단항순열행렬에 의해 구성된 비실베스터 하다마드 행렬의 고유치

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Eigenvalues of Non-Sylvester Hadamard Matrices Constructed by Monomial Permutation Matrices

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요 약

본 논문에서는 단항순열행렬에 의해 구성된 다양한 비실베스터 하다마드 행렬의 고유치가 유도 되었고 이는, 새로 구성한 행렬과 실베스트 하다마드 행렬의 고유치와의 연관성을 보여준다.

Key Words : Hadamard matrix, Equivalent Hadamard matrix, Eigenvalues of Hadamard matrix, Monomial permutation matrix

ABSTRACT

In this paper, the eigenvalues of various non-Sylvester Hadamard matrices constructed by monomial permutation matrices are derived, which shows the relation between the eigenvalues of the newly constructed matrix and Sylvester Hadamard matrix.

Ⅰ. Introduction

A Hadamard matrix *A* of order *n* is an *n* x *n* square matrix of +1's and -1's such that any pair of distinct rows is orthogonal (i.e., their inner product is zero). In the Hadamard matrix invented by Sylvester (1867), placing any two columns or rows side by side gives half the adjacent cells the same sign and the half, the opposite sign. The following serves as a formal definition of Hadamard matrix.

Let N be the set of natural numbers.

Definition I.1: A Hadamard matrix H_n of order $n \in N$ is an *n* x *n* matrix of +1's and -1's such that $H_n H_n^T = n I_n$ where I_n is the *n* x *n* identity matrix and H_n^T denotes transpose of H_n.

There are various construction methods for Hadamard matrices such as Sylvester construction and Paley construction (see [1], Chapter 6). In this paper, we will modify the Sylvester construction of order 2^k, $k \in N$.

It is known that Hadamard transform is an orthogonal transform with practical purpose for representing signals and images especially for the data compression [2]. A complete set of 2^n Walsh functions of order n

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gives a Hadamard matrix H_{2^n}. Walsh Hadamard transform (WHT) is used for the Walsh representation of the data sequences in image coding and for signature sequence in the CDMA mobile communication systems.

It is known that the sampled Walsh function corresponds to row vectors of the Hadamard matrices H_n. Hadamard matrices can be used to make error-correcting codes, in particular, the Reed-Muller codes.

Let I_{2^j} and O_{2^j}, $j \in N \cup 0$ be the $(2^j$ x $2^j)$ identity matrix and zero matrix, respectively.

We also define matrices as

$$Z_{2^k} = \begin{bmatrix} I_{2^{(k-1)}} & O_{2^{(k-1)}} \\ O_{2^{(k-1)}} & -I_{2^{(k-1)}} \end{bmatrix}, S_{2^k} = \begin{bmatrix} O_{2^{(k-1)}} & I_{2^{(k-1)}} \\ I_{2^{(k-1)}} & O_{2^{(k-1)}} \end{bmatrix}$$

$$J_{2^k} = \begin{bmatrix} O_{2^{(k-1)}} & I_{2^{(k-1)}} \\ -I_{2^{(k-1)}} & O_{2^{(k-1)}} \end{bmatrix} \text{for } k \geq 1.$$

Sylvester Hadamard matrix H_n can be represented as

$$H_{2^{k+1}} = H_2 \otimes H_{2^k} \text{ for } k \geq 1$$

where $\otimes$ denotes the Kronecker product and

$$H_2 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}.$$

Upto now, only the eigenvalues of the Sylvester Hadamard matrix $H_{2^{k+1}}$ are known as

i) 2^k eigenvalues are $+2^{(k+1)/2}$,
ii) 2^k eigenvalues are $-2^{(k+1)/2}$.

It is interesting to investigate the eigenvalues of Hadamard matrices other than Sylvester-type [3], [4].

The following definitions and theorems will be used in Section II.

A monomial matrix (sometimes called scaled permutation matrix) has exactly one nonzero entry in every row and column.

Definition I.2: [5] Matrices A and B are said to be *equivalent Hadamard matrices*, if $B = PAQ$, where P and Q are *monomial permutation matrices* with elements -1 and +1.

Let $n \in N$ be a dimension of matrix A. A^*, $\overline{A}$ and A^T denote the complex conjugate and transpose, complex conjugate, and transpose of A, respectively. Further let

$$J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$$

be a *standard involutory permutation (sip) matrix*, which has the properties $J^{-1} = -J$ and $J^T = -J$.

Definition I.3: (Adjoint) The adjoint of the $r \times c$ matrix A is the $r \times c$ matrix such that

$$A*_{i,j} = \overline{A_{j,i}}\ ,\quad 1 \leq i \leq r,\ 1 \leq j \leq c.$$

Let C be the set of complex numbers.

An operator A is called *self-adjoint* or *Hermitian* if $A = A^*$ and *normal* if $AA^* = A^*A$.

It can be easily seen that for every n x n invertible Hermitian matrix L, the formula

$$[x, y] = (Lx, y),\ x, y \in C^n$$

determines an indefinite scalar product on C^n.

The L-*adjoint* $A^{[*]}$ of A is the unique matrix, which satisfies $[Ax, y] = [x, A^{[*]}y]$ for all $x, y \in C^n$. Let $A^*: C^n \to C^n$ be the usual adjoint of A (i.e. $(x, A^*y) = (Ax, y)$ for all $x, y \in C^n$). It follows that

$$A^{[*]} = L^{-1}A^*L.$$

Now, it is natural to describe a matrix as L-*selfadjoint* (or selfadjoint with respect to [.,.]) if $A = A^{[*]}$.

Proposition I.4: [6] The set of eigenvalues $\lambda(A)$ of an L-selfadjoint matrix A, is symmetric relative to the real axis, i.e. $\lambda_0 \in \lambda(A)$ implies $\overline{\lambda_0} \in \lambda(A)$. Moreover, in the Jordan normal form of A, the size of the Jordan blocks with eigenvalue λ_0 are equal to the sizes of Jordan blocks with eigenvalue $\overline{\lambda}_0$.

In the next section, we will investigate the eigenvalues and eigenvectors of the non-Sylvetser Hadamard matrices, which are particularly equivalent to the Sylvester Hadamard matrices.

Ⅱ. Main Results

Let $\widetilde{P_4}$ = diag { 1,1,1,-1} be a 4x4 monomial permutation matrix.

In order to construct the non-Sylvester Hadamard matrices, we define the non- Sylvester Hadamard matrix $\widetilde{R_4}$ as

$$\widetilde{R_4} = \widetilde{P_4} H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 S_2 & -H_2 S_2 \end{bmatrix}.$$

Then,

$$\lambda(\widetilde{R_4}) = \{-2, 2, -1 + j\sqrt{3}, -1 - j\sqrt{3}\}$$

is the set of all eigenvalues of the matrix $\widetilde{R_4}$.

Let

$$\widetilde{P}_{2^{k+1}} = \widetilde{P}_{2^k} \otimes I_2.$$

Further, let

$$\widehat{P_8} = \begin{bmatrix} I_2 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 \\ 0 & 0 & I_2 & 0 \\ 0 & 0 & 0 & -Z_2 \end{bmatrix}$$

and $\widehat{P}_{2^{k+1}} = \widehat{P_8} \otimes I_{2^{k-2}} \quad (k \geq 3)$.

Then we can define the matrix $\widetilde{R}_{2^{k+1}}$ as follows.

Definition II.1: For $k \geq 1$, we define a class of non-Sylvester Hadamard matrices as

$$\widetilde{R}_{2^{k+1}} = \widetilde{P}_{2^{k+1}} \cdot H_{2^{K+1}} = \begin{bmatrix} H_{2^k} & H_{2^k} \\ H_{2^k} S_{2^k} & -H_{2^k} S_{2^k} \end{bmatrix}.$$

Note that the sign of a quarter rows of the matrix $H_{2^{k+1}}$ is changed to yield $\widetilde{R}_{2^{k+1}}$.

Similarly, we can also define a matrix using $\widehat{P}_{2^{k+1}}$ as follows.

Definition II.2: For $k \geq 2$, we define a matrix as

$$\widehat{P}_{2^{k+1}} = \widehat{P}_{2^{k+1}} \cdot H_{2^{K+1}} = \begin{bmatrix} \widetilde{R}_{2^k} & \widetilde{R}_{2^k} \\ \widetilde{R}_{2^k} S_{2k} & \widetilde{R}_{2^k} S_{2^k} \end{bmatrix}.$$

Remark II.3: Since $\widetilde{P}_{2^{k+1}}$ and $\widehat{P}_{2^{j+1}}$ are both monomial permutation matrices according to Definition I.2, the matrices $\widetilde{R}_{2^{k+1}}$ and $\widetilde{R}_{2^{j+1}}$ are equivalent to the Sylvetser Hadamard matrix $H_{2^{k+1}}$ and $H_{2^{j+1}}$ for $k \geq 1$ and $j \geq 2$, respectively.

Definition II.4: For a given n, the complex numbers z which satisfies

$$z^n = 1 \qquad (n \in N_0)$$

are called the complex n-th roots of unity. There are n different n-th roots of unity.

Theorem II.5: Let $\lambda(H_{2^{k+1}}),\ k \geq 1$, and λ_H be the set of eigenvalues and a eigenvalue of a Sylvester Hadamard matrix $H_{2^{k+1}}$ of order $n = 2^{k+1}$, respectively.

The spectrum of the matrix $\widetilde{R}_{2^{k+1}}$ for $k \geq 2$ can be expressed as

i) $2^{(k+1)/2}$, 2^{k-1} times
ii) $-2^{(k+1)/2}$, 2^{k-1} times
iii) $2^{(k-1)/2} + j\,2^{(k+1)/2} \sin\frac{\pi}{3}$, 2^{k-2} times
iv) $2^{(k-1)/2} - j\,2^{(k+1)/2} \sin\frac{\pi}{3}$, 2^{k-2} times
v) $-2^{(k-1)/2} + j\,2^{(k+1)/2} \sin\frac{\pi}{3}$, 2^{k-2} times
vi) $-2^{(k-1)/2} - j\,2^{(k+1)/2} \sin\frac{\pi}{3}$, 2^{k-2} times

where $j = \sqrt{-1}$.

Proof: We first prove the case i) and ii).

Let $v_i^{(1)}$ and $v_i^{(2)}$, $1 \leq i \leq 2^k$ be eigenvectors of the corresponding eigenvalues $2^{\frac{k+1}{2}}$ and $-2^{\frac{k+1}{2}}$ of $H_{2^{k+1}}$, respectively.

It is clear that any linear combinations of $v_i^{(1)}$ (or $v_i^{(2)}$) are also eigenvectors of the corresponding eigenvalue $2^{\frac{k+1}{2}}$ (or $-2^{\frac{k+1}{2}}$).

Using the elementary column operations for $v_i^{(1)}$ (or $v_i^{(2)}$), $1 \le i \le 2^k$, $2^{k+1} \times 2^{k+1}$ matrix consisting of 2^{k+1} eigenvectors given as

$$V=[\, v_1^{(1)},\ v_2^{(1)}, \cdots, v_{2^k}^{(1)}\ \ v_1^{(2)}, v_2^{(2)}, \cdots, v_{2^k}^{(2)} \,]$$

can be modified as

where

$$B_{2^k} = \text{diag}\,\{ \lambda_1, \cdots, \lambda_{2^{k-1}}, -\lambda_1, \cdots, -\lambda_{2^{k-1}} \}$$
$$= \text{diag}\,\{ 2^{\frac{k+1}{2}}, \cdots, 2^{\frac{k+1}{2}}, -2^{\frac{k+1}{2}}, \cdots, -2^{\frac{k+1}{2}} \}.$$

From this, we can infer that half-spectrums of the matrices $\widetilde{R}_{2^{k+1}}$ and $H_{2^{k+1}}$ for $k \ge 1$ are the same.

$$V=\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots\cdots & a_{1,2^k} & a_{1,2^k+1} & a_{1,2^k+2} & \cdots\cdots & a_{1,2^{k+1}} \\ a_{2,1} & a_{2,2} & \cdots\cdots & \cdots & a_{2,2^k+1} & \cdots & \cdots\cdots & a_{2,2^{k+1}} \\ \vdots & \cdots & \cdots\cdots & \cdots & \cdots & \cdots & \cdots\cdots & \vdots \\ & \cdots & \cdots\cdots & \cdots & \cdots & \cdots & \cdots\cdots & \\ a_{2^k,1} & \cdots & \cdots\cdots & \cdots & a_{2^k,2^k+1} & \cdots & \cdots\cdots & a_{2^k,2^{k+1}} \\ a_{2^k+1,1} & a_{2^k+1,2} & \cdots\cdots & \cdots & a_{2^k+1,2^k+1} & a_{2^k+1,2^k+2} & \cdots\cdots & a_{2^k+1,2^{k+1}} \\ 0 & a_{2^k+2,2} & \cdots\cdots & \cdots & 0 & a_{2^k+2,2^k+2} & \cdots\cdots & \vdots \\ \vdots & \cdots & \cdots\cdots & \cdots & \vdots & \cdots & \cdots\cdots & \vdots \\ 0 & 0 & \cdots\cdots & \cdots & 0 & 0 & \cdots\cdots & \vdots \\ 0 & 0 & 0\ \cdots & a_{2^{k+1},2^k} & 0 & 0 & 0\ \cdots & a_{2^{k+1},2^{k+1}} \end{bmatrix}.$$

By choosing the first 2^{k-1} columns and 2^k+1-st column through $3 \cdot 2^{k-1}$-th columns in (2.13), for $k \ge 1$, we have the $2^{k+1} \times 2^k$ matrix as

$$X_{2^{k+1}}=\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,2^{k-1}} & a_{1,2^k+1} & a_{1,2^k+2} & \cdots & a_{1,3\cdot 2^{k-1}} \\ a_{2,1} & a_{2,2} & \cdots & \cdots & a_{2,2^k+1} & a_{2,2^k+2} & \cdots & a_{2,3\cdot 2^{k-1}} \\ \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots \\ & & \cdots & \cdots & \cdots & \cdots & & \\ a_{2^k+1,1} & a_{2^k+1,2} & \cdots & \cdots & a_{2^k+1,2^k+1} & a_{2^k+1,2^k+2} & \cdots & a_{2^k+1,3\cdot 2^{k-1}} \\ \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \end{bmatrix}$$

where the last 2^{k-1} rows are zero.

It is clear that the first 2^{k-1} columns are still eigenvectors of the corresponding eigenvalue $2^{(k+1)/2}$ and the last 2^{k-1} columns the corresponding eigenvalue $-2^{(k+1)/2}$ of $H_{2^{k+1}}$. Thus we have the following relations

$$X_{2^{k+1}}^T \widetilde{P}_{2^{k+1}} = X_{2^{k+1}}^T,$$

and

$$\begin{aligned} X_{2^{k+1}}^T \widetilde{R}_{2^{k+1}} &= X_{2^{k+1}}^T \widetilde{P}_{2^{k+1}} H_{2^{k+1}} \\ &= X_{2^{k+1}}^T H_{2^{k+1}} \\ &= B_{2^k} X_{2^{k+1}}^T, \end{aligned}$$

Let $A = H_{2^{k-1}}$ and $M=\begin{bmatrix} A & A \\ A & -A \end{bmatrix}$. Then $MS=\begin{bmatrix} A & A \\ -A & A \end{bmatrix}$. We obtain

$$H_{2^{k+1}} = \begin{bmatrix} M & M \\ M & -M \end{bmatrix} = \begin{bmatrix} A & A & A & A \\ A & -A & A & -A \\ A & A & -A & -A \\ A & -A & -A & A \end{bmatrix}$$ and

$$\widetilde{R}_{2^{k+1}} = \begin{bmatrix} M & M \\ MS & -MS \end{bmatrix} = \begin{bmatrix} A & A & A & A \\ A & -A & A & -A \\ A & A & -A & -A \\ -A & A & A & -A \end{bmatrix}.$$

Since $AA = 2^{k-1} I_{2^{k-1}}$, $A(-A) = -I_{2^{k-1}}$, and $MM = 2^k I_{2^k}$, it is easily verified that

$$H_{2^{k+1}}^6 = \widetilde{R}_{2^{k+1}}^6 = (2^{k+1})^3 I_{2^{k+1}}. \tag{2.17}$$

Equation (2.17) tells us that eigenvalues of $\widetilde{R}_{2^{k+1}}$ are complex 6-th roots on the circle, whose radius is the absolute value of the eigenvalues of the Sylvester Hadamard matrix. It is known that if λ is a complex eigenvalue of real matrix, then $\overline{\lambda}$ is also eigenvalue of the real matrix. Thus $2^{\frac{k+1}{2}} e^{\pm j\frac{\pi}{3}}$ are the eigenvalues of $\widetilde{R}_{2^{k+1}}$.

Since the trace of $\widetilde{R}_{2^{k+1}}$ is equal to zero, the sum of all eigenvalues of $\widetilde{R}_{2^{k+1}}$ should be zero. Using the above properties, the distribution of the eigenvalues can be derived. △

Corollary II.6: Let λ_{2^k} be an eigenvalue of the matrix $\widetilde{R}_{2^k}$, which is not an eigenvalue of the Sylvester Hadamard matrix H_{2^k} for $k \geq 1$. Then, we have

$$\lambda_{2^{k+1}} = \lambda_{2^k}\sqrt{2}.$$

Theorem II.7: Let $\lambda(H_{2^{k+1}}),\quad k \geq 1,$ and λ_H be the set of eigenvalues and a eigenvalue of a Sylvester Hadamard matrix $H_{2^{k+1}}$ of order $n = 2^{k+1}$, respectively.

Then the spectrum of the matrix $\hat{R}_{2^{k+1}}$ for $k \geq 2$ can be expressed as

i) $2^{\frac{k+1}{2}}$, 2^{k-1} times

ii) $-2^{\frac{k+1}{2}}$, 2^{k-1} times

iii) $\pm 2^{\frac{k+1}{2}} e^{\pm j\frac{\pi}{4}}$ or $\pm 2^{\frac{k+1}{2}} e^{j\frac{\pi}{4}}$, 2^k times.

Proof: We first prove the cases i) and ii). It is clear that 2^{k-1} rows/columns of the matrix $\hat{P}_{2^{k+1}}$ have negative signs compared with the identity matrix. Let V' be the matrix swapped the fourth 2^{k-1} rows and the last (eighth) 2^{k-1} of V. Then we can modify the matrix V' into V' in the form of (2.12) by elementary column operation. By swapping the fourth 2^{k-1} rows and the last 2^{k-1} of V' and applying the similar method used in the proof of the previous theorem, we can easily prove the cases i) and ii). Similarly to Theorem II.5, we can derive

$$H_{2^{k+1}}^8 = \hat{R}_{2^{k+1}}^8 = (2^{k+1})^4 I_{2^{k+1}},\ k \geq 2. \quad (2.20)$$

This implies that the eigenvalues of the matrices $\hat{R}_{2^{k+1}}$ for $k \geq 2$ are complex 8-th roots on the circle, whose radius is the absolute value of the eigenvalues of the Sylvester Hadamard matrix. △

We can also construct various non- Sylvester-Hadamard matrices using different monomial permutation matrices. For example, we can change the rows of a quarter or half of the Sylvester-Hadamard matrices.

The eigenvalues will be created depending on which rows of the Hadamard matrices are changed. In case the sign with a quarter of rows is changed, the same result as that case in Theorem II.4 is observed.

As the case when the sign with a half of rows is changed, let us define a new matrix as

$$G_{2^{k+1}} = \begin{bmatrix} H_{2^k} & H_{2^k} \\ -H_{2^k} & H_{2^k} \end{bmatrix} \quad \text{for } k \geq 0$$

where H_{2^k} is a Hadamard matrix of order 2^k and $H_1 = 1$. From (2.21), it follows immediately that

$$G_{2^{k+1}} = G_2 \otimes H_{2^k}.$$

These matrices $G_{2^{k+1}}$ are also equivalent to the Sylvester-Hadamard matrices.

Next, let us investigate the eigenvalues and eigenvectors of the matrix of $G_{2^{k+1}}$.

Theorem II.8: Let X_n be an eigenvector of the matrix H_n associated with eigenvalue $\tilde{\lambda}_n$.

Further, let λ_{n+1} be an eigenvalue of the matrix G_{n+1}. Then,

$$\lambda_{n+1} = \tilde{\lambda}_n\,(1+j) \text{ or } \tilde{\lambda}_n\,(1-j)$$

and the corresponding eigenvector is

$$\begin{bmatrix} j\,X_n \\ X_n \end{bmatrix} \text{ or } \begin{bmatrix} X_n \\ j\,X_n \end{bmatrix},$$

respectively.

Proof: Let $X_{n+1} = \begin{bmatrix} aX_n \\ bX_n \end{bmatrix},\ a, b \in C.$

$$\begin{aligned} G_{n+1}X_{n+1} &= \begin{bmatrix} H_n & H_n \\ -H_n & H_n \end{bmatrix} \begin{bmatrix} aX_n \\ bX_n \end{bmatrix} \\ &= \begin{bmatrix} aH_nX_n + bH_nX_n \\ -aH_nX_n + bH_nX_n \end{bmatrix} \\ &= \begin{bmatrix} (a+b)H_nX_n \\ (-a+b)H_nX_n \end{bmatrix} \\ &= \tilde{\lambda}_n \begin{bmatrix} (a+b)X_n \\ (-a+b)X_n \end{bmatrix}. \end{aligned}$$

In order for X_{n+1} to be an eigenvector of G_{n+1}, we have

$$\tilde{\lambda}_n \begin{bmatrix} (a+b)X_n \\ (-a+b)X_n \end{bmatrix} = \lambda_{n+1} \begin{bmatrix} aX_n \\ bX_n \end{bmatrix}.$$

From (2.27), we can infer that $\lambda_{n+1} = \tilde{\lambda}_n(1 \pm j)$ for $a=1,\ b=j$ or $a=j,\ b=1$, respectively. △

Let $\tilde{J}_n = \begin{bmatrix} 0 & I_n \\ I_n & 0 \end{bmatrix}$ be a Hermitian matrix. Then, it can be easily confirmed that

$$\tilde{J}_n \cdot G^{*}_{n+1} \cdot \tilde{J}_n = G_{n+1}.$$

Thus, the matrix G_{n+1} is a J-selfadjoint matrix, and according to the proposition I.4, the spectrum $\sigma(G_{n+1})$ of a J-selfadjoint matrix G_{n+1} is symmetric relative to the real axis, i.e.

$$\lambda \in \sigma(G_{n+1}) \text{ implies } \overline{\lambda} \in \sigma(G_{n+1}).$$

Ⅲ. Conclusions

We have presented various non-Sylvester Hadamard matrices and provided their eigenvalues. The equivalent Hadamard matrix constructed by the multiplication of the Hadamard matrix and a certain monomial permutation matrix were found to show the interesting similarity with the Hadamard matrix. For example, half of the eigenvalues of one of these equivalent Hadamard matrices were found to be the same as that of the Sylvester Hadamard matrix. We have shown that the eigenvaules of this matrix were determined by the monomial permutaion matrix. We found that the eigenvalues are determined depending on which rows of the Hadamard matrices are changed.

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