Interference Alignment Aided by Relays for the Quasi-Static X Channel

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***Abstract*—In this paper, two relay-aided interference alignment (IA) schemes are proposed for the quasi-static $M \times 2$ X channel. The first scheme utilizes one full-duplex relay and the second scheme utilizes two half-duplex relays. In the proposed schemes, transmitters transmit in every time slot and relays operate in amplify-and-forward (AF) mode. By confirming the linear independence between the desired signals and interference and also among the desired signals, it is shown that the proposed schemes can achieve $2M/(M+1)$ degrees of freedom (DoF) which is the maximum DoF of $M \times 2$ X channel.**

***Index Terms*—Full-duplex relaying, half-duplex relaying, interference alignment (IA), relay-aided interference alignment, X channel**

I. Introduction

Interference alignment (IA) has been studied to achieve the optimal multiplexing gain in the interference channel. Especially, the IA schemes for general K user interference channel and $M \times N$ X channel were proposed in [1] and [2]. By extending desired symbol size in time domain, they can asymptotically achieve the maximum degrees of freedom (DoF) for each channel.

To achieve the maximum DoF, a relay-aided approach was investigated for 3-user interference channel [3], which can achieve $3/2$ DoF. But these schemes assume that the channel varies every symbol time. Since all transmitters and receivers need global channel state information and a lot of feedback information is necessary, these schemes may not be practical for time-varying channel. Therefore, research on IA in the slow fading environment is needed from the practical viewpoint.

In order to implement IA scheme in the slow fading environment, multiple channels can be used such as multiple carriers or multiple antennas. Since these resources are limited, IA scheme with time extension is still efficient to support many users.

Nourani et al. proposed the relay-aided IA schemes for quasi-static X channel and interference channel in [4] and [5]. They used full-duplex relays with some memories and proved that the beamforming signal vector for IA can be designed in the quasi-static environment. These schemes suggested the possibility of IA in the quasi-static environment but full-duplex relay is difficult to implement in general. In fact, half-duplex relays are widely used because transmitted signal from a relay can be fedback to itself as a self interference in full-duplex mode. Therefore, research on relay-aided IA scheme using half-duplex relay is necessary.

In this paper, a simple full-duplex relay-aided IA scheme is introduced, and then, IA schemes with two half-duplex relays will be proposed for the quasi-static X channel. In addition to that, by checking whether the desired and interference signal vectors are linearly independent each other, the maximum DoF of them will be derived.

II. System Model and Preliminaries

In this section, two system models are given for two relay-aided IA schemes and then, IA scheme for $M \times 2$ X is described.

A. System Model

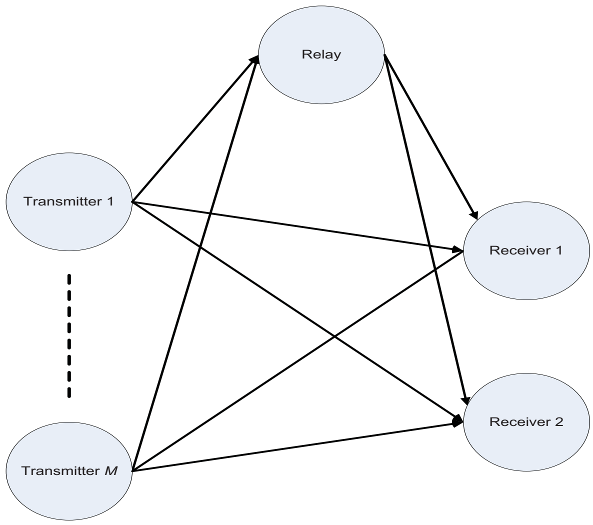
It is assumed that the number of transmitters is M and the number of receivers is 2. It is also assumed that one antenna is equipped at the every node and every channel is quasi-static Rayleigh fading channel. All transmitters and receivers know state information of all the channels. Fig. 1 shows system models for the two relay aided IA schemes.

- System model for a full-duplex relay case (Fig. 1(a)):
  In this model, a full-duplex relay operates in amplify-and-forward (AF) mode. Let $y_r(k)$ and $y_j(k)$ be the received signals at the relay and receiver j at the time slot k. Then the system model is given as

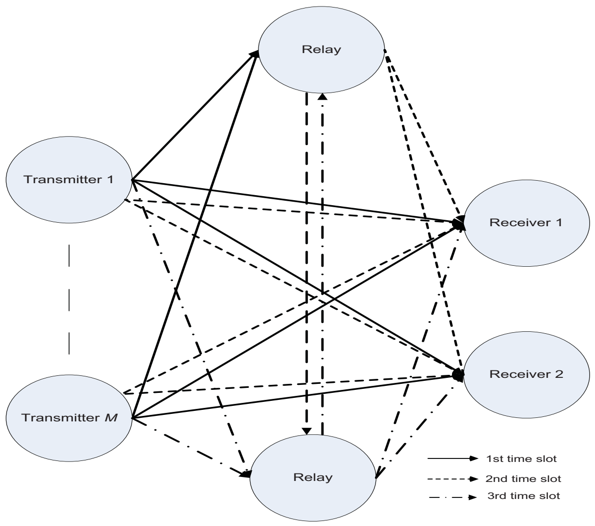
$$y_r(k) = \sum_{i=1}^{M} h_{ri} x_i(k) + n_r(k), \tag{1}$$

$$y_j(k) = \sum_{i=1}^{M} h_{ji} x_i(k) + u(k) y_r(k-1) + n_j(k), \ j = 1, 2 \tag{2}$$

  where h_{ri} and h_{jr} denote the channel coefficients between the transmitter i and the relay and between the relay and the receiver j, respectively. $n_r(k)$ and $n_j(k)$ are the additive noises at the relay and receiver j, respectively. For simplicity, it is assumed that h_{ri}, h_{jr}, $n_r(k)$, and $n_j(k)$ are independent complex Gaussian random variables with the distribution $C(0, 1)$ of zero mean and



(a)



(b)

Fig. 1. System model for the $M \times 2$ X channel: Aided by (a) a full-duplex relay and (b) two half-duplex relays

unit variance. $u(k)$ is the relay gain and $u(k) = 0$ when $k = 0$.
This model is simpler than that in [4] since a relay does not cumulatively store the symbols in every time slot and forwards only one symbol. In this model, each transmitter uses $M + 1$ symbol extension to transmit a data stream because it can achieve $2M/(M + 1)$ DoF which is the maximum DoF of X channel. The achievability will be proved in Section III.

- System model for the two half-duplex relay case (Fig. 1(b)):
In this model, two relays are used between the transmitters and receivers. The received signals at the relays are given as

$$\begin{aligned} y_{r1}(2k-1) &= \sum_{i=1}^{M} h_{(r1)i} x_i(2k-1) \\ &\quad + gu(2k-1) y_{r2}(2k-2) + n_{r1}(k) \text{ at relay 1}, \\ y_{r2}(2k) &= \sum_{i=1}^{M} h_{(r2)i} x_i(2k) \\ &\quad + gu(2k) y_{r1}(2k-1) + n_{r2}(2k) \text{ at relay 2} \end{aligned} \quad (3)$$

and the received signals at the receivers are also given as

$$\begin{aligned} y_j(k) &= \sum_{i=1}^{M} h_{ji} x_i(k) + u(k-1) h_{j(r2)} y_{r2}(k-1) \\ &\quad + n_j(k), j = 1, 2, \text{ and } k = \text{odd}, \\ y_j(k) &= \sum_{i=1}^{M} h_{ji} x_i(k) + u(k-1) h_{j(r1)} y_{r1}(k-1) \\ &\quad + n_j(k), j = 1, 2, \text{ and } k = \text{even} \end{aligned} \quad (4)$$

where $x_i(\cdot)$ is the transmitted signal from transmitter i. $h_{(r1)i}$, $h_{j(r1)}$, $h_{(r2)i}$, and $h_{j(r2)}$ are defined similarly as h_{ij} and g is the channel coefficient between two relays. It is assumed that they are mutually independent with the distribution $C(0, 1)$. n_{r1}, and n_{r2} are the noises at relay 1 and 2, respectively, with the distribution $C(0, 1)$.
In this model, the transmitters transmit new signals at every time slot and each relay operates in AF mode (non-orthogonal AF). But if one relay transmits the signal, the other should receive. In other words, they transmit and receive by turns. Since the received signal at one relay is from the transmitters and the other relay, it contains all the past and current data symbols. Note that each transmitter also uses $M + 1$ symbol extension to transmit a data stream.

B. Preliminaries: Interference Alignment for $M \times 2$ X Channel

In this subsection, IA scheme in [2] will be described. In [2], it was proved that the maximum DoF of $M \times N$ X channel is $MN/(M + N - 1)$ and $2M/(M + 1)$ DoF can obtained for $M \times 2$ X channel by using $M + 1$ symbol extension. The steps to design beamforming vectors are given as follows.

First of all, we should design linearly independent beamforming vectors b_{11} and b_{21} from the transmitter 1 to receivers 1 and 2, respectively. And then, the other beamforming vectors are designed using the following relations.

$$H_{2i} b_{1i} = H_{21} b_{11}, \ i = 2, \ldots, M, \text{ for receiver 1} \quad (5)$$

$$H_{1i} b_{2i} = H_{11} b_{21}, \ i = 2, \ldots, M, \text{ for receiver 2} \quad (6)$$

where b_{ji} and H_{ji} denote the $(M+1) \times 1$ beamforming vector and the channel matrix between transmitter i and receiver j. $x_i(k)$ can be obtained by $b_{1i,k} + b_{2i,k} = x_i(k)$ where $b_{ji,k}$ is the kth element of b_{ji}. In (5) and (6), the equality means that two vectors are placed in the same direction.

The received signal spaces at each receiver can be given as follows

$$\Gamma_1 = \begin{bmatrix} H_{11}b_{11} & H_{12}b_{12} & \cdots & H_{1M}b_{1M} & H_{11}b_{21} \end{bmatrix} \quad (7)$$

$$\Gamma_2 = \begin{bmatrix} H_{21}b_{21} & H_{22}b_{22} & \cdots & H_{2M}b_{2M} & H_{21}b_{11} \end{bmatrix}. \quad (8)$$

Multiplying Γ_1 and Γ_2 by H_{11}^{-1} and H_{21}^{-1}, respectively, (7) and (8) can be written as

$$\Delta_1 = \begin{bmatrix} b_{11} & H_{11}^{-1}H_{12}b_{12} & \cdots & H_{11}^{-1}H_{1M}b_{1M} & b_{21} \end{bmatrix} \quad (9)$$

$$\Delta_2 = \begin{bmatrix} b_{12} & H_{21}^{-1}H_{22}b_{22} & \cdots & H_{21}^{-1}H_{1M}b_{2M} & b_{11} \end{bmatrix}. \quad (10)$$

If these matrices have full rank, each receiver can detect all desired signal vectors by zero-forcing filter. This is called perfect IA in this paper.

In [2], it was proved that these matrices have full rank and guarantee $2M/(M+1)$ DoF. However, it was assumed that every channel is time-varying and the equivalent channel matrix has the diagonal form in [2]. Thus, its analysis cannot be applied to this paper directly. Finally, it should be proved that Δ_1 and Δ_2 have full rank in the system models of the proposed schemes and the proof will be given in the next section.

III. Achievability of Proposed IA Schemes

In this section, we investigate the equivalent channel matrix of each system model and confirm that perfect IA can be achieved by the proposed relay aided IA schemes.

A. Aided by A Full-Duplex Relay

The equivalent channel matrix between transmitter i and receiver j in Fig. 1(a) is given as

$$H_{ji} = \begin{bmatrix} h_{ij} & & & & \mathbf{0} \\ u(1)h_{ri}h_{jr} & h_{ij} & & & \\ & u(2)h_{ri}h_{jr} & h_{ij} & & \\ & & \ddots & \ddots & \\ \mathbf{0} & & & u(M)h_{ri}h_{jr} & h_{ij} \end{bmatrix}. \quad (11)$$

In this $(M+1)\times(M+1)$ square matrix, the main diagonal elements and subdiagonal elements are not zero but the other elements are zero.

If a column vector of Δ_1 in (9) is linearly dependent, it can be written as

$$0 = \sum_{i\in\{1,\ldots,M\}} a_i T_{1i} b_{11} + a_{M+1} b_{21} \quad (12)$$

where T_{1i} denotes $H_{11}^{-1}H_{1i}H_{2i}^{-1}H_{21}$ and it has the following form.

$$T_{1i} = \begin{bmatrix} \mathcal{K}_1^{(i)} & & & & \mathbf{0} \\ c_{11}\mathcal{K}_2^{(i)} & \mathcal{K}_1^{(i)} & & & \\ c_{12}\mathcal{K}_3^{(i)} & c_{22}\mathcal{K}_2^{(i)} & \mathcal{K}_1^{(i)} & & \\ \vdots & \cdots & \cdots & \ddots & \\ c_{1M}\mathcal{K}_{M+1}^{(i)} & c_{2M}\mathcal{K}_M^{(i)} & \cdots & c_{MM}\mathcal{K}_2^{(i)} & \mathcal{K}_1^{(i)} \end{bmatrix} \quad (13)$$

where $\mathcal{K}_l^{(i)}$ is the random variable from T_{1i} and $c_{ij} = \prod_{k=i}^{j} u(k)$. Since $\mathcal{K}_l^{(i)\prime}$s are too complex to describe, the detail expressions are omitted in this paper. But note that they are continuous random variables because h_{ij}, h_{ri}, and h_{jr} are continuous random variables. $\mathcal{K}_l^{(i)\prime}$s are correlated but not linearly related, that is $\mathcal{K}_l^{(i)} \neq c\mathcal{K}_k^{(j)}$ when $i \neq j$ or $l \neq k$ because T_{1i} contains H_{2i}^{-1} which can be obtained by nonlinear computation.

In order to satisfy (12), a non-zero vector $\mathbf{a} = (a_1, a_2, ..., a_{M+1})$ satisfying the following system of equations should exist.

$$\begin{aligned} &\sum_{i=1}^{M} a_i \mathcal{K}_1^{(i)} b_{11,1} + a_{M+1} b_{21,1} = 0 \\ &c_{11}\sum_{i=1}^{M} a_i \mathcal{K}_2^{(i)} b_{11,1} + \sum_{i=1}^{M} a_i \mathcal{K}_1^{(i)} b_{11,2} + a_{M+1} b_{21,2} = 0 \\ &\vdots \\ &c_{1M}\sum_{i=1}^{M} a_i \mathcal{K}_{M+1}^{(i)} b_{11,1} + c_{2M}\sum_{i=1}^{M} a_i \mathcal{K}_M^{(i)} b_{11,2} \\ &+ \cdots + \sum_{i=1}^{M} a_i \mathcal{K}_1^{(i)} b_{11,M+1} + a_{M+1} b_{21,M+1} = 0. \end{aligned} \quad (14)$$

It can be rewritten as

$$\mathcal{C}_f \mathbf{a} = 0 \quad (15)$$

where $\mathcal{C}_f$ is given as (16) in the next page.

If $\mathcal{C}_f$ is nonsingular, (14) can have only one solution $\mathbf{a} = \mathbf{0}$. In [6], it was proved that a continuous real random matrix is almost surely nonsingular regardless of distributions of its entries. This analysis can be applied to $\mathcal{C}_f$ since it is a continuous complex random matrix. Therefore, (14) is a system of equations which consists of $M+1$ independent equations and there is no solution except for zero vector $\mathbf{a}$. Thus (12) cannot be satisfied and receiver 1 can detect every desired signals by zero-forcing filter. For receiver 2, this proof can be performed in the same way.

Finally, by using a full-duplex relay, $2M/(M+1)$ DoF can be achieved.

B. Aided by Two Half-Duplex Relays

Considering arbitrary time extension, the equivalent channel matrix of this scheme between transmitter i and receiver j is given as (17)

Similarly to the previous subsection, T_{1i} can be obtained as (18) where $M \bmod 2$ means modulo 2 reduction of M. $\mathcal{K}_{l,m}^{(i)\prime}$s are correlated continuous random variables but they are not linearly related, that is, $\mathcal{K}_{l,m}^{(i)} \neq c\mathcal{K}_{k,n}^{(j)}$ when $i \neq j$, $l \neq k$, or $m \neq n$ because T_{1i} contains H_{2i}^{-1} which can be obtained by non-linear computation.

In order to satisfy (12), a non-zero vector $\mathbf{a} = (a_1, a_2, ..., a_{M+1})$ satisfying the following system of equations

$$\mathcal{C}_f = \begin{bmatrix} \mathcal{K}_1^{(1)} b_{11,1} & \cdots & \mathcal{K}_1^{(M)} b_{11,1} & b_{21,1} \\ c_{11}\mathcal{K}_2^{(1)} b_{11,1} + \mathcal{K}_1^{(1)} b_{11,2} & \cdots & c_{11}\mathcal{K}_2^{(M)} b_{11,1} + \mathcal{K}_1^{(M)} b_{11,2} & b_{21,2} \\ & \vdots & & \\ \sum_{j=1}^{M} c_{jM}\mathcal{K}_{M+2-j}^{(1)} b_{11,j} + \mathcal{K}_1^{(1)} b_{11,M+1} & \cdots & \sum_{j=1}^{M} c_{jM}\mathcal{K}_{M+2-j}^{(M)} b_{11,j} + \mathcal{K}_1^{(M)} b_{11,M+1} & b_{21,M+1} \end{bmatrix} \quad (16)$$

$$H_{ji} = h_{ji} \begin{bmatrix} 1 & & & & \mathbf{0} \\ \frac{u(1)h_{(r1)i}h_{j(r1)}}{h_{ji}} & 1 & & & \\ \frac{u(2)u(1)gh_{(r1)i}h_{j(r2)}}{h_{ji}} & \frac{u(2)h_{(r2)i}h_{j(r2)}}{h_{ji}} & 1 & & \\ \frac{u(3)u(2)u(1)g^2h_{(r1)i}h_{j(r1)}}{h_{ji}} & \frac{u(3)u(2)gh_{(r2)i}h_{j(r1)}}{h_{ji}} & \frac{u(3)h_{(r1)i}h_{j(r1)}}{h_{ji}} & 1 & \\ \vdots & \cdots & \cdots & \ddots & \ddots \end{bmatrix} \quad (17)$$

$$T_{1i} = \begin{bmatrix} \mathcal{K}_1^{(i)} & & & & & \mathbf{0} \\ c_{11}\mathcal{K}_{2,0}^{(i)} & \mathcal{K}_1^{(i)} & & & & \\ c_{12}\mathcal{K}_{3,0}^{(i)} & c_{22}\mathcal{K}_{2,1}^{(i)} & \mathcal{K}_1^{(i)} & & & \\ c_{13}\mathcal{K}_{4,0}^{(i)} & c_{23}\mathcal{K}_{3,1}^{(i)} & c_{33}\mathcal{K}_{2,0}^{(i)} & \mathcal{K}_1^{(i)} & & \\ \vdots & \cdots & \cdots & \ddots & \ddots & \\ c_{1M}\mathcal{K}_{(M+1),0}^{(i)} & c_{2M}\mathcal{K}_{M,1}^{(i)} & \cdots & \cdots & c_{MM}\mathcal{K}_{2,(M-1)\mathrm{mod}2}^{(i)} & \mathcal{K}_1^{(i)} \end{bmatrix} \quad (18)$$

$$\mathcal{C}_h = \begin{bmatrix} \mathcal{K}_1^{(1)} b_{11,1} & \cdots & \mathcal{K}_1^{(M)} b_{11,1} & b_{21,1} \\ c_{11}\mathcal{K}_{2,0}^{(1)} b_{11,1} + \mathcal{K}_1^{(1)} b_{11,2} & \cdots & c_{11}\mathcal{K}_{2,0}^{(M)} b_{11,1} + \mathcal{K}_1^{(3)} b_{11,2} & b_{21,2} \\ & \vdots & & \\ \sum_{j=1}^{M} c_{jM}\mathcal{K}_{M+2-j,(j-1)\mathrm{mod}2}^{(1)} b_{11,j} + \mathcal{K}_1^{(1)} b_{11,M+1} & \cdots & \sum_{j=1}^{M} c_{jM}\mathcal{K}_{M+2-j,(j-1)\mathrm{mod}2}^{(M)} b_{11,j} + \mathcal{K}_1^{(M)} b_{11,M+1} & b_{21,M+1} \end{bmatrix} \quad (21)$$

should exist.

$$\begin{aligned} &\sum_{i=1}^{M} a_i \mathcal{K}_1^{(i)} b_{11,1} + a_{M+1} b_{21,1} = 0 \\ &c_{11} \sum_{i=1}^{M} a_i \mathcal{K}_{2,0}^{(i)} b_{11,1} + \sum_{i=1}^{M} a_i \mathcal{K}_1^{(i)} b_{11,2} + a_{M+1} b_{21,2} = 0 \\ &\qquad \vdots \\ &c_{1M} \sum_{i=1}^{M} a_i \mathcal{K}_{(M+1),0}^{(i)} b_{11,1} + c_{1(M-1)} \sum_{i=1}^{M} a_i \mathcal{K}_{M,1}^{(i)} b_{11,2} + \cdots \\ &+ c_{MM} \sum_{i=1}^{M} a_i \mathcal{K}_{2,(M\mathrm{mod}2)}^{(i)} b_{11,M} + \sum_{i=1}^{M} a_i \mathcal{K}_1^{(i)} b_{11,M+1} \\ &+ a_{M+1} b_{21,M+1} = 0. \end{aligned} \quad (19)$$

It can be written as

$$\mathcal{C}_h \mathbf{a} = 0 \quad (20)$$

where $\mathcal{C}_h$ is given as (21). Similarly to the full-duplex relay case, it can be seen that (19) is a system of equations which consists of $M+1$ independent equations. Therefore, receiver 1 can detect every desired signal from the received signal. This proof can also be applied to the receiver 2. Therefore, this scheme can also achieve the maximum DoF of $M \times 2$ X channel.

Equation (17) is similar to that of the relay aided IA scheme in [4] which can guarantee $2M/(M+1)$ DoF. Actually, this is the converted version of a full-duplex relay aided IA scheme to the two half-duplex relay aided one. But the proposed scheme needs only one memory at each relay and does not use full-duplex relay.

IV. CONCLUSION

In this paper, two relay-aided IA schemes were proposed for the quasi-static $M \times 2$ X channel which were IA schemes aided by a full-duplex relay and two half-duplex relays. It was proved that the proposed schemes can achieve the perfect IA and, especially, it could be seen that two half-duplex relays can replace a full-duplex relay for IA in the quasi-static $M \times 2$ X channel.

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REFERENCES

[1] V. Cadambe and S. Jafar "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[2] V. Cadambe and S. Jafar, "Interference alignment and the degrees of freedom of wireless X networks," *IEEE Trans. Inf. Theory,* vol. 55, no. 9, pp. 3893–3908, Sep. 2009.

[3] K. Gomadam, V. Cadambe, and S. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," arXiv:cs.IT/0803.3816, Mar. 2008

[4] B. Nourani, S. Motahari, and A. Khandani, "Relay-aided interference alignment for the quasi-static X channel," in *Proc. IEEE ISIT*, Jun. 2009.

[5] B. Nourani, S. Motahari, and A. Khandani, "Relay-aided interference alignment for the quasi-static interference channel," in *Proc. ISIT*, Jun. 2010.

[6] X. Feng and Z. Zhang, "The rank of a random matrix," *Appl. Math. Comput.*, vol. 185, no. 1, pp. 689–694, Feb. 2007.