# Combining Interference Alignment and Alamouti Codes for Quasi-Static MIMO X Channel

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***Abstract*—In this paper, a new combining scheme of interference alignment and Alamouti codes is proposed for the quasi-static MIMO $K \times 2$ X channel. In fact, a brilliant scheme was already proposed in [1] for $2 \times 2$ MIMO X channel. However, it is difficult to extend for more general case. Therefore, for $K \times 2$ MIMO X channel, a simple transmission and decoding schemes is proposed in this paper. The transmission scheme is based on the interference alignment concept and Alamouti-code and the decoding scheme exploits the zero-forcing and decoupling method of two Alamouti-codes.**

## I. Introduction

Recently, the interference alignment (IA) has received attention for interference management. Especially, for $K$-user interference channel and $M \times N$ X channel, IA schemes are proposed in [2], [3] which achieve the maximum degrees of freedom (DOF) asymptotically.

From the viewpoint of the throughput, DOF is a good measure for the spectral efficiency. However, the reliability is also important factor for wireless communications and the diversity gain is the best-known measure for the reliability [4]. To achieve the more diversity gain, many space-time codes (STCs) have been introduced until now. Among STCs, the space-time block codes (STBCs) have been studied by many engineers because of their simple structure and Alamouti-code is the first STBC [5].

In the environment where much interference exist, both of throughput and reliability should be satisfied therefore, the combination of IA and STCs can be a good trial.

Li, Jafarkhani and Syed Ali Jafar proposed the combination scheme of IA and Alamouti code in [1]. Their scheme achieve the maximum DOF of two-user MIMO X channel with two antennas and diversity order two. It is a great advance because its reliability was improved greatly compared to the conventional IA scheme. However, it was the result only for the two-user X channel and until now, for $K \times 2$ MIMO X channel, the maximum DOF and diversity order are open problems. Therefore, in this paper, for $K \times 2$ X channel, a simple combination scheme of IA and Alamouti code is proposed. Two decoding schemes are also combined, which are the zero-forcing and decoupling scheme (DS) in [6]. In [6], the decoupling scheme was introduced for two-user multiple access channel, where the receiver can decouple two independent Almouti-codes when they are received simultaneously. In this paper, each receiver converts $K \times 2$ X channel into some independent multiple access channels by using zero-forcing and then, it uses DS in order to decouple two Alamouti codes .

## II. System Model

In this paper, $K \times 2$ MIMO X channel is considered. At each receiver, the received signal is given as

$$\mathbf{Y}_1 = \sum_{i=1}^{K} \mathbf{H}_{1i}\mathbf{A}_{1i}\mathbf{C}_{1i} + \sum_{i=1}^{K} \mathbf{H}_{1i}\mathbf{A}_{2i}\mathbf{C}_{2i} + \mathbf{n}_1 \quad (1)$$

$$\mathbf{Y}_2 = \sum_{i=1}^{K} \mathbf{H}_{2i}\mathbf{A}_{2i}\mathbf{C}_{2i} + \sum_{i=1}^{K} \mathbf{H}_{2i}\mathbf{A}_{1i}\mathbf{C}_{1i} + \mathbf{n}_2. \quad (2)$$

where $\mathbf{H}_{ji}$, $\mathbf{A}_{ji}$, and $\mathbf{C}_{ji}$ is the channel matrix, precoding matrix, and Alamouti code between transmitter $i$ and receiver $j$. Alamouti code is given as [5]

$$\mathbf{C}_{ji} = \begin{bmatrix} c_{ji}^{1} & -c_{ji}^{2*} \\ c_{ji}^{2} & c_{ji}^{1*} \end{bmatrix}. \quad (3)$$

$\mathbf{n}_j$ is an additive white Gaussian noise matrix whose entries are Gaussian random variables with zero mean and variance $N$. It is assumed that each node have $K$ antennas and thus $\mathbf{H}_{ji}$ is a $K \times K$ matrix and $\mathbf{A}_{ji}$ is a $K \times 2$ matrix.

## III. Desired Signal and Interference Alignment

In [6], it was shown that if each user transmits Alamouti code for two-user multiple access channel, the receiver can decouple two Alamouti code and decode one by one. In [1], this scheme is exploited and it is also used in this paper.

However, in $K \times 2$ MIMO X channel, each receiver should receive $K$ desired signals and $K$ interference signals thus, if each receiver uses only only DS in [6], all desired signals cannot be decoded. Therefore, in order to handling interference signals and the desired signals of more than two, IA and Alamouti code should be combined and two decoding method for both transmission scheme, zero-forcing and DS in [6], should be also combined.

First, for the desired signal space and interference alignment, we assume that $2K$ dimensional spaces are reserved at each receiver as follows.

$$\begin{bmatrix} D_1^j & D_2^j & \cdots & D_{\frac{K}{2}}^j & I_1^j & I_2^j & \cdots I_{\frac{K}{2}}^j \end{bmatrix} \quad (4)$$

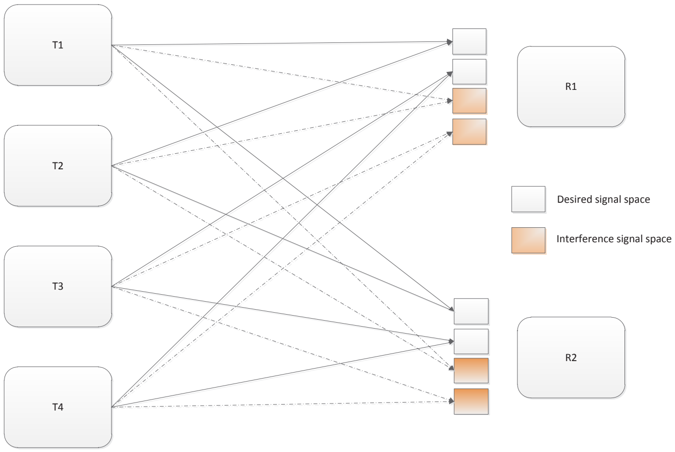


Fig. 1. $4 \times 2$ X channel with signal and interference alignment.

where the columns of $D_m^j, m \in \{1, 2, .., K/2\}$'s and $I_n^j, n \in \{1, 2, .., K/2\}$'s are the bases for the spaces of desired and interference signals at the receiver $j$, respectively, and $D_m^j$ and $I_m^j$ are the $K \times 2$ matrix. For simplicity, let $K$ be a even number in this paper.

The conditions for the precoding are given as

- IA condition

$$\mathbf{H}_{2(2k-1)}\mathbf{A}_{1(2k-1)} = \mathbf{H}_{2(2k)}\mathbf{A}_{1(2k)} = I_k^1 \quad (5)$$
$$\mathbf{H}_{1(2k-1)}\mathbf{A}_{2(2k-1)} = \mathbf{H}_{1(2k)}\mathbf{A}_{2(2k)} = I_k^2 \quad (6)$$
$$k \in \{1, 2, .., K/2\}$$

  where the equality means that the column spaces of matrices in both side are equal. In order to use DS, two desired signals should occupy only two-dimensional space therefore, only IA condition is not sufficient and the following condition is also necessary.

- Desired signal space alignment (DSSA) condition

$$\mathbf{H}_{1(2k-1)}\mathbf{A}_{1(2k-1)} = \mathbf{H}_{1(2k)}\mathbf{A}_{1(2k)} = D_k^1 \quad (7)$$
$$\mathbf{H}_{2(2k-1)}\mathbf{A}_{2(2k-1)} = \mathbf{H}_{2(2k)}\mathbf{A}_{2(2k)} = D_k^2 \quad (8)$$
$$k \in \{1, 2, .., K/2\}$$

If we let $D_k^2 = I_k^1$, the precoding matrix $\mathbf{A}_{ji}$ can be obtained easily. $\mathbf{A}_{11}$ and $\mathbf{A}_{21}$ can be obtained as follows

$$\mathbf{A}_{11} = eig_2(\mathbf{H}_{2i}^{-1}\mathbf{H}_{22}\mathbf{H}_{21}^{-1}\mathbf{H}_{11})$$
$$\mathbf{A}_{21} = eig_2(\mathbf{H}_{11}^{-1}\mathbf{H}_{12}\mathbf{H}_{22}^{-1}\mathbf{H}_{21}) \quad (9)$$

where $eig_2()$ denotes the matrix consisting of two eigenvectors of the matrix. Also, the other precoding matrices can be obtained similarly.

In Fig. 4, the combination of the desired signal space alignment and IA for $4 \times 2$ MIMO X channel is shown. The solid and the dashed lines describe the desired and interference signals, respectively. To implement this system, eight antennas are needed at each node. Let $R_k^1$ be the zero-forcing matrix for $D_k^j$ at the receiver 1 then, we have

$$R_k^{1*}\mathbf{Y}_1$$
$$= R_k^{1*}\sum_{i=1}^{K}\mathbf{H}_{1i}\mathbf{A}_{1i}\mathbf{C}_{1i} + R_k^{1*}\sum_{i=1}^{K}\mathbf{H}_{1i}\mathbf{A}_{2i}\mathbf{C}_{2i} + R_k^{1*}\mathbf{n}_1 \quad (10)$$
$$= R_k^{1*}\mathbf{H}_{1(2k-1)}\mathbf{A}_{1(2k-1)}\mathbf{C}_{1(2k-1)} + R_k^{1*}\mathbf{H}_{1(2k)}\mathbf{A}_{1(2k)}\mathbf{C}_{1(2k)} + R_k^{1*}\mathbf{n}_1. \quad (11)$$

In (10), it can be seen that the channel is converted into the two-user multiple access channel, thus DS can be used in order to decouple two Alamouti codes from $R_k^{1*}\mathbf{Y}_1$. We conduct this process for the other $D_{k'\neq k}^1$, then all desired signal can be decoded. This decoding procedure can be applied to the receiver 2 similarly. In fact, the output of DS for (11) is similar to that in [1] and [6]. Therefore we can conjecture that the proposed scheme can achieve diversity order 2.

## IV. Conclusion

In this paper, we propose a combining scheme of IA and Alamouti codes for quasi-static $K \times 2$ MIMO X channels. By designing the precoding matrix to satisfy DSSA and IA conditions and using zero-forcing, we can convert a $K \times 2$ X channel into $K$ independent multiple access channels. And then, from the output of zero-forcing, two Alamouti codes can be decoded successively by using DS. In fact, the proposed scheme requires many antennas at each node therefore, as the further work, the effort to reduce the number of antennas is needed.

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