A New SLM OFDM Scheme With Low Complexity for PAPR Reduction

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***Abstract*—In this letter, the authors introduce a new selected mapping (SLM) orthogonal frequency division multiplexing (OFDM) scheme with low computational complexity. The proposed SLM scheme transforms an input symbol sequence into a set of OFDM signals by multiplying the phase sequences to the signal after a certain intermediate stage of inverse fast Fourier transform (IFFT). Then, the OFDM signal with the lowest peak-to-average power ratio (PAPR) is selected for transmission. The new SLM OFDM scheme reduces the computational complexity, while it shows almost the same performance of PAPR reduction as that of the conventional SLM OFDM scheme.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), selected mapping (SLM).**

I. Introduction

An ORTHOGONAL frequency division multiplexing (OFDM) system has been proposed as a standard for the next-generation mobile radio communication system. OFDM signals have efficient spectral bandwidth, and the performance of the OFDM system over frequency-selective fading channels is better than that of the single carrier modulation system. One of the major drawbacks of OFDM system is that the OFDM signal can have high peak to average power ratio (PAPR). The high PAPR brings on signal distortion in the nonlinear region of high power amplifier (HPA), and the signal distortion induces the degradation of bit error rate (BER). Various techniques [1]–[6] have been proposed for reducing the PAPR. The simple and widely used method is clipping the signal to limit the PAPR below a threshold level, but it causes both in-band distortion and out-of-band radiation. Block coding [2], the encoding of an input data into a code word with low PAPR, is another well-known technique to reduce PAPR, but it incurs the rate decrease.

In [5], selected mapping (SLM) and partial transmit sequence (PTS) were proposed to lower the PAPR, with relatively small increase in redundancy but without any signal distortion. In SLM, an input symbol sequence is multiplied by each of the predetermined sequences, called phase sequences, to yield alternative input symbol sequences. Each of these alternative input symbol sequences is inverse fast Fourier transformed (IFFTed), and then, the one with the lowest PAPR is selected for transmission. In PTS, the input symbol sequence is partitioned into a number of disjointed subblocks. IFFT is applied to each subblock, and the signals of subblocks are summed after they are multiplied by distinct rotating factors. Then, the PAPR is computed for each set of rotating factors and compared. It is known that SLM is more advantageous than PTS if the amount of redundancy is limited, but the computational complexity of SLM is larger than that of PTS. It is mainly the increase in the computational complexity that restricts the implementation of SLM for the OFDM system with large carriers. The computational complexity of the conventional SLM is increased in proportion to the number of phase sequences, simply because the alternative input symbol sequences that should be individually IFFTed for PAPR comparison are generated at the initial stage of IFFT. Now, at this point, a simple but natural question can be raised: "What if we generate the alternative input symbol sequences at some intermediate stage of IFFT, not the initial stage, so that the complexity increase is mitigated?" Then, on the contrary, the immediate next question emerges: "Isn't it true that the PAPR reduction effect is being reduced as this intermediate stage approached to the last stage?" This letter stems from these two questions.

This letter is organized as follows: In Section II, the definition of PAPR and the conventional SLM OFDM scheme are described. Section III introduces a new SLM OFDM scheme and discusses the computational complexity issue. The simulation results are shown in Section IV, and finally, the concluding remarks are given in Section V.

II. Conventional SLM OFDM Scheme

In the discrete time domain, an OFDM signal a_t of N carriers can be expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi \frac{n}{N} t}, \quad 0 \le t \le N-1$$

where $\mathbf{A} = [A_0 A_1 \dots A_{N-1}]$ is an input symbol sequence and t stands for a discrete time index.

The PAPR of an OFDM signal, defined as the ratio of the maximum to the average power of the signal, can be expressed as

$$\text{PAPR}(\mathbf{a}) \triangleq \frac{\underset{0\le t\le N-1}{\text{Max}} |a_t|^2}{\text{E}[|a_t|^2]}$$

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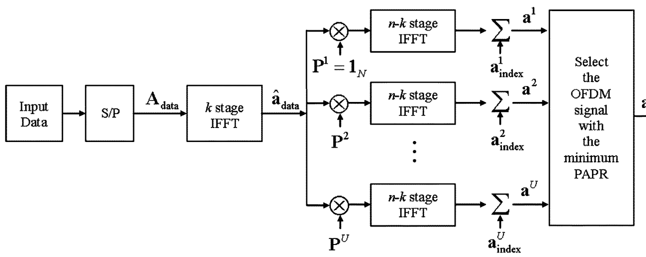

Fig. 1. Block diagram of the new SLM OFDM scheme.

where $\mathrm{E}[\cdot]$ denotes the expected value, and $\mathbf{a} = [a_0 a_1 \ldots a_{N-1}]$. In the conventional SLM OFDM scheme [5], alternative symbol sequences $\mathbf{A}^u = [A_0^u A_1^u \ldots A_{N-1}^u], 1 \leq u \leq U$ are generated by multiplying the phase sequences $\mathbf{P}^u = [P_0^u P_1^u \ldots P_{N-1}^u], 1 \leq u \leq U$ to the input symbol sequence $\mathbf{A} = [A_0 A_1 \ldots A_{N-1}]$. We use the expression $\mathbf{A}^u = \mathbf{A} \otimes \mathbf{P}^u$ to represent the component-wise multiplication, i. e., $A_n^u = A_n P_n^u, 0 \leq n \leq N-1$. Each symbol of the phase sequences is set to have unit magnitude to preserve the power, and the first phase sequence $\mathbf{P}^1$ is usually all one sequence $\mathbf{1}_N$. For the ease of implementation, P_n^u is usually selected from $\{\pm 1\}$. The OFDM signal $\mathbf{a}^{\tilde{u}} = \mathrm{IFFT}\{\mathbf{A}^{\tilde{u}}\}$ with the lowest PAPR is selected for transmission, where $\tilde{u}$ is expressed as

$$\tilde{u} = \arg \min_{1 \leq u \leq U} (\mathrm{PAPR}(\mathbf{a}^u)).$$

III. New SLM OFDM Scheme

A. New SLM OFDM Scheme

Unlike the conventional SLM scheme, where different phase sequences are multiplied to an input symbol sequence, in the proposed scheme, they are multiplied to the "so-called" intermediate signal, which is the partially IFFTed input symbol sequence. Fig. 1 shows the block diagram of the new SLM OFDM scheme. In this scheme, the $N(=2^n)$ point IFFT based on decimation-in-time algorithm is partitioned into two parts. The first part is the first k stages of IFFT, and the second part is the remaining $n-k$ stages. A set of OFDM signals is generated by multiplying different phase sequences $\mathbf{P}^u, 1 \leq u \leq U$ to the intermediate signal $\hat{\mathbf{a}}_{\mathrm{data}}$ after the kth stage of IFFT. Compared to the conventional SLM scheme, the computational load of the new scheme is much relieved since the intermediate signal $\hat{\mathbf{a}}_{\mathrm{data}}$ is used in common.

The information on the phase sequence used for the transmitted signal must be conveyed to the receiver in the SLM scheme. In the conventional SLM scheme, this information, represented as an index symbol sequence $\mathbf{A}_{\mathrm{index}}$, is added to the data symbol sequence $\mathbf{A}_{\mathrm{data}}$ to form the input symbol sequence $\mathbf{A}$, i.e., $\mathbf{A} = \mathbf{A}_{\mathrm{data}} + \mathbf{A}_{\mathrm{index}}$.

However, in the scheme of this letter, this summing operation (in fact, it is equivalent to augmentation) is not done at the symbol sequence stage but rather at the final stage after the IFFT operations, as shown in Fig. 1. Usually, the index information is encoded for error detection and correction due to its importance. For example, in M-QAM signalling, when the encoder code rate is R and the number of phase sequences is U, the number of index symbols to transmit is $\lceil \log_M U/R \rceil$, where $\lceil x \rceil$ denotes the smallest integer exceeding or equal to x. Thus, $\lceil \log_M U/R \rceil$ elements of $\mathbf{A}_{\mathrm{data}}$ are set to zero to reserve the index information, and $N - \lceil \log_M U/R \rceil$ elements of $\mathbf{A}_{\mathrm{index}}$ are set to zero. The index signals $\mathbf{a}_{\mathrm{index}}^u = \mathrm{IFFT}\{\mathbf{A}_{\mathrm{index}}^u\}, 1 \leq u \leq U$ are stored in the memory. Then, the index signal $\mathbf{a}_{\mathrm{index}}^u$ is added after IFFT of $\mathbf{A}_{\mathrm{index}}^u$. The new SLM OFDM signal $\mathbf{a}^u$ can be written as

$$\begin{aligned}\mathbf{a}^u &= \mathrm{IFFT}_{k+1}^n\{\mathbf{P}^u \otimes \mathrm{IFFT}_1^k(\mathbf{A}_{\mathrm{data}})\} + \mathrm{IFFT}_1^n\{\mathbf{A}_{\mathrm{index}}^u\} \\ &= \mathrm{IFFT}_{k+1}^n\{\mathbf{P}^u \otimes \hat{\mathbf{a}}_{\mathrm{data}}\} + \mathbf{a}_{\mathrm{index}}^u\end{aligned}$$

where IFFT_i^j indicates IFFT from i stage to j stage, and the size of IFFT is $N = 2^n$.

B. Phase Sequences of New SLM OFDM Scheme

The amount of computational complexity reduction of the new SLM OFDM scheme is increasing as the intermediate stage k approaches the last stage n. As in the conventional SLM OFDM scheme, we put the restriction on the phase sequences to be $\{\pm 1\}$ sequences. Intuitively, and by the numerical analysis, we may say that the performance of PAPR reduction degrades as k approaches n. This tradeoff between the computational complexity and the performance of PAPR reduction should be considered in finding the optimal stage k.

For $N = 2^n$, an N point IFFT can be computed from two $N/2$ point transforms, the $N/2$ point IFFT from two $N/4$ point transforms, and so on. According to the successive doubling method, the transform at the kth stage consists of 2^{n-k} blocks, where each block corresponds to 2^k point IFFT. Thus, if we define $\mathbf{T}_i$ to be $N \times N$ symmetric matrix representing ith stage of IFFT, then the N point IFFTed signal $\mathbf{a}$ can be expressed as $\mathbf{a}^T = \mathbf{T}_n \mathbf{T}_{n-1} \ldots \mathbf{T}_2 \mathbf{T}_1 \mathbf{A}^T$. Multiplying a phase sequence $\mathbf{Q}$ to the input symbol sequence $\mathbf{A}$ corresponds to multiplying an $N \times N$ diagonal matrix $\tilde{\mathbf{Q}}$ whose diagonal entries form the phase sequence $\mathbf{Q}$. Therefore, the output signal $\mathbf{a}_{\mathbf{Q}}$ corresponding to the phase sequence $\mathbf{Q}$ in the conventional SLM scheme can be expressed as

$$\mathbf{a}_{\mathbf{Q}}^T = \mathbf{T}_n \mathbf{T}_{n-1} \ldots \mathbf{T}_2 \mathbf{T}_1 \tilde{\mathbf{Q}} \mathbf{A}^T \tag{1}$$

whereas the output signal $\mathbf{a}'_{\mathbf{P}}$ corresponding to the phase sequence $\mathbf{P}$ in the proposed scheme can be expressed as

$$\mathbf{a}_{\mathbf{P}}^{\prime T} = \mathbf{T}_n \ldots \mathbf{T}_{k+1} \tilde{\mathbf{P}} \mathbf{T}_k \ldots \mathbf{T}_1 \mathbf{A}^T. \tag{2}$$

In general, the OFDM signal in the scheme given in (2) can not be expressed in the form given in (1). If so, the phase sequence matrix $\tilde{\mathbf{Q}}$ must be

$$\tilde{\mathbf{Q}} = (\mathbf{T}_k \ldots \mathbf{T}_1)^{-1} \tilde{\mathbf{P}} (\mathbf{T}_k \ldots \mathbf{T}_1) \tag{3}$$

but in general, given a $\{\pm 1\}$ diagonal matrix $\tilde{\mathbf{P}}$, $\tilde{\mathbf{Q}}$ in (3) is not guaranteed to have $\{\pm 1\}$ entries and not even guaranteed to be a diagonal matrix. If we view the matrix $\mathbf{T}_i$ as a $2^{n-i} \times 2^{n-i}$ block matrix, where each block is a $2^i \times 2^i$ submatrix, then it is not difficult to see that $\mathbf{T}_i$ is a block identity matrix, which tells us that the matrix $(\mathbf{T}_k \ldots \mathbf{T}_1)$ can be considered as a $2^{n-k} \times 2^{n-k}$ block identity matrix. Thus, if we make $\tilde{\mathbf{P}}$ as a $2^{n-k} \times 2^{n-k}$ diagonal block matrix, i. e., each $2^k \times 2^k$ subblock of $\tilde{\mathbf{P}}$ is either $\pm \mathbf{I}_{2^k}$, then from (3), we have $\tilde{\mathbf{Q}} = \tilde{\mathbf{P}}$ and $\mathbf{Q} = \mathbf{P}$.

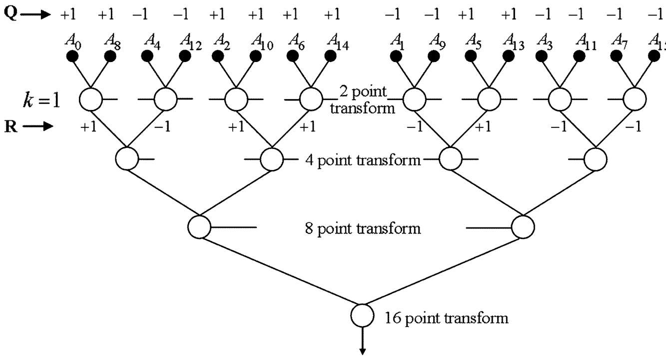

Fig. 2. Example of the phase sequence generation and the successive doubling method when $N = 16$ and $k = 1$ are used.

In other words, if we choose a phase sequence of length 2^n such that each of 2^{n-k} subsequences of length 2^k is either all 1 or all -1 sequence, then multiplying the phase sequence at the intermediate stage k is equivalent to doing so at the initial stage, as in the conventional SLM scheme. This somewhat resolves the issue of performance degradation due to the multiplication at the intermediate stage. Let $\mathbf{R} = [R_0 R_1 \ldots R_{2^{n-k}-1}]$ be a sequence of length 2^{n-k}, called the constituent phase sequence. Then the phase sequence $\mathbf{P}$ of length 2^n is given as follows:

$$P_{l \cdot 2^k} = P_{l \cdot 2^k + 1} = \cdots = P_{l \cdot 2^k + 2^k - 1} = R_l \qquad 0 \le l \le 2^{n-k} - 1.$$

The remaining task is the selection of $\mathbf{R}$, which is the constituent phase sequence. In the new SLM OFDM scheme, we choose $\mathbf{R}$ as the rows of a cyclic Hadamard matrix $\mathbf{H}_{2^{n-k}}$ constructed from an m-sequence. There are two reasons why we use the rows of a cyclic Hadamard matrix. The first reason is because they themselves are good candidates for phase sequences in the conventional SLM scheme [4], and the second is because the numerical analysis shows that there is only a negligible performance degradation when we extend the rows of a cyclic Hadamard matrix $\mathbf{H}_{2^{n-k}}$ by repeating 2^k times and use them as phase sequences, compared to using rows of a cyclic Hadamard matrix $\mathbf{H}_{2^n}$.

Fig. 2 shows an example for $n = 4, k = 1$ and $\mathbf{R} = [+1 -1 +1 +1 -1 +1 -1 -1]$. By repeating each element of $\mathbf{R}$, two times, $\mathbf{P} = \mathbf{Q} = [+1 +1 -1 -1 +1 +1 +1 +1 -1 -1 +1 +1 -1 -1 -1 -1]$. For $N = 16$, the input symbol is bit-reversed before the IFFT as the following order $\{0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15\}$. Multiplying the phase sequence to the output signal of the first stage is identical to multiplying the phase sequence $\mathbf{S} = [+1 -1 +1 -1 -1 +1 +1 +1 -1 +1 -1 +1 -1 -1 +1 +1 -1]$ to the unordered data symbol sequence $\mathbf{A}_{\text{data}}$. Consequently, the receiver can recover the data symbol sequence $\mathbf{A}_{\text{data}}$ by multiplying the phase sequence $\mathbf{S}$ to the alternative symbol sequence $\mathbf{A}_{\text{data}} \otimes \mathbf{S}$, which is obtained by Fourier-transforming the received signal. This shows that even though we modified IFFT in the transmitter for the computation reduction, the FFT part in the receiver need not be changed.

TABLE I
COMPUTATIONAL COMPLEXITY REDUCTION RATIO OVER THE CONVENTIONAL SLM OFDM SCHEME

$n-k$	CCRR(%)											
	$N = 256 (n = 8)$			$N = 1024 (n = 10)$			$N = 2048 (n = 11)$			$N = 8192 (n = 13)$		
	$U = 4$	$U = 8$	U=16	$U = 4$	$U = 8$	U=16	$U = 4$	$U = 8$	U=16	$U = 4$	$U = 8$	U=16
3	47	55	59	53	61	66	55	64	68	58	67	72
4	38	44	47	45	53	56	48	56	60	52	61	65
5	28	33	35	38	44	47	41	48	51	46	54	58
6	19	22	23	30	35	38	34	40	43	40	47	50

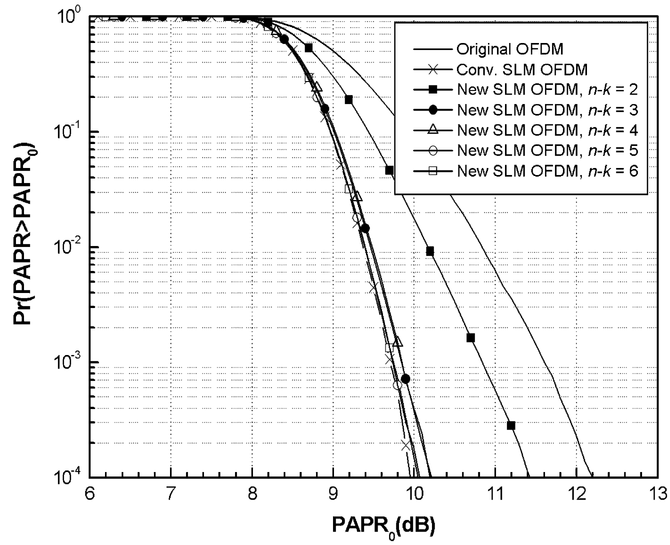

Fig. 3. CCDF of the PAPR of new and conventional SLM OFDM scheme for some stages of multiplication with phase sequences when $N = 2048, U = 4$, and 16-QAM constellation are used.

C. Computational Complexity

When the number of carriers is $N = 2^n$, the numbers of complex multiplication n_{mul} and complex addition n_{add} of the conventional SLM OFDM scheme are given by $n_{\text{mul}} = 2^{n-1} nU$ and $n_{\text{add}} = 2^n nU$, where U is the total number of phase sequences.

If the phase sequences are multiplied after the kth stage of IFFT, the numbers of complex computations of the new SLM OFDM scheme are given by $n_{\text{mul}} = 2^{n-1} n + 2^{n-1}(n-k)(U-1)$ and $n_{\text{add}} = 2^n n + 2^n (n-k)(U-1)$.

The computational complexity reduction ratio (CCRR) of the new SLM OFDM scheme over the conventional SLM OFDM scheme is defined as

$$\text{CCRR} = \left(1 - \frac{\text{Complexity of new SLM}}{\text{Complexity of conventional SLM}}\right) \times 100 = \left(1 - \frac{1}{U}\right) \frac{k}{n} \times 100.$$

Table I gives the CCRR of the new SLM OFDM scheme over the conventional SLM OFDM scheme with typical values of U, k, and n.

IV. SIMULATION RESULTS

Simulations are performed for the OFDM system of the *IEEE* standard 802.16 [7] for the mobile wireless metropolitan area network (WMAN). The OFDM system specified in *IEEE* 802.16 has 2048 carriers with QPSK, 16-QAM, and 64-QAM constellation. The number of used carriers is 1702. Of the

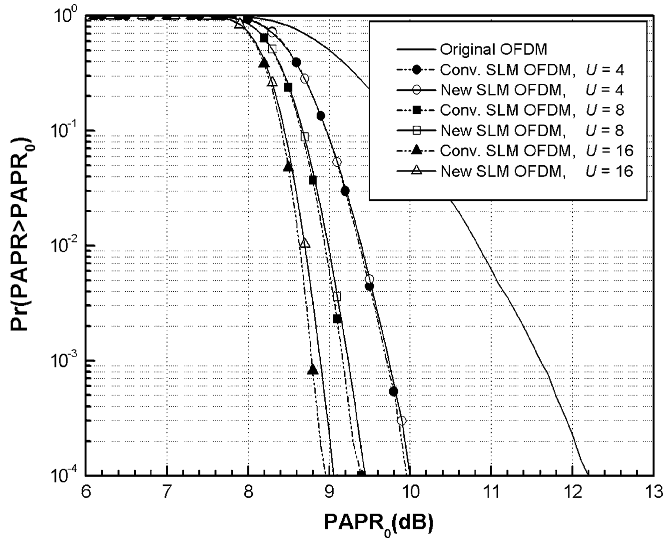

Fig. 4. Performance comparison of the conventional SLM OFDM scheme and the new SLM OFDM scheme when $N = 2048, n - k = 5$, and 16-QAM constellation are used.

remaining 346 carriers, 345 carriers are set to zero to shape the power spectral density of the transmit signal, and one carrier is used for DC. The 100 000 input symbol sequences are generated randomly with uniform distribution. Figs. 3 and 4 illustrate the probability that the PAPR of the OFDM signal exceeds the given threshold. Fig. 3 shows the simulation results, as the stage of multiplication with phase sequences is varied for $n - k = 2, 3, 4, 5, 6$. The new SLM OFDM scheme with 2048 carriers has almost the same performance compared to the conventional SLM OFDM scheme when $n - k$ is 5. By the simulation results, we can say that the optimal value for $n - k$ does not depend on the number of carriers, and the optimal value is around 5 when the number of carriers is between 256 and 8192. Fig. 4 shows a comparison of the performance between the conventional SLM OFDM scheme and the new SLM OFDM scheme with $n - k = 5$ and 16-QAM constellation. As one can see, the new SLM OFDM has almost the same performance of PAPR reduction as that of the conventional SLM OFDM scheme. In the case of $n - k = 5$, the new SLM OFDM system reduces the computational complexity by 41% $\sim 51\%$ as the number of sequences U increases from 4 to 16.

V. Conclusion

A new SLM OFDM scheme with low computational complexity has been proposed, and its performance is analyzed in reference to the standard of *IEEE* 802.16 for mobile WMAN. The simulation results show the new SLM OFDM scheme with 2048 carriers reduces the computational complexity by 51% for $n - k = 5$, while it has almost the same performance of PAPR reduction as that of the conventional SLM OFDM scheme. The computational complexity reduction ratio increases as the number of carriers increases, which makes the proposed scheme more suitable for the high-data-rate OFDM systems.

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