Compressive Sensing Based Clipping Noise Cancellation for OFDM Systems

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***Abstract*—In this paper, we propose clipping noise cancellation scheme using compressive sensing (CS) for orthogonal frequency division multiplexing (OFDM) systems. The proposed scheme exploits the data tones to reconstruct the clipping noise. In the data tones, transmitted data and the clipping noise are mixed together. To extract the clipping noise from the data tones accurately, in the proposed scheme, only the data tones with high reliability instead of the whole data tones are exploited in reconstructing the clipping noise. For reconstructing the clipping noise using a fraction of the data tones at the receiver, the CS technique is applied. Numerical analysis shows that the proposed scheme performs well for clipping noise cancellation of OFDM systems.**

I. Introduction

Many peak-to-average power ratio (PAPR) reduction schemes for the Orthogonal frequency division multiplexing (OFDM) systems have been proposed [1]-[8]. The simplest method for PAPR reduction of the OFDM signals is clipping [6]-[8]. Clipping at the Nyquist sampling rate has been used for low complexity applications but suffers from peak regrowth after digital-to-analog (D/A) conversion. It is known that clipping of the oversampled OFDM signals reduces the peak regrowth after D/A conversion. But, it causes out-of-band radiation which has to be filtered. Furthermore, clipping of OFDM signals causes clipping noise which has sparsity in time domain. There are several schemes to mitigate this clipping noise in each of the two sampling rate cases [9]-[11]. The scheme in [9] shows good clipping noise cancellation performance, but it requires iterative maximum likelihood (ML) estimation for all tones with clipping and filtering at the receiver, which causes lots of computation.

Compressive sensing (CS) is a sampling method that converts input signal in high dimension into the signal lying in the smaller dimension [12]-[15]. In general, it is not enough to recover an unknown signal using compressed observations in the reduced dimension. Nevertheless, if the input signal has sparsity, its reconstruction can be achieved at the receiver using CS reconstruction algorithm. In this context, the clipping noise at the receiver can be reconstructed by CS reconstruction algorithm.

In this paper, we propose a new clipping noise cancellation method without data rate loss by using CS. The proposed scheme exploits compressed observations of the clipping noise underlying in the data tones to reconstruct the clipping noise, which is different from [16] and [17]. In [16] and [17], the reserved tones and pilot tones are exploited, respectively. In this case, the transmitted data and the clipping noise are mixed in the data tones. To extract the clipping noise from the data tones accurately, the proposed scheme exploits the partial data tones with high reliability instead of the whole data tones. And, the number of compressed observations is adjustable in the sense that the optimal number of compressed observations can be selected according to the noise level. When the noise level is high, exploiting the partial data tones is a better choice than exploiting the whole data tones as will be shown in numerical analysis. By doing so, we successfully overcomes the weakness of CS reconstruction against noise. Also, the proposed scheme can select the partial data tones with low-complexity method, which is implemented with a simple decision region. Thus, the proposed scheme has much lower computational complexity than [18], which uses a complex metric to select the reliable tones. Numerical analysis shows that the proposed method can mitigate the clipping noise well in both cases of clipping at Nyquist sampling rate and clipping and filtering at oversampling rate.

II. System Model

Let $X = (X(0), X(1), ..., X(N-1))^T$ be an input symbol sequence. The continuous time baseband OFDM signal can be represented as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi kt}{T}\right), \qquad 0 \le t \le T \quad (1)$$

where N is the number of subcarriers and T is the symbol duration. Let $\Delta t_L = T/LN$ be a sampling interval, where L is oversampling factor. Then the discrete time OFDM signal sampled at time $n\Delta t_L$ can be expressed as

$$x_L(n) = x(n\Delta t_L), \qquad n = 0, 1, ..., LN-1. \quad (2)$$

An OFDM signal sequence with oversampling factor L can also be obtained by padding X with $(L-1)N$ zeros and processing the inverse discrete Fourier transform (IDFT).

The PAPR of the OFDM signal sequence $x_L(n)$ with oversampling factor L is defined as the ratio of the peak-to-average power of the signal as

$$\mathrm{PAPR} = \frac{\max_{0\le n\le LN-1} |x_L(n)|^2}{E[|x_L(n)|^2]} \tag{3}$$

where $E[\cdot]$ denotes the expected value.

To reduce peak regrowth at the D/A conversion, clipping and filtering are performed on the OFDM signal sequence oversampled by a factor L at the transmitter. The clipping operation is given as

$$\bar{x}_L(n) = \begin{cases} x_L(n), & |x_L(n)| \le A \\ A\exp\{j\arg(x_L(n))\}, & |x_L(n)| > A \end{cases} \tag{4}$$

where A is the clipping threshold. Then the clipping ratio (CR) is defined as

$$\mathrm{CR} = 20\log\frac{A}{\sigma}\ [\mathrm{dB}] \tag{5}$$

where $\sigma = \sqrt{E[|x_L(n)|^2]}$.

It is shown in [7] that the clipped signal $\bar{x}_L(n)$ can be modeled in two different ways. The first one is an additive model where the clipped signal $\bar{x}_L(n)$ is considered as the sum of the sampled signal $x_L(n)$ and the clipping noise $c_L(n)$, and the second one is an attenuated model where $\bar{x}_L(n)$ is viewed as the sum of an attenuated component $\alpha x_L(n)$ and clipping noise component $d_L(n)$, that is,

$$\bar{x}_L(n) = \begin{cases} x_L(n) + c_L(n), & \text{for additive model} \\ \alpha x_L(n) + d_L(n), & \text{for attenuated model} \end{cases} \tag{6}$$

where $n = 0, 1, ..., LN-1$ and the attenuation α is given as [7]

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}\gamma}{2}\mathrm{erfc}(\gamma) \tag{7}$$

and $\gamma = \sqrt{10^{\mathrm{CR}/10}}$.

In order to remove the out-of-band radiation due to clipping operation, the clipped signal $\bar{x}_L(n)$ in time domain is converted back to the frequency domain by taking LN-point discrete Fourier transform (DFT) and being filtered. Then we can have $\bar{X}(k)$ as

$$\bar{X}(k) = \begin{cases} X(k) + C(k), & \text{for additive model} \\ \alpha X(k) + D(k), & \text{for attenuated model} \end{cases} \tag{8}$$

where $k = 0, 1, ..., N-1$. In [7], $D(k)$ is assumed as a complex Gaussian random variable with zero mean.

Let $x(n) + c(n),\ n = 0, ..., N-1$, denote the IDFT of (8) for the additive model, where $x(n)$ is Nyquist sampled OFDM signal and $c(n)$ is clipping noise. For the case of $L = 1$, we do not use filtering and $c(n)$ is equal to $c_1(n)$. Then the clipping noise $c(n)$ can be considered as K-sparse signal having K nonzero elements. We define the sparsity ratio as K/N. Now we can exploit this sparsity by CS method. If L is larger than 1, $c(n)$ is not equal to $c_L(n)$ but $c(n)$ can be considered as nearly K-sparse signal, implying that the amplitudes of remaining $N-K$ elements are close to zero, but not exactly zero. Even in this case, CS method to cancel the clipping noise can also be applied effectively.

III. Clipping Noise Cancellation for OFDM Signals Using CS

A. Formulation to CS Problem

The received symbol using the additive model in (8) can be expressed in frequency domain as

$$Y(k) = H(k)(X(k) + C(k)) + Z(k), \quad 0 \le k \le N-1 \tag{9}$$

where $X(k)$ and $Y(k)$ denote the input and received symbols, $H(k)$ denotes the frequency domain channel response, and $Z(k)$ denotes the additive white Gaussian noise (AWGN) with variance N_0 in frequency domain. In matrix form, it can be rewritten as

$$Y = \mathbf{H}(X + C) + Z \tag{10}$$

where $\mathbf{H} = diag(H)$ and Y, X, C, Z are $N \times 1$ column vectors.

We assume perfectly known channel response $\mathbf{H}$ and perfect synchronization. After channel equalization, its output is given as

$$Y^{(eq)} = \mathbf{H}^{-1}Y = X + C + \mathbf{H}^{-1}Z. \tag{11}$$

Note that in the attenuated model, $Y^{(eq)}$ can be similarly expressed as

$$Y^{(eq)} = \alpha X + D + \mathbf{H}^{-1}Z \tag{12}$$

where the clipping noise D is $N \times 1$ column vector.

For applying CS method, we need a compressed observation vector in the reduced dimension. Our suggestion here is to select a subset of components in $Y^{(eq)}$, namely M out of N components of $Y^{(eq)}$. This can be done by multiplying $M \times N$ selection matrix $\mathbf{S}_{RR}$ consisting of some M rows of the identity matrix $\mathbf{I}_N$ to $Y^{(eq)}$ in (11). The discussion on $\mathbf{S}_{RR}$ will be made in the next subsection. Let $C = \mathbf{F}c$, where $\mathbf{F}$ is $N \times N$ DFT matrix. Then, we have

$$\mathbf{S}_{RR}Y^{(eq)} = \mathbf{S}_{RR}\mathbf{F}c + \mathbf{S}_{RR}X + \mathbf{S}_{RR}\mathbf{H}^{-1}Z. \tag{13}$$

If we subtract the estimation $\mathbf{S}_{RR}\hat{X}$ from (13), we have

$$\begin{aligned} \tilde{Y} = \mathbf{S}_{RR}Y^{(eq)} - \mathbf{S}_{RR}\hat{X} &= \mathbf{S}_{RR}\mathbf{F}c + \mathbf{S}_{RR}(X - \hat{X}) + \mathbf{S}_{RR}\mathbf{H}^{-1}Z \\ &= \mathbf{\Phi}c + \underbrace{\mathbf{S}_{RR}(X - \hat{X}) + \mathbf{S}_{RR}\mathbf{H}^{-1}Z}_{\text{noise vector}} \end{aligned} \tag{14}$$

where the matrix $\mathbf{\Phi} = \mathbf{S}_{RR}\mathbf{F}$ can be considered as $M \times N$ *measurement matrix* in CS. As one can see in [13], the measurement matrix for CS can be constructed by using the subset of rows in DFT matrix. Then the resulting equation (14) can be considered as CS problem, where the vector $\tilde{Y}$ can be considered as $M \times 1$ *compressed observation vector*, the clipping noise c as $N \times 1$ *sparse signal vector*, and the remaining vector $\mathbf{S}_{RR}(X - \hat{X}) + \mathbf{S}_{RR}\mathbf{H}^{-1}Z$ as $M \times 1$

noise vector. By using CS reconstruction algorithm, we can reconstruct c as $\hat{c}$ from the compressed observation vector $\tilde{Y}$. Then fast Fourier transform (FFT) of $\hat{c}$ is subtracted from the equalized received symbol $Y^{(eq)}$ and then the final decision is made.

B. Selection Matrix $\mathbf{S}_{RR}$

As mentioned in the previous subsection, we suggest that the $M \times 1$ compressed observation vector $\tilde{Y} = \mathbf{S}_{RR}Y^{(eq)} - \mathbf{S}_{RR}\hat{X}$. And, $\mathbf{S}_{RR}Y^{(eq)}$ in $\tilde{Y}$ is constructed from $Y^{(eq)}$ by selecting those components of $Y^{(eq)}$ that are thought to be more reliable than others. We designate a specific area in the received signal space as reliable region (RR) and define the index set $\mathcal{K}_{RR}$ as the set of component indices of the received signals that lie in the reliable region. For quaternary phase shift keying (QPSK) and 16-quadrature amplitude modulation (QAM), RR is designated as shaded area in Fig. 1. For other modulations, RR can be set in analogous way. In Fig. 1, the distance between adjacent signal points is assumed to be 2.

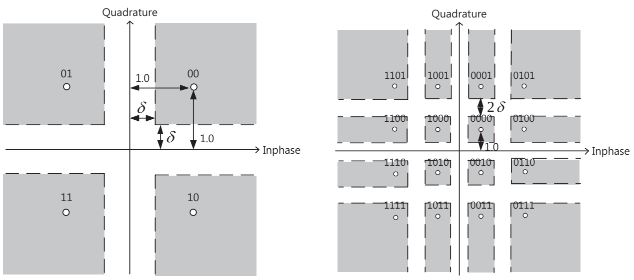

Fig. 1. Reliable regions for QPSK and 16-QAM modulations.

Prior to explain the index set $\mathcal{K}_{RR}$ in detail, we compare the additive model and the attenuated model in (8) from the viewpoint of signal-to-clipping-noise ratio (SCNR). In [7], $D(k)$ in attenuated model is uncorrelated clipping noise from $X(k)$. And, $C(k)$ in additive model is easily derived from (8) as

$$C(k) = (\alpha - 1)X(k) + D(k). \tag{15}$$

Additionally to $D(k)$, $C(k)$ in (15) also has a term $(\alpha - 1)X(k)$ correlated from $X(k)$. And thus it implies that the SCNR of the attenuated model is larger than SCNR of the additive model, that is,

$$\frac{E[|\alpha X(k)|^2]}{E[|D(k)|^2]} \geq \frac{E[|X(k)|^2]}{E[|C(k)|^2]}. \tag{16}$$

For the reason as above, we describe the index set $\mathcal{K}_{RR}$ in detail using the attenuated model. Assume that M components of $(1/\alpha)Y^{(eq)}$ fall into RR. Then the index set $\mathcal{K}_{RR}$ can be expressed as

$$\mathcal{K}_{RR} = \{k_m : \frac{1}{\alpha}Y^{(eq)}(k_m) \in \text{RR},\ 0 \leq m \leq M-1\}. \tag{17}$$

The $M \times N$ selection matrix $\mathbf{S}_{RR}$ can be obtained from the identity matrix of order N by selecting M rows corresponding to the index set $\mathcal{K}_{RR}$, where M is cardinality of the set $\mathcal{K}_{RR}$.

In our proposed scheme, for the determined $\mathbf{S}_{RR}$, we estimate $\mathbf{S}_{RR}X$. Clearly, we can also conclude that $\mathbf{S}_{RR}X$ can be more efficiently estimated by using attenuated model. That is, in order to estimate $\mathbf{S}_{RR}X$ efficiently, we decide $(1/\alpha)\mathbf{S}_{RR}Y^{(eq)}$ as $\mathbf{S}_{RR}\hat{X}$ by ML estimation. Intuitionally, at the receiver, when we decide $(1/\alpha)Y^{(eq)}(k_m)$ in RR as $\hat{X}(k_m)$, its decision error probability is lower than that when we decide $(1/\alpha)Y^{(eq)}(k \notin \mathcal{K}_{RR})$ in the outside region of RR as $\hat{X}(k \notin \mathcal{K}_{RR})$. Therefore, if we consider the signals only in the RR to reconstruct the clipping noise c, decision error of $\mathbf{S}_{RR}\hat{X}$ can be reduced as δ increases in Fig. 1.

C. Determination of δ

Accuracy of reconstruction via CS in (14) is affected by not only the noise but also the selection of compressed observations. Certainly, both the noise and the compressed observations vary according to δ in our scheme. We perform the massive simulations to find an optimal value of δ which maximizes the performance of CS reconstruction.

IV. Numerical Analysis

In this section, we evaluate the BER performance of the proposed clipping noise cancellation scheme in the AWGN channel. The K largest peaks reduction scheme, which is introduced in [16], is simulated. The distance between adjacent signal points is 2.

Fig. 2 shows the BER performance of the proposed scheme using K largest peaks reduction. There is a large benefit to use the proposed scheme compared to no clipping noise cancellation case. As K increases, the BER performance decreases and for $K > 8$, the BER performance is not closed to that of the original OFDM. Thus, we use the maximum iteration number of OMP as $0.125N$ for unknown K cases. In Fig. 2, clipping noise cancellation scheme in [16] is also given when 29 tones are positioned by a $(59, 29, 14)$ difference set [21] and reserved. It uses the sufficient number of reserved tones (i.e., the sufficient number of compressed observations) for reconstructing the clipping noise with $K = 4$ in the absence of noise. But, the scheme in [16] shows the poor BER performance due to weakness of CS reconstruction against noise. Although the clipping noise cancellation scheme in [17] is not shown in Fig. 2, it may show worse BER performance than that in [16]. The reason is that, considering the practical system, the number of pilot tones (i.e., the number of compressed observations) is smaller than 29 when $N = 64$.

V. Conclusion

In this paper, we proposed the new clipping noise cancellation scheme in OFDM using CS. In the proposed scheme, the data tones are partially exploited to reconstruct the clipping noise instead of the whole data tones. By introducing RR, we can select these partial data tones and numerical analysis shows that the proper value of δ needs to be chosen. Using the proposed clipping noise cancellation scheme for the OFDM systems, the BER performance can be improved compared to the conventional schemes.

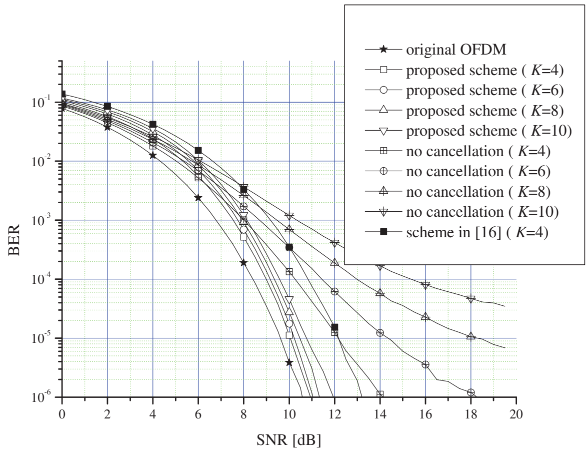

Fig. 2. BER performance of the proposed scheme and the scheme in [16] with K largest peaks reduction when $N = 64$ and QPSK is used.

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