Two-Way Relaying Schemes with Alamouti Code

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***Abstract*—This paper proposes two schemes for a two-way relay channel based on Alamouti code. The proposed schemes enable symbol-by-symbol decoding and improve the diversity gain.**

I. Introduction

In a two-way relay channel (TWRC), two source nodes simultaneously transmit their messages to each other via a helping relay node. [1] proposed a two-way relaying scheme, which requires only two time phases, i.e., MAC and BC phases. The relay receives signals from two nodes simultaneously in the MAC phase. In the BC phase the signals are linearly combined by means of superposition coding, and then, broadcast to two source nodes. Since the source nodes know their own transmitted signal, they subtract its effect to get the desired signal.

In this paper, we first show that the interference cancellation (IC) scheme based on Alamouti code in the multi-access scenario [2]-[3] can also be used for the TWRC. Then, we propose a new scheme using beamforming matrices to achieve more diversity gain.

Throughout the paper, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\dagger}$, and $\|\cdot\|$ denote the transpose of a matrix or a vector, the conjugate transpose of a matrix or a vector, and the Frobenius norm of a matrix or a vector, respectively. $\mathbf{I}_k$ represents the $k\times k$ identity matrix. Alamouti code (or matrix) $\begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix}$ is denoted by $\mathbf{A}(a,b)$.

II. Two-Way Relaying Scheme I Based on Alamouti Code

Consider the TWRC, where each node has two antennas. The TWRC consists of two source nodes, S_1 and S_2, which exchange information via a helping relay node, R. It is assumed that there is no direct connection between S_1 and S_2. In this section, we will show that an IC scheme based on Alamouti code can be applied to the TWRC.

In the MAC phase, each source transmits Alamouti code as

$$\mathbf{X}^{[t]} = \mathbf{A}(x_1^{[t]},\ x_2^{[t]}), \quad t = 1,2 \tag{1}$$

where $x_i^{[t]}$ is the i-th data symbol transmitted from S_t. The received signal matrix at R is given as

$$\mathbf{Y}^{[\mathrm{R}]} = \mathbf{H}^{[\mathrm{R}1]}\mathbf{X}^{[1]} + \mathbf{H}^{[\mathrm{R}2]}\mathbf{X}^{[2]} + \mathbf{N}^{[\mathrm{R}]} \tag{2}$$

where $\mathbf{Y}^{[\mathrm{R}]}$ is the received signal matrix at R whose entry is $y_{i,j}^{[\mathrm{R}]}$, $\mathbf{H}^{[\mathrm{R}t]}$ is the channel matrix from S_t to R, and $\mathbf{N}^{[\mathrm{R}]}$ denotes the additive white complex Gaussian noise (AWGN) matrix with zero-mean and unit-variance entries, $n_{i,j}^{[\mathrm{R}]}$, at R. Entries of $\mathbf{H}^{[\mathrm{R}t]}$, $h_{i,j}^{[\mathrm{R}t]}$, are assumed to be independent identical distributed (i.i.d.) complex Gaussian random variable. It is also assumed that channel is block fading (or constant), i.e., the channel state does not change during the transmission of each code.

(2) can be converted to the following vectorized form

$$\begin{bmatrix} \mathbf{y}_1^{[\mathrm{R}]} \\ \mathbf{y}_2^{[\mathrm{R}]} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R}1]} & \mathbf{H}_{1,1}^{[\mathrm{R}2]} \\ \mathbf{H}_{2,1}^{[\mathrm{R}1]} & \mathbf{H}_{2,1}^{[\mathrm{R}2]} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1^{[1]} \\ \mathbf{x}_1^{[2]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1^{[\mathrm{R}]} \\ \mathbf{n}_2^{[\mathrm{R}]} \end{bmatrix} \tag{3}$$

where $\mathbf{y}_i^{[\mathrm{R}]} = \begin{bmatrix} y_{i,1}^{[\mathrm{R}]} & y_{i,2}^{[\mathrm{R}]*} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{H}_{i,1}^{[\mathrm{R}t]} = \begin{bmatrix} h_{i,1}^{[\mathrm{R}t]} & h_{i,2}^{[\mathrm{R}t]} \\ h_{i,2}^{[\mathrm{R}t]*} & -h_{i,1}^{[\mathrm{R}t]*} \end{bmatrix}$, and $\mathbf{n}_i^{[\mathrm{R}]} = \begin{bmatrix} n_{i,1}^{[\mathrm{R}]} & n_{i,2}^{[\mathrm{R}]*} \end{bmatrix}^{\mathrm{T}}$. The vectors $\mathbf{x}_1^{[t]}$ denotes the symbol vector $\begin{bmatrix} x_1^{[t]} & x_2^{[t]} \end{bmatrix}^{\mathrm{T}}$. Then we can separate the equations of $\mathbf{x}_1^{[1]}$ and $\mathbf{x}_1^{[2]}$ from (3) using $\hat{\mathbf{G}}_1^{[\mathrm{R}]}$ and $\hat{\mathbf{G}}_2^{[\mathrm{R}]}$ as

$$\hat{\mathbf{G}}_1^{[\mathrm{R}]} = \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{1,1}^{[\mathrm{R}2]}\mathbf{H}_{2,1}^{[\mathrm{R}2]\dagger}}{\|\mathbf{H}_{2,1}^{[\mathrm{R}2]}\|^2} \end{bmatrix}^{\dagger} \tag{4}$$

$$\hat{\mathbf{G}}_2^{[\mathrm{R}]} = \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{1,1}^{[\mathrm{R}1]}\mathbf{H}_{2,1}^{[\mathrm{R}1]\dagger}}{\|\mathbf{H}_{2,1}^{[\mathrm{R}1]}\|^2} \end{bmatrix}^{\dagger}. \tag{5}$$

By multiplying the conjugate transpose of the 4×2 matrix $\hat{\mathbf{G}}_i^{[\mathrm{R}]}$, we have

$$\hat{\mathbf{G}}_1^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{y}_1^{[\mathrm{R}]} \\ \mathbf{y}_2^{[\mathrm{R}]} \end{bmatrix} \overset{(a)}{=} \underbrace{\hat{\mathbf{G}}_1^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R}1]} \\ \mathbf{H}_{2,1}^{[\mathrm{R}1]} \end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}1]}} \mathbf{x}_1^{[1]} + \hat{\mathbf{G}}_1^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{n}_1^{[2]} \\ \mathbf{n}_2^{[2]} \end{bmatrix} \tag{6}$$

$$\hat{\mathbf{G}}_2^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{y}_1^{[\mathrm{R}]} \\ \mathbf{y}_2^{[\mathrm{R}]} \end{bmatrix} \overset{(b)}{=} \underbrace{\hat{\mathbf{G}}_2^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R}2]} \\ \mathbf{H}_{2,1}^{[\mathrm{R}2]} \end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}2]}} \mathbf{x}_1^{[2]} + \hat{\mathbf{G}}_2^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{n}_1^{[2]} \\ \mathbf{n}_2^{[2]} \end{bmatrix} \tag{7}$$

where the equalities (a) and (b) use the fact that $\hat{\mathbf{G}}_i^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R}j]\dagger} & \mathbf{H}_{2,1}^{[\mathrm{R}j]\dagger} \end{bmatrix}^{\dagger}, i \neq j$ is a 2×2 zero matrix. The equivalent channel, $\mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}1]}$ and $\mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}2]}$ in (6) and (7) are Alamouti matrices. Therefore, R can decode four symbols $x_i^{[t]}$ using by symbol-by-symbol Alamouti decoding in [4].

In the BC phase, R broadcast the following Alamouti code.

$$\mathbf{X}^{[\mathrm{R}]} = \mathbf{A}(\hat{x}_1^{[1]} + \hat{x}_1^{[2]},\ \hat{x}_2^{[1]} + \hat{x}_2^{[2]}) \tag{8}$$

where $\hat{x}_i^{[t]}$ denotes the estimated symbol at R. The received signal matrices at S_1 and S_2 are given as

$$\mathbf{Y}^{[i]} = \mathbf{H}^{[i\mathrm{R}]}\mathbf{X}^{[\mathrm{R}]} + \mathbf{N}^{[i]}, \quad i = 1, 2 \tag{9}$$

Then the sources can decode $\hat{x}_1^{[1]} + \hat{x}_1^{[2]}$ and $\hat{x}_2^{[1]} + \hat{x}_2^{[2]}$ by using Alamouti decoding. Since the sources know the symbol they have transmitted in the MAC phase, they can cancel this contribution and decode the desired symbols.

III. Two-Way Relaying Scheme II Based on Alamouti Code

In order to achieve more diversity gain, our proposed scheme utilizes beamforming matrices. In the MAC phase, the transmitted block code at each source is designed as

$$\mathbf{X}^{[t]} = \mathbf{P}^{[t]}\mathbf{A}(x_1^{[t]},\ x_2^{[t]}). \tag{10}$$

As shown in (10), each source encodes data symbols using Alamouti code followed by $\mathbf{P}^{[t]}$. Suppose that the beamforming matrices $\mathbf{P}^{[t]}$ are designed to satisfy the following alignment conditions

$$\mathbf{H}^{[\mathrm{R}1]}\mathbf{P}^{[1]} = \mathbf{H}^{[\mathrm{R}2]}\mathbf{P}^{[2]}. \tag{11}$$

Then the received signal matrix at R is given as

$$\begin{aligned}\mathbf{Y}^{[\mathrm{R}]} &= \mathbf{H}^{[\mathrm{R}1]}\mathbf{X}^{[1]} + \mathbf{H}^{[\mathrm{R}2]}\mathbf{X}^{[2]} + \mathbf{N}^{[\mathrm{R}]} \qquad (12)\\ &= \mathbf{H}^{[\mathrm{R}1]}\mathbf{P}^{[1]}\mathbf{A}(x_1^{[1]} + x_1^{[2]},\ x_2^{[1]} + x_2^{[2]}) + \mathbf{N}^{[\mathrm{R}]}\end{aligned}$$

(12) can be converted to the following vectorized form

$$\mathbf{y}^{[\mathrm{R}]} = \begin{bmatrix}\mathbf{y}_1^{[\mathrm{R}]}\\ \mathbf{y}_2^{[\mathrm{R}]}\end{bmatrix} = \underbrace{\begin{bmatrix}\mathbf{H}_{1,1}^{[\mathrm{R}1]}\\ \mathbf{H}_{2,1}^{[\mathrm{R}1]}\end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}]}}\begin{bmatrix}x_1^{[1]} + x_1^{[2]}\\ x_2^{[1]} + x_2^{[2]}\end{bmatrix} + \begin{bmatrix}\mathbf{n}_1^{[\mathrm{R}]}\\ \mathbf{n}_2^{[\mathrm{R}]}\end{bmatrix} \tag{13}$$

where $\mathbf{y}_i^{[\mathrm{R}]} = \begin{bmatrix}y_{i,1}^{[\mathrm{R}]} & y_{i,2}^{[R]*}\end{bmatrix}^{\mathrm{T}}$, $\mathbf{H}_{i,1}^{[\mathrm{R}1]} = \begin{bmatrix}h_{i,1}^{[\mathrm{R}1]} & h_{i,2}^{[\mathrm{R}1]}\\ h_{i,2}^{[\mathrm{R}1]*} & -h_{i,1}^{[\mathrm{R}1]*}\end{bmatrix}$, $h_{i,j}^{[\mathrm{R}1]}$'s are entries of $\mathbf{H}^{[\mathrm{R}1]}\mathbf{P}^{[1]}$, and $\mathbf{n}_i^{[2]} = \begin{bmatrix}n_{i,1}^{[2]} & n_{i,2}^{[2]*}\end{bmatrix}^{\mathrm{T}}$.

In the BC phase, soft decision value at R can be obtained and re-encoded by using Alamouti code as

$$\begin{bmatrix}\hat{x}_1^{[\mathrm{R}]}\\ \hat{x}_2^{[\mathrm{R}]}\end{bmatrix} = \mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}]\dagger}\mathbf{y}^{[\mathrm{R}]} \tag{14}$$

$$\mathbf{X}^{[\mathrm{R}]} = \mathbf{A}(\hat{x}_1^{[\mathrm{R}]},\ \hat{x}_2^{[\mathrm{R}]}) \tag{15}$$

The received signal matrices at S_1 and S_2 have the same structure as (9), then the sources can decode by using the same symbol-by-symbol decoding in the previous section.

IV. Simulation Results

It is assumed that the channel is Rayleigh block fading. All channel coefficients and noise at S_1, S_2, and R are assumed to be complex Gaussian random variables $\mathcal{CN}(0, 1)$. Quadrature phase-shift keying (QPSK) is used and the average transmit power per symbol at each node is set to P.

Fig. 1 compares the symbol error rate (SER) performance of two-way relaying schemes I and II. Fig. 1 shows that the scheme II outperforms the scheme I. In the MAC phase of the scheme I, R have (6) and (7) after the cancellation, i.e., multiplying $\hat{\mathbf{G}}_i^{[\mathrm{R}]\dagger}$. (6) and (7) can be viewed as the received signal for a point-to-point Alamouti scheme, which consists of a double-antenna transmitter and a single-antenna receiver, i.e., the equation (12) in [4]. On the other hand, the scheme II does not require the cancellation in the MAC phase and (13) can be viewed as the received signal for the point-to-point Alamouti scheme where both the transmitter and the receiver have two antennas, i.e., the equation (14) in [4]. Therefore, the scheme II achieves a diversity order of four, while the scheme I achieves a diversity order of two, which can be verified by the slope of the SER curve in high SNR region in Fig. 1.

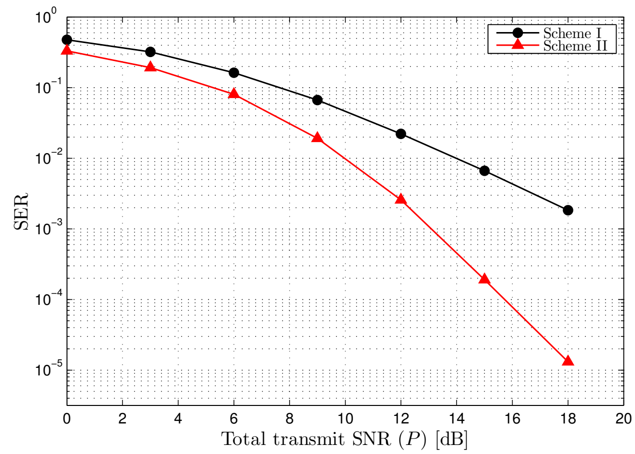

Fig. 1. SER performance

V. Conclusions

We propose a method on how to apply Alamouti code to the TWRC. The interference cancellation method based on Alamouti code for the multi-access scenario can be used for the TWRC, which enables the nodes to perform symbol-by-symbol decoding and achieve diversity order of two.

In order to achieve more diversity gain, we propose a new two-way relaying scheme based on Alamouti code which utilizes beamforming matrices to align signals at the relay node. From the simulation results, it is shown that the proposed scheme achieves a diversity order of four.

References

[1] B. Rankov and A. Wittneben, "Spectral efficient signaling for half-duplex relay channels," *in Proc. 39th Asilomar Conf. Signals, Systems, Comput.*, pp. 1066-1071, Nov. 2005.

[2] A. F. Naguib, N. Seshadri, and A. R. Calderbank, "Applications of space-time block codes and interference suppression for high capacity and high data rate wireless systems," *in Proc. 32nd Asilomar Conf. Signals, Systems, Comput.*, pp. 1803-1810, Oct. 1998.

[3] A. Stamoulis, N. Al-Dhahir and A. R. Calderbank, "Further results on interference cancellation and space-time block codes," *in Proc. 35th Asilomar Conf. Signals, Systems, Comput.*, pp. 257-261, Nov. 2001.

[4] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Aug. 2008.