composed of one source (S), one destination (D), and M relays ($\mathsf{R}_m,\ m = 1, \cdots, M$). In the second phase transmission, the best relay selection scheme is considered.

The following notations are used in this section. The Alamouti code $\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$ is denoted by $\mathbf{A}(a, b)$. Also, for any 2×2 matrix $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, the 4×2 matrix $\mathbf{B}'$ and the vector $cv(\mathbf{B})$ are defined as $\mathbf{B}' = \begin{bmatrix} b_{11} & b_{21}^* & b_{12} & b_{22}^* \\ b_{21} & -b_{11}^* & b_{22} & -b_{12}^* \end{bmatrix}^{\mathrm{T}}$ and $cv(\mathbf{B}) = \begin{bmatrix} b_{11} & b_{21}^* & b_{12} & b_{22}^* \end{bmatrix}^{\mathrm{T}}$.

The total transmit power P in the network is defined as the sum of the transmit power P_1 at S and the total transmit power P_2 at the relays. The channel gains of each link $\mathsf{S} \to \mathsf{D}$, $\mathsf{S} \to \mathsf{R}_m$, and $\mathsf{R}_m \to \mathsf{D}$ are assumed to be Rayleigh-faded, i.e., $f_0^{ij} \sim \mathcal{CN}(0, \sigma_{\mathsf{SD}}^2)$, $f_m^{ij} \sim \mathcal{CN}(0, \sigma_{\mathsf{SR}_m}^2)$, and $g_m^{ij} \sim \mathcal{CN}(0, \sigma_{\mathsf{R}_m\mathsf{D}}^2)$, where f_0^{ij}, f_m^{ij}, and g_m^{ij}, $i, j = 1, 2$, $m = 1, \cdots, M$, denote the path gain from the ith transmit antenna at S to the jth receive antenna at D, from the ith transmit antenna at S to the jth receive antenna at R_m, and from the ith transmit antenna at R_m to the jth receive antenna at D, respectively. These path gains are represented as the channel matrices $\mathbf{F}_0 = [f_0^{ij}]$, $\mathbf{F}_m = [f_m^{ij}]$, and $\mathbf{G}_m = [g_m^{ij}]$.

The signal transmission in the cooperative networks is composed of two phases. In the first phase, S transmits the

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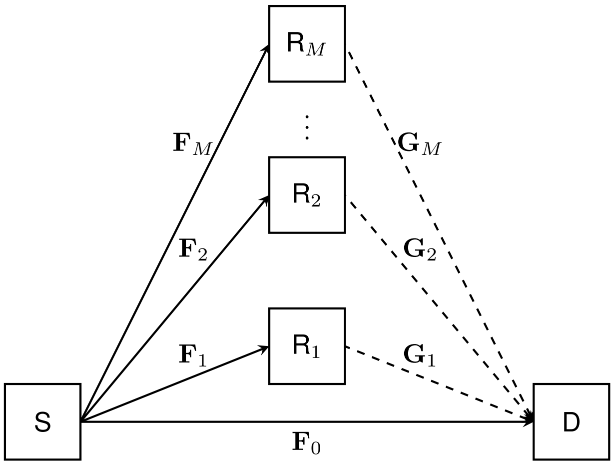

Fig. 1. Cooperative communication network composed of one source (S), M relays (R_m), and one destination (D) with two antennas in each node. In the second phase, one of M relays is only participated in the transmission.

signals using Alamouti code to $\mathsf{R}_m,\ m = 1, \cdots, M$ and D. The received signals at R_m and D are represented, respectively, as

$$\mathbf{Y}_{\mathsf{R}_m} = \sqrt{\frac{P_1}{2}}\mathbf{X}\mathbf{F}_m + \mathbf{N}_{\mathsf{R}_m}, \quad m = 1, \cdots, M$$
$$\mathbf{Y}_{\mathsf{D}1} = \sqrt{\frac{P_1}{2}}\mathbf{X}\mathbf{F}_0 + \mathbf{N}_{\mathsf{D}1} \tag{1}$$

where $\mathbf{X} = \mathbf{A}(x_1, x_2)$ is the transmit codeword for the message vector $\mathbf{x} = [x_1\ x_2]^{\mathrm{T}}$ at S in the first phase, $\mathbf{F}_0$ and $\mathbf{F}_m$ denote the channel matrices of S $\rightarrow$ D and S $\rightarrow$ R_m, respectively, and $\mathbf{N}_{\mathsf{R}_m}$ and $\mathbf{N}_{\mathsf{D}1}$ are the 2×2 additive white Gaussian noise (AWGN) matrices with zero-mean and unit-variance entries. The equation (1) can be also rewritten as the vector-form

$$cv(\mathbf{Y}_{\mathsf{R}_m}) = \begin{bmatrix} y_{11}^{(\mathsf{R}_m)} & y_{21}^{(\mathsf{R}_m)*} & y_{12}^{(\mathsf{R}_m)} & y_{22}^{(\mathsf{R}_m)*} \end{bmatrix}^{\mathrm{T}}$$
$$= \sqrt{\frac{P_1}{2}}\mathbf{F}'_m\mathbf{x} + cv(\mathbf{N}_{\mathsf{R}_m})$$
$$cv(\mathbf{Y}_{\mathsf{D}1}) = \begin{bmatrix} y_{11}^{(\mathsf{D}1)} & y_{21}^{(\mathsf{D}1)*} & y_{12}^{(\mathsf{D}1)} & y_{22}^{(\mathsf{D}1)*} \end{bmatrix}^{\mathrm{T}}$$
$$= \sqrt{\frac{P_1}{2}}\mathbf{F}'_0\mathbf{x} + cv(\mathbf{N}_{\mathsf{D}1}).$$

In contrast to the conventional multiple-relay transmission [8], the signal in the second phase of the best relay selection scheme is transmitted from only one relay $\mathsf{R}_{\hat{m}}$ according to the selection criterion

$$\hat{m} = \arg\max_{m} \left\{ \frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1} \right\} \tag{2}$$

where $\gamma_{m,1} = P_1\|\mathbf{F}_m\|^2/2$ and $\gamma_{m,2} = P_2\|\mathbf{G}_m\|^2/2$. This selection criterion stems from the maximization of the ergodic capacity given by

$$C = \frac{1}{2}\log_2\left(1 + \gamma_0 + \max_{m} \frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1}\right). \tag{3}$$

Then, the selected $\hat{m}$th relay $\mathsf{R}_{\hat{m}}$ transmits the following codeword to D

$$\mathbf{X}_{\mathsf{R}_{\hat{m}}} = \mathbf{A}(\tilde{x}_{\hat{m},1}, \tilde{x}_{\hat{m},2}) = \begin{bmatrix} \tilde{x}_{\hat{m},1} & \tilde{x}_{\hat{m},2} \\ -\tilde{x}^*_{\hat{m},2} & \tilde{x}^*_{\hat{m},1} \end{bmatrix}.$$

In the second phase, the destination D receives the signal from the $\hat{m}$th relay as

$$\mathbf{Y}_{\mathsf{D}2} = \sqrt{\frac{P_2}{2}}\mathbf{X}_{\mathsf{R}_{\hat{m}}}\mathbf{G}_{\hat{m}} + \mathbf{N}_{\mathsf{D}2}$$

where $\mathbf{G}_{\hat{m}}$ is the channel matrix of $\mathsf{R}_{\hat{m}} \rightarrow$ D and $\mathbf{N}_{\mathsf{D}2}$ denotes the 2×2 AWGN matrix with zero-mean and unit-variance entries. Converting the matrix equation into the vector form gives us the following alternative expression

$$cv(\mathbf{Y}_{\mathsf{D}2}) = \frac{\sqrt{P_1P_2}}{2}\lambda_{\hat{m}}\|\mathbf{F}_{\hat{m}}\|^2\mathbf{G}'_{\hat{m}}\mathbf{x}$$
$$+ \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\mathbf{G}'_{\hat{m}}\mathbf{F}'^{\dagger}_{\hat{m}}cv(\mathbf{N}_{\mathsf{R}_{\hat{m}}}) + cv(\mathbf{N}_{\mathsf{D}2})$$

where

$$\lambda_{\hat{m}} = \sqrt{\frac{2}{\|\mathbf{F}_{\hat{m}}\|^2(P_1\|\mathbf{F}_{\hat{m}}\|^2 + 2)}}.$$

The received signal at D in both phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D}1}) \\ cv(\mathbf{Y}_{\mathsf{D}2}) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}}\begin{bmatrix} \mathbf{F}'_0 \\ \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\|\mathbf{F}_{\hat{m}}\|^2\mathbf{G}'_{\hat{m}} \end{bmatrix}}_{\mathbf{H}}\mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}_{\mathsf{D}1}) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix}}_{\mathbf{n}} \tag{4}$$

where $cv(\mathbf{N}_{\mathsf{D}})$ means the equivalent noise at D in the vector form given by

$$cv(\mathbf{N}_{\mathsf{D}}) = \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\mathbf{G}'_{\hat{m}}\mathbf{F}'^{\dagger}_{\hat{m}}cv(\mathbf{N}_{\mathsf{R}_{\hat{m}}}) + cv(\mathbf{N}_{\mathsf{D}2}).$$

The ML decoder for the best relay selection can be found in [4] since it is the same as the one for a single relay case.

III. PEP and Diversity Order

The proposed scheme uses only one relay in the second phase, i.e., the relay $\mathsf{R}_{\hat{m}}$ selected according to the selection criterion in (2) transmits the signals with power P_2 in the second phase. Assume that the uniform power allocation is used between S and $\mathsf{R}_{\hat{m}}$, i.e., $P_1 = P_2 = P/2$. Let γ_0, $\gamma_{\hat{m},1}$, and $\gamma_{\hat{m},2}$ be the SNRs of S $\rightarrow$ D, S $\rightarrow$ $\mathsf{R}_{\hat{m}}$, and $\mathsf{R}_{\hat{m}}$ $\rightarrow$ D links defined by

$$\gamma_0 = P\|\mathbf{F}_0\|^2/4, \gamma_{\hat{m},1} = P\|\mathbf{F}_{\hat{m}}\|^2/4, \gamma_{\hat{m},2} = P\|\mathbf{G}_{\hat{m}}\|^2/4.$$

Then, the instantaneous end-to-end SNR for best relay selection can be expressed as

$$\gamma_{\mathrm{eq}} = \gamma_0 + \max_{m} \frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1}$$
$$= \gamma_0 + \frac{\gamma_{\hat{m},1}\gamma_{\hat{m},2}}{\gamma_{\hat{m},1} + \gamma_{\hat{m},2} + 1} \tag{5}$$

where $\gamma_0 \sim \mathcal{G}\left(4, \sigma^2_{\mathsf{SD}} P/4\right)$, $\gamma_{\hat{m},1} \sim \mathcal{G}\left(4, \sigma^2_{\mathsf{SR}_{\hat{m}}} P/4\right)$, and $\gamma_{\hat{m},2} \sim \mathcal{G}\left(4, \sigma^2_{\mathsf{R}_{\hat{m}}\mathsf{D}} P/4\right)$, respectively.

For two positive numbers x and y, if we consider the constant $k > (x+y+1)/(x+y)$, the following inequality holds:

$$\frac{xy}{k(x+y)} < \frac{xy}{x+y+1} < \frac{xy}{x+y}.$$

Thus, (5) can be rewritten as

$$\gamma_0 + \frac{\mu_H(\gamma_{\hat{m},1}, \gamma_{\hat{m},2})}{2k} < \gamma_{\text{eq}} < \gamma_0 + \frac{\mu_H(\gamma_{\hat{m},1}, \gamma_{\hat{m},2})}{2} \tag{6}$$

where $\mu_H(a,b) \triangleq \dfrac{2ab}{a+b}$ and $k > 1 + (\gamma_{\hat{m},1} + \gamma_{\hat{m},2})^{-1}$.

The conditional PEP under ML decoder can be written as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}} | \mathbf{H}) \leq Q\left(\sqrt{\frac{\delta^2_{\mathbf{x}}}{2}\left(\gamma_0 + \frac{\mu_H(\gamma_{\hat{m},1}, \gamma_{\hat{m},2})}{2k}\right)}\right)$$
$$\Pr(\mathbf{x} \to \hat{\mathbf{x}} | \mathbf{H}) \geq Q\left(\sqrt{\frac{\delta^2_{\mathbf{x}}}{2}\left(\gamma_0 + \frac{\mu_H(\gamma_{\hat{m},1}, \gamma_{\hat{m},2})}{2}\right)}\right)$$

where $\delta_{\mathbf{x}} = \|\hat{\mathbf{x}} - \mathbf{x}\|$ and $k > 1 + (\gamma_{\hat{m},1} + \gamma_{\hat{m},2})^{-1}$.

The average PEP for the SDF protocol can by obtained by averaging the conditional PEP in the above equation over $\mathbf{H}$. Furthermore, Q-function is upperly and lowerly bounded by

$$\sum_{n=1}^{N} a_n \exp\left(-b_{n-1} x^2\right) \leq Q(x) \leq \sum_{n=1}^{N} a_n \exp\left(-b_n x^2\right) \tag{7}$$

where $a_n = (\theta_n - \theta_{n-1})/\pi$ and $b_n = 1/(2\sin^2\theta_n)$ for $n = 1, \cdots, N$ with $\theta_0 = 0$. And then, the PEP can be lowerly and upperly bounded

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \leq \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n}{2}\delta^2_{\mathbf{x}}\right) \mathcal{M}_{\gamma_{\max}}\left(\frac{b_n}{4k}\delta^2_{\mathbf{x}}\right)$$
$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \geq \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_{n-1}}{2}\delta^2_{\mathbf{x}}\right) \mathcal{M}_{\gamma_{\max}}\left(\frac{b_{n-1}}{4}\delta^2_{\mathbf{x}}\right)$$

where $\mathcal{M}_X(s)$ is the MGF of random variable X, i.e., the Laplace transform of X, and $\gamma_{\max} \triangleq \mu_H(\gamma_{\hat{m},1}, \gamma_{\hat{m},2})$.

Since the lower bound can have the same form as the upper bound except the index of b_n, it is enough to show the diversity order for the upper or lower bound of the PEP.

However, it is difficult to derive the maximum distribution of the harmonic mean bewteen two gamma random variables. For derivation of the diversity order, we will use the following relation:

$$\begin{aligned}\Pr\left(\gamma_{\max} \leq \gamma\right) &= \Pr\left(\gamma_1 \leq \gamma, \cdots, \gamma_M \leq \gamma\right) \\ &= \Pr\left(\lim_{r\to\infty}\left[\gamma_1^r + \cdots + \gamma_M^r \leq \gamma^r\right]\right).\end{aligned}$$

Since γ_m's are independent, the MGF of $\gamma_{\max}$ equals to the Mth power of the MGF of γ_m^r, i.e.,

$$\mathcal{M}_{\gamma_{\max}}(s) = \mathcal{M}_{\sum_{m=1}^{M}\gamma_m^r}(s) = \left[\mathcal{M}_{\gamma_m^r}(s)\right]^M$$

as $r \to \infty$. As γ_m has H-function distribution [9], so does γ_m^r. Since the MGF of γ_m is expressed in terms of H-function, the MGF of γ_m^r can be written as H-function as in the following theorem.

Theorem 1: The asymptotic MGF of $\gamma_{\max}$ is represented as

$$\mathcal{M}_{\gamma_{\max}}(s) \approx \left\{\left(\frac{\Omega}{2}\right)^{-4}\left[3 + \frac{95}{2^7}\left(\frac{\Omega}{2}\right)^{-4}\log\left(\frac{\Omega}{2}\right)\right]\right\}^M.$$

Proof: Let $Z_m = Y_m^r$ where $Y_m = \mu_H(\gamma_{m,1}, \gamma_{m,2})$ for $\gamma_{m,1}, \gamma_{m,2} \sim \mathcal{G}(K, \Omega)$. By the property of H-function distribution [9], the PDF of Z_m can be expressed by using H-function given as

$$\begin{aligned}f_{Z_m}(z) =& \frac{\sqrt{\pi}}{2^{2K-2}\Gamma^2(K)\Omega}\left(\frac{2}{\Omega}\right)^{r-1} \\ &\times \mathcal{H}^{2,0}_{1,2}\left[\left(\frac{2}{\Omega}\right)^r z \middle| \begin{matrix}(K-r+\frac{1}{2}, r) \\ (K-r, r), (2K-r, r)\end{matrix}\right]\end{aligned}$$

where $\Omega = \sigma^2 P/4$ and $\mathcal{H}^{m,n}_{p,q}\left[\cdot|\cdot\right]$ is the Fox's H-function defined in [10]. Taking the Laplace transform into the above PDF, the MGF of Z_m can be expressed as

$$\mathcal{M}_{Z_m}(s) = \frac{\sqrt{\pi}}{2^{2K-1}\Gamma^2(K)}\mathcal{H}^{1,2}_{2,2}\left[cs \middle| \begin{matrix}(1-K, r), (1-2K, r) \\ (0,1), (\frac{1}{2}-K, r)\end{matrix}\right] \tag{8}$$

where $c = (\frac{\Omega}{2})^r$. In this case, $K = 4$ is considered.

Using the result in [8], the following relation holds for for s at infinity :

$$\begin{aligned}&\mathcal{H}^{1,2}_{2,2}\left[cs \middle| \begin{matrix}(-3, r), (-7, r) \\ (0,1), (-\frac{7}{2}, r)\end{matrix}\right] \\ \approx& \sum_i{}' h_i (cs)^{\frac{a_i - 1}{\alpha_i}} + \sum_i{}'' H_{i,k} (cs)^{\frac{a_i - 1}{\alpha_i}} (\log cs) \\ =& h_1 (cs)^{-\frac{4}{r}} + \left\{H_{1,4}(cs)^{-\frac{4}{r}} + H_{2,0}(cs)^{-\frac{8}{r}}\right\}\log(cs).\end{aligned} \tag{9}$$

The constants h_1, H_{1,k_1}, and H_{2,k_2} are expressed as

$$\begin{aligned}h_1 &= \frac{1}{r}\frac{\Gamma(\frac{4}{r})\Gamma(4)}{\Gamma(\frac{1}{2})} \\ H_{1,k_1} &= \frac{1}{2r^2}\frac{\Gamma(\frac{4+k_1}{r})}{\Gamma(\frac{1}{2} - k_1)} \\ H_{2,k_2} &= \frac{1}{2r^2}\frac{\Gamma(\frac{8+k_2}{r})}{\Gamma(\frac{1}{2} - k_2)}.\end{aligned}$$

Thus, using the relation

$$\lim_{r\to\infty}\frac{\Gamma(\frac{x}{r})}{r} = \frac{1}{x} \text{ and } \lim_{r\to\infty} s^{-\frac{k}{r}} = 1,$$

the limits of each term in (9) are calculated as

$$\begin{aligned}&\lim_{r\to\infty}\left[h_1 (cs)^{-\frac{4}{r}}\right] \\ =& \lim_{r\to\infty}\left[\frac{1}{r}\frac{\Gamma(\frac{4}{r})\Gamma(4)}{\Gamma(\frac{1}{2})}\left(\frac{\Omega}{2}\right)^{-4} s^{-\frac{4}{r}}\right] \\ =& \frac{3}{2\sqrt{\pi}}\left(\frac{\Omega}{2}\right)^{-4}\end{aligned} \tag{10}$$

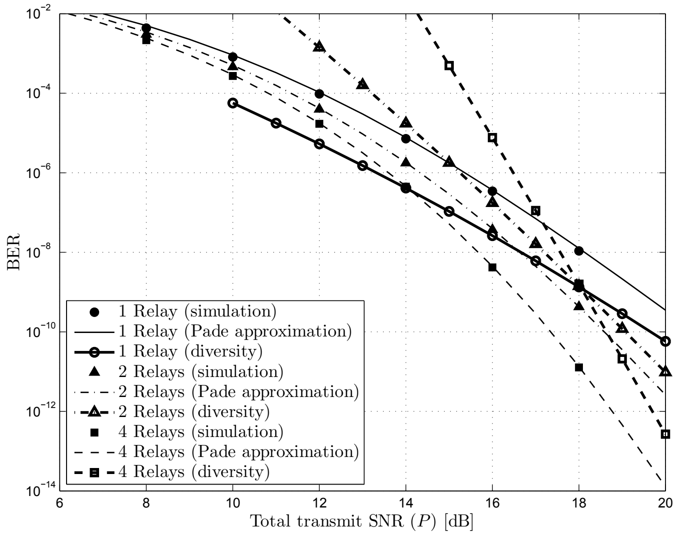

Fig. 2. BERs and diversity order of SDF cooperative network with the proposed best relay selection (The diversity plots are scaled for the clarity of view).

$$
\begin{aligned}
&\lim_{r\to\infty}\left[H_{1,4}(cs)^{-\frac{8}{r}}\log(cs)\right]\\
=&\lim_{r\to\infty}\left[\frac{1}{2r^2}\frac{\Gamma(\frac{8}{r})}{\Gamma(-\frac{7}{2})}\left(\frac{\Omega}{2}\right)^{-8}s^{-\frac{8}{r}}\log\left(\frac{\Omega}{2}\right)^{r}\right]\\
=&\frac{7\cdot 5\cdot 3}{2^8\sqrt{\pi}}\left(\frac{\Omega}{2}\right)^{-8}\log\left(\frac{\Omega}{2}\right) \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
&\lim_{r\to\infty}\left[H_{2,0}(cs)^{-\frac{8}{r}}\log(cs)\right]\\
=&\lim_{r\to\infty}\left[\frac{1}{2r^2}\frac{\Gamma(\frac{8}{r})}{\Gamma(-\frac{1}{2})}\left(\frac{\Omega}{2}\right)^{-8}s^{-\frac{8}{r}}\log\left(\frac{\Omega}{2}\right)^{r}\right]\\
=&-\frac{1}{2^5\sqrt{\pi}}\left(\frac{\Omega}{2}\right)^{-8}\log\left(\frac{\Omega}{2}\right). \qquad (12)
\end{aligned}
$$

Summing up the above results, (8) is approximated as

$$
\lim_{r\to\infty}\mathcal{M}_{Z_m}(s)\approx\frac{1}{2^{10}3^2}\left(\frac{\Omega}{2}\right)^{-4}\left[3+\frac{95}{2^7}\left(\frac{\Omega}{2}\right)^{-4}\log\left(\frac{\Omega}{2}\right)\right].
$$

Let $\gamma_{\max}=\sum_{m=1}^{M}Z_m$ where Z_m's are i.i.d. random variables for $m=1,\cdots,M$. Then, the MGF for $\gamma_{\max}$ can be obtained as

$$
\mathcal{M}_{\gamma_{\max}}(s)\approx\left\{\frac{1}{2^{10}3^2}\left(\frac{\Omega}{2}\right)^{-4}\left[3+\frac{95}{2^7}\left(\frac{\Omega}{2}\right)^{-4}\log\left(\frac{\Omega}{2}\right)\right]\right\}^{M}
$$

as $r\to\infty$. ■

From the above result, we can conclude that the diversity order of $\mathsf{S}\to\mathsf{R}_{\hat{m}}\to\mathsf{D}$ is $4M$. Since the diversity order of $\mathsf{S}\to\mathsf{D}$ is four, the diversity order of the cooperative network with the proposed best relay selection under ML decoder is $4(M+1)$.

Fig. 2 shows the analytical and numerical results for the cooperative network with the proposed best relay selection when the uniform power allocation between S and $\mathsf{R}_{\hat{m}}$ is used, i.e., $P_1=P_2=P/2$. The 'Pade approximation' results are obtained using Padé approximation technique [11]. As the number of the relays increases, the BER performance of the proposed scheme is always enhanced. And also, diversity orders from PEP are plotted. From this result, we can confirm that the proposed scheme has full diversity order.

IV. Conclusion

In this paper, the performance of best relay selection scheme has been shown. The PEP and diversity of the proposed relay selection scheme has been derived. And it has been shown that it has full diversity. From the numerical results, it has been shown that the best relay selection scheme has full diversity order.

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