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Oversampling Analysis of CORR Metric for Selected Mapping Scheme of OFDM Signals

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***Abstract*—It is known that single point correlation (CORR) metric [4] in the selected mapping (SLM) scheme outperforms the PAPR metric in terms of BER performance of orthogonal frequency division multiplexing (OFDM) signals. In this paper, oversampling effect of OFDM signal is analyzed when CORR metric is used in the SLM scheme in the presence of nonlinear high power amplifier. However, our oversampling effect analysis by using the correlation coefficients of oversampled OFDM signals for CORR computation shows that the CORR metric by two times oversampling is enough to achieve the same bit error rate (BER) performance with four times or 16 times oversampling in the SLM scheme. Simulation result shows that BER performance with two times oversampling is almost the same as that of four and 16 times oversampling.**

I. Introduction

Recently, orthogonal frequency division multiplexing (OFDM) has received lots of attention in the wireless communications. Besides PAPR, many other metrics are proposed to improve the bit error rate (BER) performance in the presence of nonlinear high power amplifier (HPA). It is known that intermodulation distortion (IMD) [1], distortion-to-signal power ratio (DSR) [2], mean squared error (MSE) [3], and single point correlation (CORR) [4] metrics outperform the PAPR metric in terms of BER performance of OFDM signals. Except for the PAPR metric, CORR metric shows the lowest computational complexity while its BER performance is almost the same as that of other metrics.

The rest of this paper is organized as follows. In Section II, we briefly review the CORR metric and show that CORR is the optimum metric in terms of BER performance. In Section III, oversamplig effect of CORR metric is analyzed in SLM scheme in the presence of nonlinear HPA by computing CORR metric and Pearson correlation coefficient. Simulation result and conclusion is given in Section IV.

II. Overview of SLM Scheme with CORR Metric

Binary data sequences are modulated by M-ary quadrature amplitude modulation (QAM) constellation. Then, $(L-1)N$ zeros are padded to the end (or middle) of an input symbol sequence. In general, it is known that four times oversampling is enough for estimating continuous signal. Then, the input symbol sequence is IFFTed. The nth time domain sample of OFDM signal sequence can be expressed as

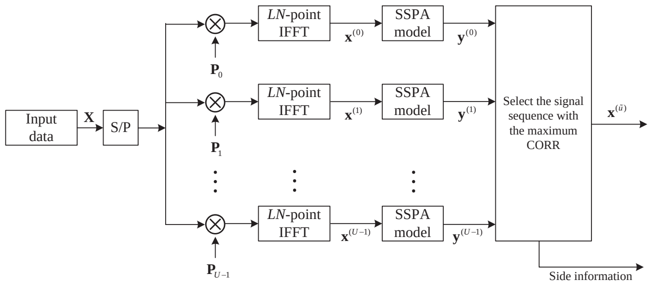

Fig. 1. A block diagram of SLM scheme with CORR metric.

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi kn}{LN}} \tag{1}$$

where $n = 0, 1, \cdots, LN-1$.

Fig. 1 shows a block diagram of SLM scheme with CORR metric in the presence of nonlinear HPA. This scheme generates U alternative OFDM symbol sequences $\mathbf{X}^{(u)}$ by multiplying U different phase sequences to an input symbol sequence componentwisely. Let $\mathbf{P}^{(u)} = \{P_0^{(u)}, P_1^{(u)}, \cdots, P_{N-1}^{(u)}\}$ be the uth phase sequence with $P_k^{(u)} = e^{j\phi_k^{(u)}}$, where $\phi_k^{(u)} \in [0, 2\pi)$, $k = 0, 1, \cdots, N-1$, and $u = 0, 1, \cdots, U-1$. It is customary to use $P_k^{(u)} = \pm 1$.

Each alternative OFDM symbol sequence is LN-point IFFTed as $\mathbf{x}^{(u)} = \{x_0^{(u)}, x_1^{(u)}, \cdots, x_{LN-1}^{(u)}\}$ and passes through the solid state power amplifier (SSPA) model. It is known that the accuracy of SSPA model used for choosing the phase sequence has no big impact in the OFDM system. Therefore, the polynomial model which has relatively low computational complexity is used for SSPA model.

The polynomial model is usually expressed as a third order nonlinearity. Thus, nth element of the output of the SSPA

TABLE I

COEFFICIENTS OF THE EQUATIONS REQUIRED TO COMPUTE CORR METRIC WHEN L TIMES OVERSAMPLING IS USED

Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$L=1$	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
$L=2$	0.04	−0.13	−0.13	0.04	−0.13	1.4	0.4	−0.13	−0.13	0.4	0.4	−0.13	0.04	−0.13	−0.13	0.04
$L=4$	0.09	−0.33	−0.30	0.09	−0.33	2.3	0.94	−0.3	−0.3	0.94	1.3	−0.33	0.09	−0.3	−0.33	0.09
$L=16$	0.37	−1.42	−1.25	0.36	−1.42	7.72	3.94	−1.25	−1.25	3.94	6.72	−1.42	0.36	−1.25	−1.42	0.37

model can be expressed as

$$y_n^{(u)} \approx \alpha_1 x_n^{(u)} + \alpha_3 x_n^{(u)} |x_n^{(u)}|^2 \tag{2}$$

where $x_n^{(u)}$ and $y_n^{(u)}$ are uth alternative input and output OFDM signal sequences of HPA, respectively. The polynomial coefficients are used as $\alpha_1 = 1$ and $\alpha_3 = -0.1769$ [4]. Then, this scheme calculates U CORR values between input $\mathbf{x}^{(u)}$ and output $\mathbf{y}^{(u)}$ of the SSPA model. The CORR computation of uth alternative OFDM signal sequence is expressed as

$$R_{xy}^{(u)} = \sum_{n=0}^{LN-1} x_n^{(u)} y_n^{*(u)} = \alpha_1 \sum_{n=0}^{LN-1} |x_n^{(u)}|^2 + \alpha_3 \sum_{n=0}^{LN-1} |x_n^{(u)}|^4 \tag{3}$$

where $(\cdot)^*$ indicates complex conjugation. Among U $R_{xy}^{(u)}$, the signal with the maximum CORR is selected for transmission. Side information should be transmitted to the receiver.

III. OVERSAMPLING EFFECT OF CORR IN SLM SCHEME

A. Expression of Oversampled Signal

Oversampled signal can be achieved by linear combination of Nyquist rate samples [5]. The oversampling operation is called interpolator. The impulse response of an interpolator for the L times oversampling is defined as

$$h_L[n] = \frac{\sin(\pi n/L)}{\pi n/L}. \tag{4}$$

Since ideal interpolator (low-pass filter) cannot be implemented, we use finite filter of length I in practice. Then, the output of the interpolator can be expressed as

$$\tilde{x}_L[n_L] = \sum_{k=\lceil (n_L - LI)/L \rceil}^{\lfloor (n_L + LI)/L \rfloor} x[k] h_L[n_L - Lk] \tag{5}$$

where $\tilde{x}_L[n_L]$ is the estimated n_Lth element of L times oversampled signal with finite filter of length I and $x[k]$ denotes kth element of Nyquist rate signal. In this paper, we assume $I = 2$ for simple expression.

To show the oversampling effect of CORR, we can only express 16 samples for CORR computation in (3) among total $16N$ samples without loss of generality. Then, (3) can be rewriten as

$$\bar{R}_{\tilde{x}\tilde{y}}^{(u)} = \sum_{s=0}^{15} \tilde{x}_{16}[16m+s]^{(u)} \tilde{y}_{16}^*[16m+s]^{(u)} \tag{6}$$

where $0 \leq m \leq N-1$.

There are 16 signal sequences to be added and we assign alphabets, (a) to (p), to each term of signal sequence in (6), that is, (a) = $\tilde{x}_{16}[16m]^{(u)} \tilde{y}_{16}^*[16m]^{(u)}$, (b) = $\tilde{x}_{16}[16m+1]^{(u)} \tilde{y}_{16}^*[16m+1]^{(u)}$, and (p) = $\tilde{x}_{16}[16m+15]^{(u)}$ $\tilde{y}_{16}^*[16m+15]^{(u)}$. For better understanding, we write (i) as (7) by using (5).

Now, consider that arbitrary L times oversampling is used for CORR computation. For example, we only need (a) for Nyquist rate sampling. Furthermore, three signal sequences (a) + (i), (a) + (e) + (i) + (m), and (a) + (b) + $\cdots$ + (p) are needed to compute (3) when two times, four times, and 16 times oversamplings are used, respectively.

TABLE II

PEARSON CORRELATION COEFFICIENT BETWEEN SEQUENCES WITH DIFFERENT L

	$L=1$ and $L=16$	$L=2$ and $L=16$	$L=4$ and $L=16$
r	0.6023	0.9133	0.9839

B. Correlation between Signal Sequences derived from CORR Computation

To compute the correlation between signal sequences in {(a), (a) + (i), (a) + (e) + (i) + (m), (a) + (b) + $\cdots$ + (p)} derived from (6) when different $L \in \{1, 2, 4, 16\}$ are used, we only consider the coefficients of each signal sequence.

Table I shows the coefficients of each signal sequence. Indices are arranged to each coefficient in numerical order (1 to 16). To compute the correlation value between signal sequences, we use Pearson correlation coefficient (r) and regard 16 coefficients in Table I as a sequence.

Table II shows the Pearson correlation coefficients between two signal sequences obtained from Table I when different L is used. When comparing $L = 1$ and $L = 16$, r is 0.6023, which shows low correlation. On the other hand, the value of r when comparing $L = 2$ and $L = 16$ and $L = 4$ and $L = 16$ are 0.9133 and 0.9839, respectively, which show very high correlation. These results show that the probability of choosing the same phase sequence as that of 16 times oversampling case is very high when two times or four times oversampling are used. Therefore, we can expect that two times oversampling can be used for CORR computation instead of four times or 16 times oversampling. Table III shows the computational complexity of CORR SLM when $N = 256$.

TABLE III

COMPARISON OF COMPUTATIONAL COMPLEXITY REQUIRED FOR METRIC COMPUTATION OF CORR SLM SCHEME WHEN $N = 256$.

	Original		Proposed	
$N=256$	RM	3,073	RM	1,537
	RA	3,071	RA	3,071

$$
\begin{aligned}
(\mathrm{i}) &= \tilde{x}_{16}[16m{+}8]^{(u)}\tilde{y}^{*}_{16}[16m{+}8]^{(u)} \\
&= 0.04x[16m{-}16]^{(u)}y^{*}[16m{-}16]^{(u)} - 0.13x[16m{-}16]^{(u)}y^{*}[16m]^{(u)} - 0.13x[16m{-}16]^{(u)}y^{*}[16m{+}16]^{(u)} + 0.04x[16m{-}16]^{(u)}y^{*}[16m{+}32]^{(u)} \\
&\quad - 0.13x[16m]^{(u)}y^{*}[16m{-}16]^{(u)} + 0.4x[16m]^{(u)}y^{*}[16m]^{(u)} + 0.4x[16m]^{(u)}y^{*}[16m+16]^{(u)} - 0.13x[16m]^{(u)}y^{*}[16m+32]^{(u)} \\
&\quad - 0.13x[16m{+}16]^{(u)}y^{*}[16m{-}16]^{(u)} + 0.4x[16m{+}16]^{(u)}y^{*}[16m]^{(u)} + 0.4x[16m{+}16]^{(u)}y^{*}[16m{+}16]^{(u)} - 0.13x[16m{+}16]^{(u)}y^{*}[16m{+}32]^{(u)} \\
&\quad + 0.04x[16m{+}32]^{(u)}y^{*}[16m{-}16]^{(u)} - 0.13x[16m{+}32]^{(u)}y^{*}[16m]^{(u)} - 0.13x[16m{+}32]^{(u)}y^{*}[16m{+}16]^{(u)} + 0.04x[16m{+}32]^{(u)}y^{*}[16m{+}32]^{(u)}.
\end{aligned} \tag{7}
$$

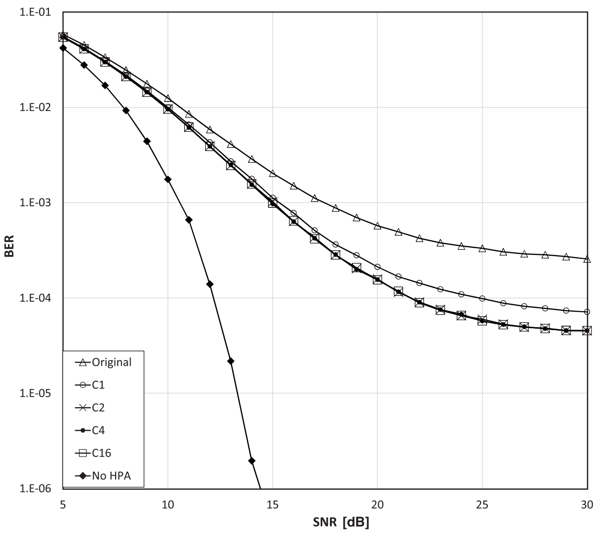

Fig. 2. BER performances of SLM schemes with CORR metric when $N = 256$, $U = 4$, and $L = 1, 2, 4$, and 16 with OBO = 4 dB.

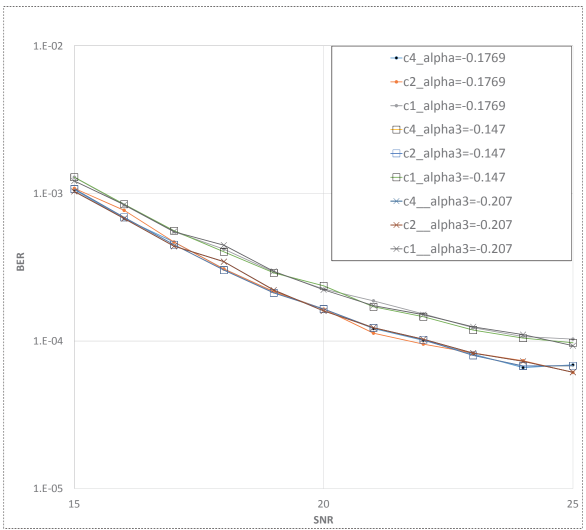

Fig. 3. BER performance of CORR SLM schemes with different α_3's.

IV. Simulation Results and Conclusion

In this section, we derive the BER performances of SLM schemes with different L times oversamplings are used for CORR computation, where $N = 256$, $U = 4$, and OBO $= 4$ dB. Also, nonlinear HPA and AWGN channel is considered and 16 times oversampling is used for transmission to estimate continuous OFDM signal. We assume the perfect knowledge of side information.

In Fig. 2, Original, CL, and No HPA indicate OFDM signal without SLM scheme, OFDM signal with CORR computation (L times oversampling), and OFDM signal without HPA, respectively. C2, C4, and C16 show almost the same BER performances, while C1 is degraded compared to them. In Fig. 3, various values of α_3 was simulated for CORR SLM scheme and it was found that there was no impact.

In this paper, we analyze the oversampling effect when CORR metric is used for SLM schemes in the presence of nonlinear HPA. By deriving the oversampled signals which are obtained by linear combination of Nyquist rate samples, each signal sequence for CORR computation for $L = \{1, 2, 4, 16\}$ can be obtained.

Simulation result shows that BER peformance of two times oversampling for CORR metric is almost the same as that of four or 16 times oversampling. On the other hand, Nyquist sampling rate is degraded from two, four, and 16 times oversampling. The computational complexity of two times oversampling is reduced to $1/8$ as that of 16 times oversampling when computing CORR metric.

Consequently, two times oversampling for CORR computation is enough to achieve the same BER peformance as that of 16 times oversampling.

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References

[1] M. R. D. Rodrigues and I. J. Wassell, "IMD reduction with SLM and PTS to improve the error-probability performance of nonlinearly distorted OFDM signals," *IEEE Trans. Veh. Technol.,* vol. 55, pp. 537–548, Mar. 2006.

[2] E. Al-Dalakta, A. Al-Dweik, A. Hazmi, C. Tsimenidis, and B. Sharif, "Efficient BER reduction technique for nonlinear OFDM transmission using distortion prediction," *IEEE Trans. Veh. Technol.*, vol. 61, pp. 2330–2336, Jun. 2012.

[3] D. Park and H. Song, "A new PAPR reduction technique of OFDM system with nonlinear high power amplifier," *IEEE Trans. Consum. Electron.,* vol. 53, pp. 327–332, May 2007.

[4] E. Al-Dalakta, A. Al-Dweik, A. Hazmi, C. Tsimenidis, and B. Sharif, "PAPR reduction scheme using maximum cross correlation," *IEEE Commun. Lett.,* vol. 16, no. 12, pp. 2032–2035, Dec. 2012.

[5] C. L. Wang, S. J. Ku, and C. J. Yang, "A low-complexity PAPR estimation scheme for OFDM signals and its application to SLM-based PAPR reduction," *IEEE J. Sel. Topics Signal Process.,* vol. 4, no. 3, pp. 637–645, Jun. 2010.