A New Selected Mapping Scheme for PAPR Reduction in OFDM Systems

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***Abstract*—In this paper, a new SLM scheme for peak to average power ratio (PAPR) reduction in orthogonal frequency division multiplexing (OFDM) systems is proposed. This scheme is special case of selected mapping (SLM) scheme. The proposed SLM scheme generates alternative OFDM signal sequences by exploiting the intermediate OFDM signal sequence in inverse fast Fourier transform (IFFT). By using this technique, the proposed SLM scheme achieves similar PAPR reduction performance with much lower computational complexity and no bit error rate (BER) degradation. the performance of the proposed SLM scheme is verified through the simulations.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation method utilizing the orthogonality of subcarriers. OFDM has been adopted as a standard modulation method in several wireless communication systems such as digital audio broadcasting (DAB), digital video broadcasting (DVB), IEEE 802.11 wireless local area network (WLAN), and IEEE 802.16 wireless metropolitan area network (WMAN). Similarly to other multicarrier schemes, OFDM has the high peak to average power ratio (PAPR) problem which makes its straightforward implementation quite costly. Thus, it is highly desirable to reduce the PAPR of OFDM signals.

Over the last decades, various techniques to reduce the PAPR of OFDM signals have been proposed such as clipping [1], active constellation extension (ACE) [2], coding [3], tone reservation (TR) [4], partial transmit sequence (PTS) [5],[6], and selected mapping (SLM) [7]. In SLM scheme, an input symbol sequence is multiplied by each of phase rotation vectors to generate alternative input symbol sequences. Then alternative input symbol sequences are inverse fast Fourier transformed (IFFTed) and the one with the minimum PAPR is selected for transmission. In PTS scheme, an input symbol sequence is partitioned into a number of disjoint subblocks. Then, IFFT is applied to each subblock and the resulting IFFTed subblocks are multiplied by phase rotation vectors and summed. Among them, the one with the minimum PAPR is selected for transmission. SLM and PTS schemes require many IFFTs, which causes high computational complexity. Many PAPR reduction schemes with lower complexity than the conventional SLM scheme have been proposed [5],[6],[7], but they have several shortcomings such as degradation of PAPR reduction performance and bit error rate (BER) performance.

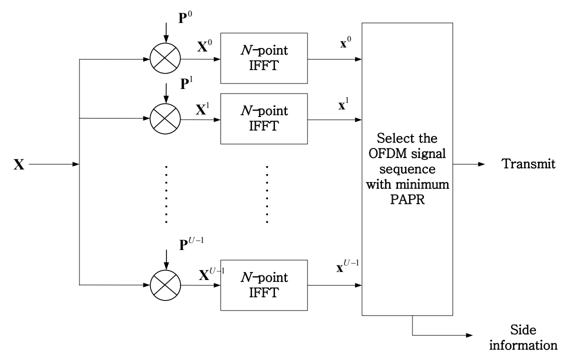


Fig. 1. A block diagram of the conventional SLM scheme.

In this paper, a new low-complexity SLM scheme is introduced, which generates alternative OFDM signal sequences by properly cyclically shifting the connections in each subblock at an intermediate stage in IFFT of input symbol sequence without additional IFFTs. This scheme has lower computational complexity compared with the conventional SLM without degradation of BER performance. Also, it is shown that its PAPR reduction performance can approach to that of the conventional SLM scheme by using randomly generated phase rotation vectors.

II. The Conventional SLM Scheme

In this paper, we use the upper case $\mathbf{X} = \{X(0), X(1), ..., X(N-1)\}$ for the input symbol sequence and the lower case $\mathbf{x} = \{x(0), x(1), ..., x(N-1)\}$ for the OFDM signal sequence. The relation of the input symbol sequence $\mathbf{X}$ and the OFDM signal sequence $\mathbf{x}$ can be expressed by IFFT as

$$\mathbf{x} = \mathrm{IFFT}(\mathbf{X}) \tag{1}$$

where $x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)W^{-kn}$, $W = e^{\frac{-j2\pi}{N}}$, and $j = \sqrt{-1}$. The conventional SLM scheme was proposed by Müller and Huber [8], as described in Fig. 1, which generates U statistically independent alternative OFDM signal sequences $\mathbf{x}^u$, $0 \le u \le U-1$, for the same input symbol sequence $\mathbf{X}$. To generate U alternative OFDM signal sequences, U distinct phase rotation vectors known to both transmitter and receiver are used, which are $\mathbf{P}^u$=$\{P^u(0),\ P^u(1), \cdots,\ P^u(N-1)\}$ where $P^u(k) = e^{j\phi^u(k)}$, $\phi^u(k) \in [0,\ 2\pi)$, $0 \le u \le U-1$. Note that each element of the phase rotation vector $\mathbf{P}^u$ is the power of the primitive K-th root of unity $e^{j\frac{2\pi}{K}}$ and, in general, $K = 2$ or 4 is used. An input symbol sequence $\mathbf{X}$ is multiplied by each phase rotation vector $\mathbf{P}^u$ element by element. Then, one input symbol sequence $\mathbf{X}$ is represented by U different alternative input symbol sequences $\mathbf{X}^u$, where $X^u(k) = X(k)P^u(k)$. These U different alternative input symbol sequences are IFFTed to generate U alternative OFDM signal seuences $\mathbf{x}^u = \text{IFFT}(\mathbf{X}^u), 0 \le u \le U-1$, and the PAPR values of U alternative OFDM signal sequences are calculated. Then the alternative OFDM signal sequence $\mathbf{x}^{\tilde{u}}$ having the minimum PAPR is selected for transmission as follows.

$$\tilde{u} = \underset{0 \le u \le U-1}{\arg\min} \left(\frac{\max|x^u(n)|^2}{E[|x^u(n)|^2]} \right). \tag{2}$$

III. A New SLM Scheme with Low Complexity

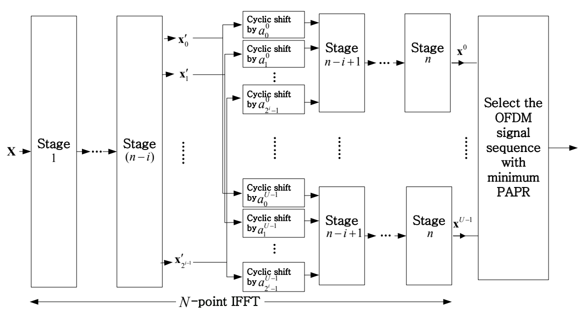


Fig. 2. A block diagram of the proposed SLM scheme($n = \log_2 N$).

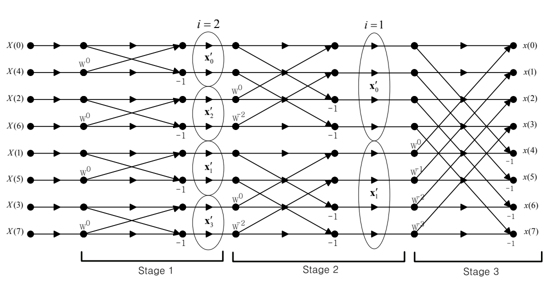


Fig. 3. Subblock partitions at stage 1 (i.e. $i = 2$) and stage 2 (i.e. $i = 1$) of IFFT when $N = 8$.

Fig. 2 shows a block diagram of the proposed SLM scheme. The N input symbols $X(k)$, $0 \le k \le N-1$, are processed by the ordinary N-point decimation-in-frequency IFFT with radix-2 algorithm up to stage $(n-i)$, where $n = \log_2 N$ and i is the number of remaining stages until finishing the IFFT. The intermediate OFDM signal sequence $\mathbf{x}'$ at the stage $(n-i)$ is divided into 2^i subblocks $\mathbf{x}'_0, \mathbf{x}'_1, ..., \mathbf{x}'_{2^i-1}$. A subblock $\mathbf{x}'_m$ is composed of 2^{n-i} outputs at the stage $(n-i)$ of IFFT, which is equivalent to the 2^{n-i}-point IFFT using the input symbol sequence $X(k)$ satisfying $k \bmod 2^i = m$. Fig. 3 shows an example of subblock partitions when $N = 8$ and $i = 1,\ 2$ in decimation-in-frequency IFFT. The elements in each subblock of $\mathbf{x}'_0, \mathbf{x}'_1, ..., \mathbf{x}'_{2^i-1}$ are cyclically shifted upward by the predetermined integer numbers, $a_0^j, a_1^j, ..., a_{2^i-1}^j$, $0 \le j \le U-1$, respectively, to generate the jth alternative OFDM signal sequence. Then these cyclically shifted 2^i subblocks become the inputs to the stage $(n-i+1)$ of N-point IFFT to generate U alternative OFDM signal sequences, $\mathbf{x}^0, \mathbf{x}^1, ..., \mathbf{x}^{U-1}$. Finally, the OFDM signal sequence having the minimum PAPR among U alternative OFDM signal sequences is selected for transmission. Compared with the conventional SLM scheme, the proposed scheme can reduce the amount of computations for IFFTs to make U alternative OFDM signal sequences, which will be explained in the next section.

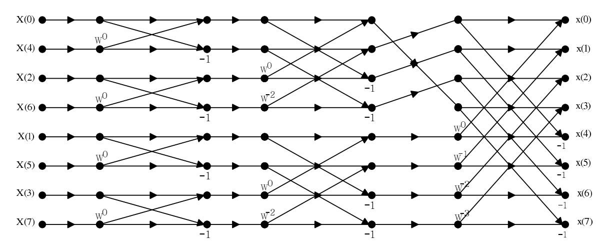


Fig. 4. An alternative OFDM signal sequence by the proposed scheme for $N = 8$, $i = 1$, and $a_0 = 1$ and $a_1 = 0$.

Fig. 4 shows an example of the proposed scheme using $a_0 = 1$ and $a_1 = 0$ for $N = 8$ and $i = 1$. Clearly, the original OFDM signal sequence can be generated by using $a_0 = 0$ and $a_1 = 0$. Other alternative OFDM signal sequences can be generated by simply changing the shift values a_0 and a_1. If $i = 2$, each of four subblocks, $\mathbf{x}'_0, \mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3$ is cyclically shifted and the last two stages of IFFT are performed as the ordinary IFFT. The value i can be any one of $1, 2, ...$,and $\log_2 N - 1$. As i increases, the PAPR reduction performance improves but the computational complexity also increases, which will be explained in the following sections.

The amount of side information for the proposed scheme is $\lceil \log_2 U \rceil$ which is the same as that of the conventional SLM. If the side information is received without error at the receiver, input symbol sequence can be obtained by multiplying the phase rotation vector corresponding to $\mathbf{P}^u$. Therefore, there is no BER degradation in the proposed SLM scheme.

IV. Computational Complexity of the Proposed SLM Scheme

In this section, the computational complexity of the proposed scheme is compared with that of the conventional SLM scheme. When the number of subcarriers is $N = 2^n$, the number of complex multiplications and complex additions

TABLE I

CCRR(%) OF THE PROPOSED SCHEME COMPARED OVER THE CONVENTIONAL SLM.

N	64			256			1024		
U	4	8	16	4	8	16	4	8	16
$i=1$	62.5	72.9	78.1	65.6	76.6	82.0	67.5	78.8	84.4
$i=2$	50.0	58.3	62.5	56.3	65.6	70.3	60.0	70.0	75.0
$i=3$	37.5	43.8	46.9	46.9	54.7	58.6	52.5	61.3	65.6
$i=4$	25.0	29.2	31.3	37.5	43.8	46.9	45.0	52.5	56.3

required for the conventional SLM scheme can be derived as follows. Using radix-2 algorithm, an N-point IFFT requires $(N/2)\log_2 N$ complex multiplications and $N\log_2 N$ complex additions. Therefore, the total number of complex multiplications and complex additions for the conventional SLM scheme using U alternative OFDM signal sequences are $U(N/2)\log_2 N$ and $UN\log_2 N$, respectively. In the proposed scheme, if the cyclic shifts are performed at the stage $(n-i)$, the number of complex multiplications and complex additions are $(N/2)\log_2 N + (U-1)(i/n)(N/2)\log_2 N$ and $N\log_2 N + (U-1)(i/n)N\log_2 N$, respectively. Note that the number of complex multiplications and complex additions have the same reduction ratio. Therefore, the computational complexity reduction ratio (CCRR) of the proposed scheme over the conventional SLM scheme is derived only for complex multiplication, which is defined as

$$\begin{aligned} \text{CCRR} &= \left(1 - \frac{\text{Complexity of the proposed scheme}}{\text{Complexity of the conventional SLM}}\right) \times 100 \ (\%) \\ &= 1 - \frac{n+(U-1)i}{nU} \ (\%) = \frac{(n-i)(U-1)}{nU} \ (\%). \end{aligned} \tag{3}$$

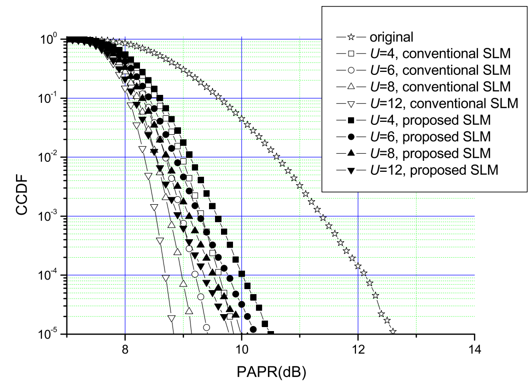
As shown in Table I, the proposed scheme has much lower computational complexity than that of the conventional SLM scheme. For example, when $i = 3, N = 1024$, and $U = 4$, the computational complexity of the proposed scheme reduces by almost 50 percent compared with the conventional SLM scheme with almost similar PAPR reduction performance. It is clear that the CCRR is large when N is large and i is small, but there appears a small amount of degradation in the PAPR reduction performance as will be shown in next section.

V. SIMULATION RESULTS

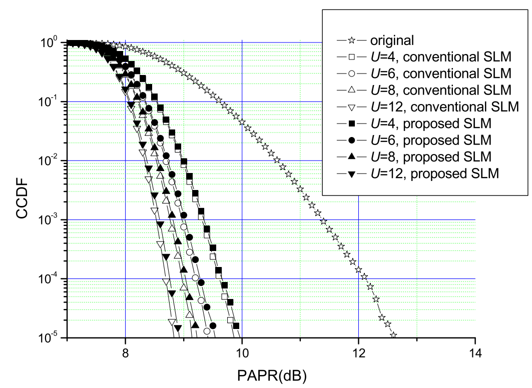
For the numerical analysis, 10^6 input symbol sequences are randomly generated from the full ensemble of the possible input symbol sequences, 16-quadrature amplitude modulation (16-QAM) is assumed, and oversampling is not considered. To compare the PAPR reduction performance, complementary cumulative distribution function (CCDF) of the PAPR is used. The CCDF of the PAPR denotes the probability that the PAPR exceeds a given threshold. The CCDF of the PAPR is derived as

$$P(PAPR > z) = 1 - (1 - \exp(-z))^N. \tag{4}$$

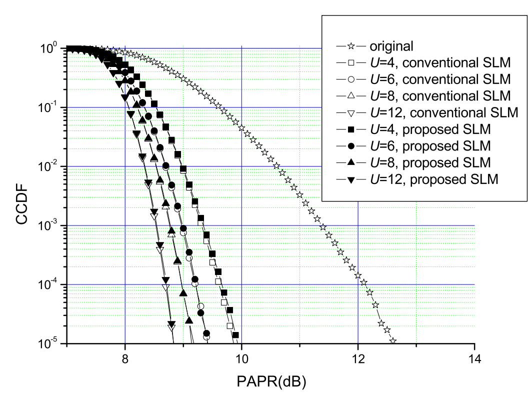
Fig. 5 compares the PAPR reduction performance of the proposed SLM scheme using random cyclic shift values with that of the conventional SLM scheme when $N = 1024$ and



(a)



(b)



(c)

Fig. 5. PAPR reduction performance of the proposed SLM scheme when $N = 1024$ and 16-QAM (a) $i = 1$, (b) $i = 2$, (c) $i = 3$.

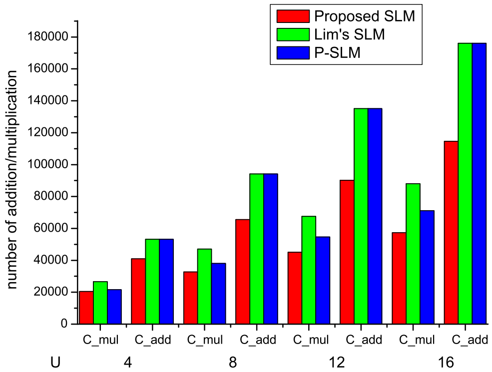


Fig. 6. Comparison of the computational complexity of the proposed SLM, P-SLM, and Lim's SLM when $N = 2048$ and 16-QAM.

$i = 1, 2, 3$. Fig. 5 shows that the PAPR reduction performance of the proposed SLM scheme becomes better as i increases. It is observed from Fig. 5 that the PAPR reduction performance of the proposed SLM scheme becomes closer to that of the conventional SLM scheme as i increases. When $i = 3$, both schemes show almost the same PAPR reduction performance. Since the performance of the proposed SLM scheme is lower bounded by that of the conventional SLM scheme and the computational complexity increases as i increases, the appropriate value of i is 2 or 3.

Fig. 6 shows that the comparison of the computational complexity of the proposed SLM scheme, Lim's SLM scheme [7], and P-SLM scheme [9]. Each scheme has the almost same PAPR reduction performance closely to the conventional SLM scheme when $N = 2048$ and 16-QAM is used. The proposed SLM scheme has the lowest computational complexity among these SLM schemes.

VI. Conclusion

A new SLM scheme with low computational complexity is proposed. This scheme shows almost the same PAPR reduction performance compared with that of the conventional SLM scheme. In the proposed scheme, performing U IFFTs in the conventional SLM scheme is exchanged to operating one IFFT up to $(n - i)$ stages, which is common to generating all alternative OFDM signal sequences. Then, each subblock in the stage $(n-i)$ of IFFT is cyclically shifted by predetermined shift value in the proposed scheme. Since the cyclic shift of each subblock can be expressed as an equivalent phase rotation vector, the proposed scheme is a special case of the conventional SLM scheme. Simulation results show that the proposed scheme can achieve almost the similar PAPR reduction performance as the conventional SLM scheme with much lower computational complexity and no degradation of BER performance.

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