A Low-Complexity PTS Scheme Using Adaptive Selection of Dominant Time-Domain Samples in OFDM Systems

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***Abstract*—In orthogonal frequency division multiplexing (OFDM) systems, partial transmit sequence (PTS) scheme has been widely studied to mitigate the peak-to-average power ratio (PAPR) problem of OFDM signals. In this paper, an improved method to select dominant time-domain samples for PTS scheme is proposed to calculate the PAPR of OFDM signals efficiently. In the proposed selection method, it is performed adaptively at each time-domain sample to rotate time-domain subbolck samples. The PTS scheme using the proposed selection method can achieve almost the optimal PAPR reduction performance of PTS schemes with much lower computational complexity.**

***Keywords*—Adaptive sample rotation; dominant time-domain samples; orthogonal frequency division multiplexing (OFDM); partial transmit sequence (PTS); peak-to-average power ratio (PAPR)**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a technique of frequency division multiplexing on multiple orthogonal subcarriers. OFDM has the primary advantages such as robustness against fading channel, bandwidth efficiency, and equalization efficiency. Due to these properties, OFDM has been used in various applications such as wireless local-area network (WLAN) 802.11a/g/n and 802.16 standards, digital video broadcasting (DVB), and 4G mobile communications.

However, transmitted signals in OFDM systems have high peak-to-average power ratio (PAPR). It causes the distortion through nonlinear high power amplifier (HPA) at the transmitter and therefore results in degradation of bit error rate (BER) performance at the receiver.

For solution of PAPR problem in OFDM systems, various PAPR reduction schemes have been proposed. Partial transmit sequence (PTS) scheme [1] is one of the PAPR reduction schemes without signal distortion but has problem of high computational complexity. In general, the computational complexity of PTS schemes is related with the number of candidate OFDM signals due to generation and PAPR calculation of those signals.

To reduce the number of candidate OFDM signals for low computational complexity, various PTS schemes have been proposed [2]–[6]. Some PTS schemes have been proposed to reduce the number of PAPR calculations and comparisons of candidate OFDM signals for low computational complexity [7], [8]. These two schemes are based on method to select dominant time-domain samples for efficient PAPR calculation of the PTS schemes.

In this paper, an improved method to select dominant time-domain samples for the PTS scheme is proposed for low complexity. The proposed method is performed by adaptively rotating subblock samples to the prescribed region and estimating the maximum power of each time-domain sample. Using the adaptive selection method, the proposed PTS scheme considerably reduces the computational complexity maintaining almost the same PAPR reduction as that of the conventional PTS scheme with the optimal PAPR reduction performance.

The remainder of the paper is organized as follows. Section II introduces basic background for main result such as OFDM signal, its PAPR, and the conventional PTS scheme. In Section III, a low-complexity PTS scheme is proposed based on the adaptive selection method for dominant time-domain samples. Also, its computational complexity and PAPR reduction performance are provided in this section. Finally, Section IV concludes the paper.

II. Preliminaries

A. PAPR of OFDM Signal

In OFDM systems, serial N modulated symbols are oversampled to generate an oversampled OFDM symbol vector $\mathbf{X} = [X_0, X_1, \cdots, X_{LN-1}]$. By using inverse fast Fourier transform (IFFT), $\mathbf{X}$ can be transformed to a time-domain OFDM signal vector $\mathbf{x} = [x_0, x_1, \cdots, x_{LN-1}]^T$ represented by

$$x_n = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X_k e^{j2\pi kn/LN} \quad (1)$$

where L denotes the oversampling factor.

The PAPR of $\mathbf{x}$ is defined as

$$PAPR(\mathbf{x}) = \frac{\max_{n=0}^{LN-1} |x_n|^2}{E[|x_n|^2]} \tag{2}$$

where $E[\cdot]$ denotes the expectation. OFDM signal vectors usually have the high PAPR values. The conventional PTS scheme reduces the PAPR of OFDM signal vectors as in the following subsection.

B. Conventional PTS Scheme

In the conventional PTS scheme, an OFDM symbol vector $\mathbf{X}$ is partitioned into M disjoint subblock symbol vectors $\mathbf{X}_m = [X_{m,0}, X_{m,1}, \cdots, X_{m,LN-1}]^T$, $0 \leq m \leq M-1$, such that

$$\mathbf{X} = \sum_{m=0}^{M-1} \mathbf{X}_m \tag{3}$$

where $X_{m,n}$ is either X_n or 0. After partitioning, each $\mathbf{X}_m$ is IFFTed to the corresponding time-domain subblock signal vectors $\mathbf{x}_m = [x_{m,0}, x_{m,1}, \cdots, x_{m,LN-1}]^T$, $0 \leq m \leq M-1$.

Phase rotating factors $b_m^{(u)}$, $0 \leq m \leq M-1$, $0 \leq u \leq U-1$ are used to generate corresponding U candidate OFDM signal vectors $\mathbf{x}^{(u)} = [x_0^{(u)}, x_1^{(u)}, \cdots, x_{LN-1}^{(u)}]^T$, $0 \leq u \leq U-1$. The u-th candidate OFDM signal vector $\mathbf{x}^{(u)}$ is generated by multiplying each $b_m^{(u)}$ to the corresponding $\mathbf{x}_m$ and adding all the u-th factored subblock signal vectors, that is

$$\mathbf{x}^{(u)} = \sum_{m=0}^{M-1} b_m^{(u)} \mathbf{x}_m. \tag{4}$$

In other words, the u-th phase rotating vector defined by $\mathbf{b}^{(u)} = [b_0^{(u)}, b_1^{(u)}, \cdots, b_{M-1}^{(u)}]$ is used to generate the corresponding $\mathbf{x}^{(u)}$. In general, $b_m^{(u)}$ is given as

$$b_m^{(u)} \in \{e^{j2\pi l/W} \mid l = 0, 1, \cdots, W-1\} \tag{5}$$

where W denotes the number of phase rotating factors. Since it is assumed that $b_0^{(u)} = 1$, the number of total candidate OFDM signal vectors U is generally determined as $U = W^{M-1}$.

In the conventional PTS scheme, among the U candidate OFDM signal vectors, the OFDM signal vector $\mathbf{x}^{(u_{opt})}$ with the minimum PAPR is selected and transmitted. The index u_{opt} is obtained as

$$u_{opt} = \arg \min_{u=0}^{U-1} PAPR(\mathbf{x}^{(u)}). \tag{6}$$

Fig. 1 shows a general procedure of the conventional PTS scheme explained as above.

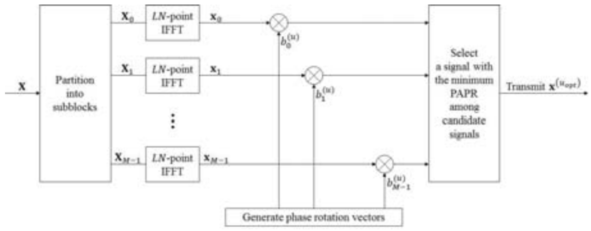

Fig. 1. A block diagram of the conventional PTS scheme.

III. Low-Complexity PTS Scheme

A. Adaptive Selection for Dominant Time-Domain Samples

In PTS schemes, the metric V_n, $0 \leq n \leq LN-1$ is defined as

$$\begin{aligned} V_n &= \max_{u=0}^{U-1} \left| x_n^{(u)} \right|^2 \\ &= \max_{u=0}^{U-1} \left| \sum_{m=0}^{M-1} b_m^{(u)} x_{m,n} \right|^2. \end{aligned} \tag{7}$$

It is known that V_n is the proper metric for selecting dominant time-domain samples which help efficiently calculating the PAPRs of the candidate OFDM signal vectors. The index set of the dominant time-domain samples by V_n is obtained as

$$\mathbf{S}_V(N_\gamma) = \{n \mid V_n \geq \gamma, \quad 0 \leq n \leq LN-1\} \tag{8}$$

where γ denotes a preset threshold satisfying condition as

$$|\mathbf{S}_V(N_\gamma)| = N_\gamma. \tag{9}$$

With only the dominant time-domain samples with indices in $\mathbf{S}_V(N_\gamma)$, the PAPRs of all candidate OFDM signal vectors can be calculated instead of the PAPR calculation with all time-domain samples. However, full search over all the corresponding U candidate samples must be still required to calculate V_n. To reduce the number of the searches to calculate V_n, an adaptive search method is proposed in the Figs. 2 and 3.

Fig. 2 shows an example of original n-th time-domain subblock signal samples in (a) and four different types of their rotation by 0° or 180° to four subplanes (shaded regions), where the first subblock signal sample $x_{0,n}$ is located in the complex plane in (b), (c), (d), and (e), respectively for $M = 4$ and $W = 2$. In Fig. 2(b), all the n-th subblock signal samples except $x_{0,n}$ are rotated to the first and the second quadrants by multiplying the phase rotating factors $(b_{0,n}, b_{1,n}, b_{2,n}, b_{3,n}) = (1, 1, -1, -1)$ to each subblock signal sample. Similarly, they are rotated to the first and the fourth quadrants by multiplying $(1, -1, 1, -1)$ as in Fig. 2(c). Figs 2(d) and 2(e) also show their other rotations to the subplanes below 45°-line and above 135°-line by multiplying $(1, -1, 1, 1)$ and $(1, 1, 1, -1)$, respectively.

Fig. 3 shows a different case with the example illustrated in Fig. 2 for $M = 4$ and $W = 2$. In Fig. 3, there is only one type of rotation of all the n-th subblock signal samples except $x_{0,n}$ to the first quadrants by multiplying $(b_{0,n}, b_{1,n}, b_{2,n}, b_{3,n}) = (1, 1, -1, -1)$. Note that in contrast with the case of Fig. 2, there always exists the only one type of rotation to one of eight quadrants defined as follows in the case of Fig. 3. The eight quadrants consist of additional four 90°-subplanes determined by using 45°-line and 135°-line as boundary lines as well as original four quadrants determined by using the horizontal line and the vertical line as boundary lines in the complex plane.

Based on Fig. 2 and Fig. 3, the proposed method to estimate V_n is explained for $W = 2$. The number of searches is adaptively determined by considering positions of all the n-th subblock signal samples. If they are located in one of eight

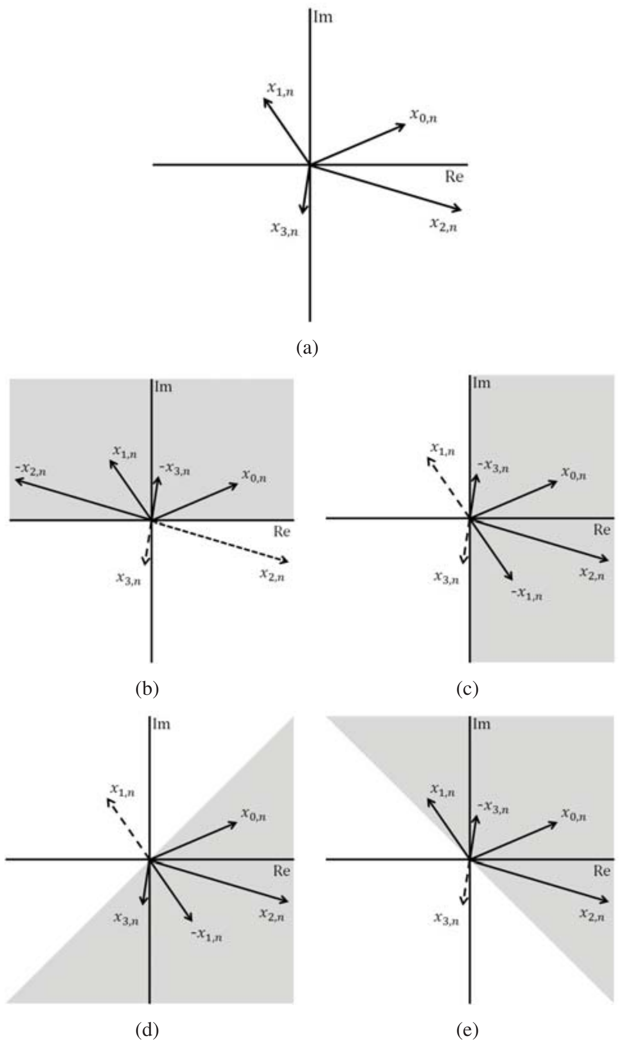

Fig. 2. The proposed method 1 for adpative phase rotation: (a) original n-th time-domain subblock signal samples and their rotation to the shaded subplane containing $x_{0,n}$ using $(b_{0,n}, b_{1,n}, b_{2,n}, b_{3,n})$ (b) $(1, 1, -1, -1)$, (c) $(1, -1, 1, -1)$, (d) $(1, -1, 1, 1)$ and (e) $(1, 1, 1, -1)$ for $M = 4$ and $W = 2$.

quadrants or its 180°-rotated quadrant in the complex plane, only one quadrant containing $x_{0,n}$ is selected and rotation of subblock signal samples to the quadrant is performed to estimate V_n. Otherwise, at most four half-plane containing $x_{0,n}$ are selected and rotations of subblock signal samples to the subplane are performed to estimate V_n. Note that the proposed method to estimate V_n in the case of $W = 4$ is similar with the case of $W = 2$.

In general, U searches are required to calculate V_n of the n-th time-domain sample. However, the computational complexity for estimating V_n is substantially reduced by using the proposed search method. With the proposed method, the powers of only C_n candidate time-domain samples are calculated at the index n. These candidate time-domain samples are denoted as $x_n^{(K_n(0))}, x_n^{(K_n(1))}, \cdots, x_n^{(K_n(C_n-1))}$, where $K_n(i) \in \{0, 1, \cdots, U-1\}$ is the selected index among indices

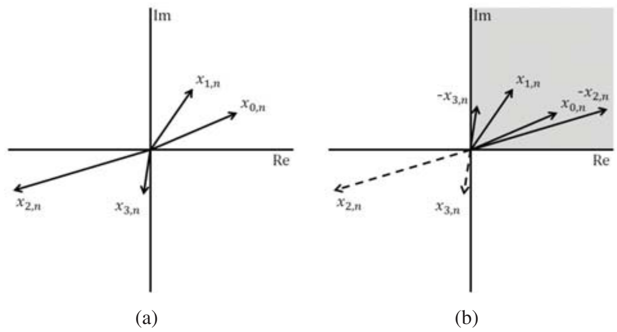

Fig. 3. The proposed method 2 for adpative phase rotation: (a) original n-th time-domain subblock signal samples and (b) their rotation to the shaded subplane containing $x_{0,n}$ using $(b_{0,n}, b_{1,n}, b_{2,n}, b_{3,n}) = (1, 1, -1, -1)$ for $M = 4$ and $W = 2$.

of U candidate time-domain samples by the proposed method. Note that the values C_n varies from $C_{\min} = 1$ to $C_{\max} = 4$ depending on the index n in the case of $W = 2$ and from $C_{\min} = 1$ to $C_{\max} = 2$ in the case of $W = 4$, respectively. By using the proposed method, an alternative metric P_n for estimating V_n is calculated as

$$P_n = \max_{c=0}^{C_n-1} \left| x_n^{(K_n(c))} \right|^2. \tag{10}$$

With P_n, the index set of dominant time-domain samples can be obtained as

$$\mathbf{S}_P(N_\gamma) = \{n \mid P_n \geq \gamma, \quad 0 \leq n \leq LN - 1\} \tag{11}$$

where γ denotes a preset threshold satisfying condition as

$$|\mathbf{S}_P(N_\gamma)| = N_\gamma. \tag{12}$$

By using the proposed method, only the time-domain subblock samples with the indices in $\mathbf{S}_P(N_\gamma)$ are used for calculating the PAPR of each candidate OFDM signal vector for the PTS scheme.

B. Low-Complexity PTS Scheme

In the proposed PTS scheme using the method in Subsections III-A, the index u_{opt} for selecting and transmitting the OFDM signal vector is obtained as

$$u_{opt} = \arg\min_{u=0}^{U-1} \frac{\max_{n \in \mathbf{S}_P(N_\gamma)} \left| \sum_{m=0}^{M-1} b_m^{(u)} x_{m,n} \right|^2}{E[|x_n|^2]} \tag{13}$$

which is based on the selection of dominant time-domain samples by using P_n. Fig. 4 shows overall procedure of the proposed PTS scheme.

The detailed procedure of the proposed PTS scheme is summarized as follows:

1) An original OFDM symbol vector $\mathbf{X}$ is partitioned into M disjoint subblock symbol vectors $\mathbf{X}_m$'s and each of them is IFFTed.
2) Calculate the metric P_n in (10) at each index n by using the methods in Figs. 2 and 3 adaptively.
3) Obtain the index set $\mathbf{S}_P(N_\gamma)$ in (11) by setting the threshold γ and the corresponding N_γ.

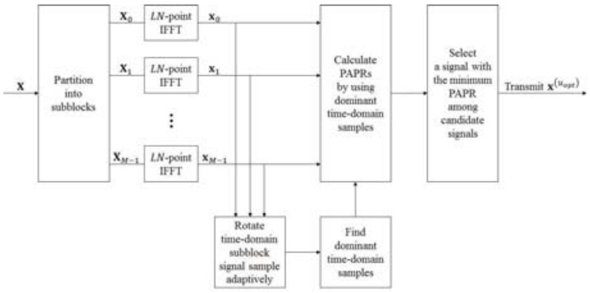

Fig. 4. A block diagram of the proposed PTS scheme.

TABLE I

COMPARISON OF COMPUTATIONAL COMPLEXITY OF THE CONVENTIONAL PTS, RC-PTS, PS-PTS, AND THE PROPOSED PTS FOR $N = 1024$, $L = 4$, $M = 8$, $W = 2$, AND $p_\gamma = 0.07$.

PTS scheme	Number of complex multiplications	PAPR for CCDF = 10^{-4}
Conventional PTS	$LNU = 524288$ (100%)	$8.4\,dB$
RC-PTS	$MLN + p_\gamma LNU = 69468$ (13.3%)	$10\,dB$
PS-PTS	$C_{\max}LN + p_\gamma LNU = 53084$ (10.1%)	$8.5\,dB$
LA-PTS	$\sum_{n=0}^{LN-1} C_n + p_\gamma LNU = 51379$ (9.8%)	$8.5\,dB$

4) Calculate the PAPRs of candidate OFDM signal vectors with only the dominant time-domain samples with the indices in $\mathbf{S}_P(N_\gamma)$.
5) Select and transmit $\mathbf{x}^{(u_{opt})}$ by using (13).

C. Computational Complexity

This subsection compares the computational complexity of the conventional PTS scheme, RC-PTS [7], PS-PTS [8], and the proposed low-complexity adaptive PTS scheme (LA-PTS). For comparison, the ratio p_γ is defined as the ratio between the number of selected time-domain samples and the number of all time-domain samples, that is

$$p_\gamma = \frac{N_\gamma}{LN}. \tag{14}$$

Note that in case of $p_\gamma = 0.07$, PS-PTS and LA-PTS achieve almost the same PAPR reduction performance as the conventional PTS at PAPR CCDF=10^{-4}.

Table II compares the computational complexity of the conventional PTS, RC-PTS, PS-PTS, and LA-PTS for $N = 1024$, $L = 4$, $M = 8$, and $W = 2$. For fair comparison, $p_\gamma = 0.07$ is set. Compared with the conventional PTS, LA-PTS shows the lowest relative computational complexity (9.8%) with achieving almost the same PAPR $8.5\,dB$ at CCDF $= 10^{-4}$. On the other hand, PS-PTS and RC-PTS show the relative computational complexity (10.1%) and (13.3%) with the PAPR $8.5\,dB$ and $10\,dB$ at CCDF $= 10^{-4}$, respectively.

D. Simulation Results

This subsection compares the PAPR reduction performance of the conventional PTS, RC-PTS, PS-PTS, and LA-PTS. Fig. 5 compares the PAPR reduction performance of these PTS schemes for $N = 1024$, $L = 4$, $M = 8$, and $W = 2$. Similar in Subsection III-C, $p_\gamma = 0.07$ is set for fair comparison. Compared with the conventional PTS, PS-PTS and LA-PTS show almost the same PAPR reduction performance which is known to be optimal, by only using 7% of time-domain samples.

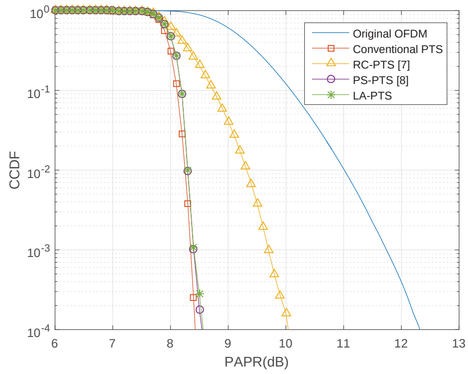

Fig. 5. Comparison of PAPR reduction performance of the conventional PTS, RC-PTS, PS-PTS, and the Proposed PTS for $N = 1024$, $L = 4$, $M = 8$, $W = 2$, and $p_\gamma = 0.07$.

IV. CONCLUSION

In this paper, a low-complexity PTS schemes using adaptive selection method of dominant time-domain samples is proposed. The proposed selection method reduces the computational complexity for calculating PAPRs of candidate OFDM signal vectors by using adaptive rotation of time-domain subblock signal vectors. The proposed PTS scheme can achieve almost the same PAPR reduction performance as that of the optimal PTS scheme with considerable computational complexity reduction.

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