A New PTS OFDM Scheme with Low Complexity for PAPR Reduction

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***Abstract*—In this paper, we introduce a new partial transmit sequence (PTS) orthogonal frequency division multiplexing (OFDM) scheme with low computational complexity. In the proposed scheme, 2^n-point inverse fast Fourier transform (IFFT) is divided into two parts. An input symbol sequence is partially transformed using the first l stages of IFFT into an intermediate signal sequence and the intermediate signal sequence is partitioned into a number of intermediate signal subsequences. Then, the remaining $n-l$ stages of IFFT are applied to each of the intermediate signal subsequences and the resulting signal subsequences are summed after being multiplied by each member of a set of W rotating vectors to yield W distinct OFDM signal sequences. The one with the lowest peak to average power ratio (PAPR) among these OFDM signal sequences is selected for transmission. The new PTS OFDM scheme reduces the computational complexity while it shows almost the same performance of PAPR reduction as that of the conventional PTS OFDM scheme.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), partial transmit sequence (PTS), peak to average power ratio (PAPR).**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) system has been considered as one of the strong standard candidates for the next generation mobile radio communication systems. Multiplexing a serial data symbol stream into a large number of orthogonal subchannel makes the OFDM signals spectral bandwidth efficient. It has been shown that the performance of OFDM system over frequency selective fading channels is better than that of the single carrier modulation system. One of the major drawbacks of OFDM system is that the OFDM signal can have high peak to average power ratio (PAPR). The high PAPR brings on the OFDM signal distortion in the nonlinear region of high power amplifier (HPA) and the signal distortion induces the degradation of bit error rate (BER).

Recently many works [1]–[3], [5], [7]–[22] have been done in developing a method to reduce the PAPR. The simple and widely used method is clipping the signal to limit the PAPR below a threshold level, but it causes both in-band distortion and out of band radiation. Block coding [2], the encoding of an input data into a codeword with low PAPR is another well-known technique to reduce PAPR, but it incurs the rate decrease.

The μ-law companding technique based on speech processing [20] has better BER performance than the clipping method. In [21], Jiang proposed a new nonlinear companding transform scheme which effectively reduces PAPR by transforming the statistics of the amplitude of the OFDM signals into the quasiuniform distribution.

Selected mapping (SLM) and partial transmit sequence (PTS) [1], [5], [8], [18], [19] were proposed to lower the PAPR with a relatively small increase in redundancy but without any signal distortion. In the SLM scheme, an input symbol sequence is multiplied by each of the phase sequences to generate alternative input symbol sequences. Each of these alternative input symbol sequences is inverse fast Fourier transformed (IFFT-ed) and then the one with the lowest PAPR is selected for transmission. In the PTS scheme, the input symbol sequence is partitioned into a number of disjoint symbol subsequences. IFFT is then applied to each symbol subsequence and the resulting signal subsequences are summed after being multiplied by a set of distinct rotating vectors. Next the PAPR is computed for each resulting sequence and then the signal sequence with the minimum PAPR is transmitted. It is known that the PTS scheme is more advantageous than the SLM scheme if the amount of computational complexity is limited, but the redundancy of the PTS scheme is larger than that of the SLM scheme. As the number of subcarriers and the order of modulation are increased, reducing the computational complexity becomes more important than decreasing redundancy.

This paper is organized as follows: In Section II, OFDM system and PTS scheme are described. Section III introduces a new PTS OFDM scheme and discusses the computational complexity issue. The simulation results are shown in Section IV, and finally, the concluding remarks are given in Section V.

II. OFDM SYSTEM AND PTS SCHEME

A. OFDM System

The OFDM signal sequence $\mathbf{a} = [a_0 a_1 \cdots a_{N-1}]^T$ using $N = 2^n$ subcarriers is expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j2\pi \frac{k}{N} t}, \quad 0 \le t \le N-1 \tag{1}$$

where $\mathbf{A} = [A_0 A_1 \cdots A_{N-1}]^T$ is an input symbol sequence and t stands for a discrete time index. If we define $\mathbf{Q}_i^j = \mathbf{T}_j \mathbf{T}_{j-1} \cdots \mathbf{T}_{i+1} \mathbf{T}_i$, where $\mathbf{T}_i$ denotes the $N \times N$ symmetric matrix representing the i-th stage of IFFT, (1) can be written as

$$\mathbf{a} = \mathbf{Q}_1^n \mathbf{A}.$$

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The PAPR of the OFDM signal sequence, defined as the ratio of the maximum divided by the average power of the signal, is expressed as

$$\mathrm{PAPR}(\mathbf{a}) \triangleq \frac{\underset{0\le t\le N-1}{\mathrm{Max}} |a_t|^2}{\mathrm{E}\left[|a_t|^2\right]}$$

where $\mathrm{E}[\cdot]$ denotes the expected value [15]. An alternative measure of the envelope variation of the OFDM signal is the crest factor ζ, which is defined as the ratio of the maximum to the root mean square of the signal envelope as follows [8]:

$$\zeta(\mathbf{a}) \triangleq \frac{\underset{0\le t\le N-1}{\mathrm{Max}} |a_t|}{\sqrt{\mathrm{E}\left[|a_t|^2\right]}}.$$

B. PTS Scheme

In PTS scheme, an input symbol sequence $\mathbf{A}$ is partitioned into V 'disjoint' symbol subsequences $\mathbf{A}_v = [A_{v,0}A_{v,1}\cdots A_{v,N-1}]^T$, $0 \le v \le V-1$ as follows:

$$\mathbf{A} = \sum_{v=0}^{V-1} \mathbf{A}_v.$$

Here, the word 'disjoint' implies that for each given k, $0 \le k \le N-1$, $A_{v,k} = 0$ except for at most a single v. In other words, the support sets of $\mathbf{A}_v$s are disjoint. The signal subsequence $\mathbf{a}_v = [a_{v,0}a_{v,1}\cdots a_{v,N-1}]^T$ is generated by applying inverse fast Fourier transform (IFFT) to each symbol subsequence $\mathbf{A}_v$, often called a subblock. Each signal subsequence $\mathbf{a}_v$ is then multiplied by an unit magnitude constant r_v^w chosen from a given alphabet $\mathbb{Z}$, which is usually $\mathbb{Z} = \{\pm 1\}$ or $\mathbb{Z} = \{\pm 1, \pm j\}$, and summed to result in a PTS OFDM signal sequence $\mathbf{a}^w = [a_0^w a_1^w \cdots a_{N-1}^w]^T$, which can be expressed as

$$\mathbf{a}^w = \sum_{v=0}^{V-1} r_v^w \mathbf{a}_v$$

where the vector $\mathbf{r}^w = [r_0^w r_1^w \cdots r_{V-1}^w]$, $0 \le w \le W-1$, $W = |\mathbb{Z}|^{V-1}$ is called a rotating vector. The PAPR of $\mathbf{a}^w$ is computed for each of W rotating vectors and compared. The one with the minimum PAPR is chosen for transmission. The index $\tilde{w}$ of the corresponding rotating vector $\mathbf{r}^{\tilde{w}}$ is expressed as

$$\tilde{w} = \underset{0\le w\le W-1}{\mathrm{argmin}} \ \underset{0\le t\le N-1}{\mathrm{Max}} \left| \sum_{v=0}^{V-1} r_v^w a_{v,t} \right|.$$

The subblock partitioning sequence is defined as a sequence $\mathbf{S} = [S_0 S_1 \cdots S_{N-1}]^T$, $S_k \in \{0, 1, \cdots, V-1\}$ such that $S_k = v$ if $A_{v,k} = A_k$. In other words, $\mathbf{S}$ is used to allocate the k-th component A_k of an input symbol sequence $\mathbf{A}$ to the v-th symbol subsequence $\mathbf{A}_v$ if $S_k = v$. Let the v-th subblock index sequence $\mathbf{B}_v = [B_{v,0}B_{v,1}\cdots B_{v,N-1}]^T$, $0 \le v \le V-1$ be generated as follows:

$$B_{v,k} = \begin{cases} 1, & S_k = v \\ 0, & S_k \ne v. \end{cases}$$

Then the v-th symbol subsequence $\mathbf{A}_v$ is expressed as

$$\mathbf{A}_v = \tilde{\mathbf{B}}_v \mathbf{A}$$

where $\tilde{\mathbf{B}}_v$ is an $N \times N$ diagonal matrix whose diagonal entries form the subblock index sequence $\mathbf{B}_v$. Then, the output signal sequence $\mathbf{a}^w$ is written as

$$\mathbf{a}^w = \sum_{v=0}^{V-1} r_v^w \mathbf{Q}_1^n \tilde{\mathbf{B}}_v \mathbf{A}.$$

The known subblock partitioning methods can be classified into three categories. The first and simplest category is called an adjacent method which allocates N/V successive symbols to the same subblock. The second category is based on interleaving. In this method, the symbols with distance V are allocated to the same subblock. The last one is called a random partitioning method in which the input symbol sequence is partitioned randomly.

For example, let us partition an input symbol sequence $\mathbf{A}$ of length 16 into 4 symbol subsequences. Then, $\mathbf{S} = [0000111122223333]^T$ is used as a subblock partitioning sequence for the adjacent method, $\mathbf{S} = [0123012301230123]^T$ for the interleaved method, and $\mathbf{S} = [1022311301203203]^T$ for the random method.

The PAPR reduction performance and the computational complexity of PTS scheme depend on the method of subblock partitioning. In other words, there is a trade-off between PAPR reduction performance and computational complexity in PTS scheme. The random partitioning is known to have the best performance in PAPR reduction. The interleaving method [5] can reduce the computational complexity of PTS scheme using Cooley-Tukey FFT algorithm, but the PAPR reduction performance is the worst.

III. New PTS OFDM Scheme

A. A New PTS OFDM Scheme

Unlike the conventional PTS scheme where input symbol sequences are partitioned at the initial stage, in the proposed scheme, the partition takes place after the first l stages of IFFT. Fig. 1 shows the block diagram of the new PTS OFDM scheme. In this scheme, the 2^n-point IFFT based on decimation-in-time algorithm is divided into two parts. The first part is the first l stages of IFFT and the second part is the remaining $n-l$ stages. In the first l stages of IFFT, the input symbol sequence $\mathbf{A}_{\mathrm{data}}$ is partially IFFT-ed to form an intermediate signal sequence $\hat{\mathbf{a}}_{\mathrm{data}}$. This intermediate signal sequence is partitioned into V intermediate signal subsequences and then the remaining $n-l$ stages of IFFT are applied to each of the intermediate signal subsequences.

Compared to the conventional PTS scheme, the computational complexity of the new scheme is much relieved since the intermediate signal sequence $\hat{\mathbf{a}}_{\mathrm{data}} = \mathbf{Q}_1^l \mathbf{A}_{\mathrm{data}}$ is used in common for IFFT of V symbol subsequence.

The index of the rotating vector used for the transmitted signal sequence must be conveyed to the receiver in PTS scheme. In the conventional PTS scheme, this information, represented as

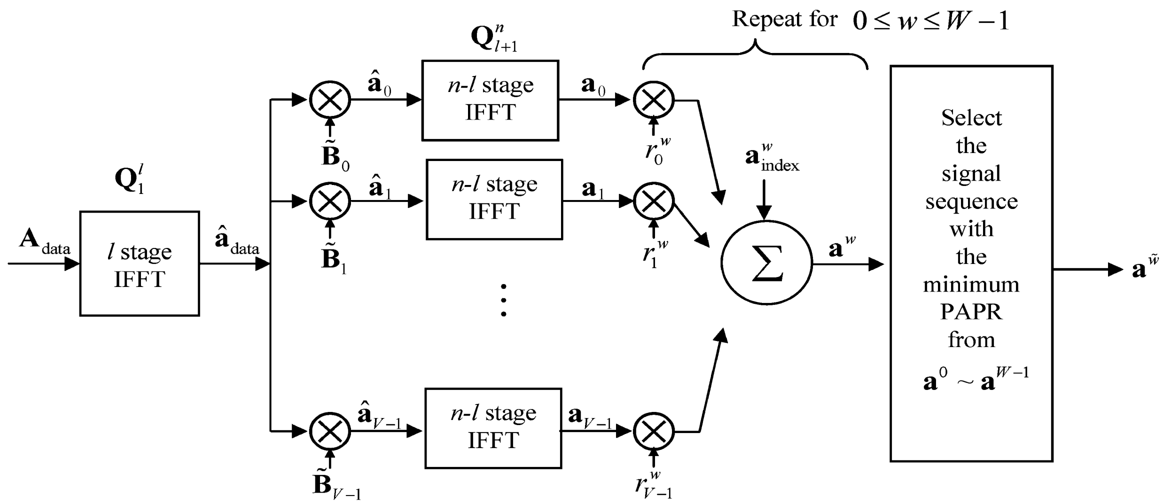

Fig. 1. Block diagram of the new PTS scheme.

an index sequence of rotating vectors $\mathbf{A}_{\text{index}}$ is added to a data symbol sequence $\mathbf{A}_{\text{data}}$ to form the input symbol sequence $\mathbf{A}$, i.e., $\mathbf{A} = \mathbf{A}_{\text{data}} + \mathbf{A}_{\text{index}}$. But in our scheme, this summing operation (in fact, it is equivalent to augmentation) is not done at the symbol sequence stage but at the final stage after the IFFT operations as shown in Fig. 1. Usually the index information is encoded for error detection and correction due to its importance. In M-QAM signalling, when encoder code rate is R and the number of rotating vectors is W, the number of index symbols to transmit is $\lceil \log_M(W/R) \rceil$, where $\lceil x \rceil$ denotes the smallest integer exceeding or equal to x. Thus, $\lceil \log_M(W/R) \rceil$ elements of $\mathbf{A}_{\text{data}}$ are set to zero to reserve the index information and $N - \lceil \log_M(W/R) \rceil$ elements of $\mathbf{A}_{\text{index}}$ are set to zero. Since the index signal sequences $\mathbf{a}_{\text{index}}^w = \mathbf{Q}_1^n \mathbf{A}_{\text{index}}^w$, $0 \le w \le W-1$ are used repeatedly, they can be stored in the memory and added to the IFFT of $\mathbf{A}_{\text{data}}$. Thus, the new PTS OFDM signal sequence $\mathbf{a}^w$ can be written as

$$\begin{aligned} \mathbf{a}^w &= \sum_{v=0}^{V-1} r_v^w \mathbf{Q}_{l+1}^n \tilde{\mathbf{B}}_v \mathbf{Q}_1^l \mathbf{A}_{\text{data}} + \mathbf{Q}_1^n \mathbf{A}_{\text{index}}^w \\ &= \sum_{v=0}^{V-1} r_v^w \mathbf{Q}_{l+1}^n \tilde{\mathbf{B}}_v \hat{\mathbf{a}}_{\text{data}} + \mathbf{a}_{\text{index}}^w. \end{aligned} \tag{2}$$

Recall that $|\mathbb{Z}|$ denotes the alphabet size of rotating vectors in Section II. The PTS scheme with V symbol subsequences and $|\mathbb{Z}|^{V-1}$ rotating vectors can be considered as a special case of the SLM scheme with $|\mathbb{Z}|^{V-1}$ phase sequences. This is because of the fact that by letting $\mathbf{P}^w = \sum_{v=0}^{V-1} r_v^w \tilde{\mathbf{B}}_v$ (2) can be rewitten as

$$\mathbf{a}^w = \mathbf{Q}_{l+l}^n \mathbf{P}^w \hat{\mathbf{a}}_{\text{data}} + \mathbf{a}_{\text{index}}^w \tag{3}$$

and the diagonal entries of each $\mathbf{P}^w$ can be considered as a phase sequence at the l-th intermediate stage [23]. The computational complexity of (3) is higher than that of (2) since the computational complexity of (3) depends on $|\mathbb{Z}|^{V-1}$ while the computational complexity of (2) depends on V.

B. A Subblock Partitioning Sequence

In this subsection, we suggest a simple but very promising subblock partitioning sequence for the case when the number of subblock is a power of 2. Let $\mathbf{M} = [M_0 M_1 \cdots M_{N-2}]^T$ be a binary m-sequence of length $N-1$, with the characteristic phase, i.e., satisfying that $M_k = M_{2k}$ [6].

For $V = 2^u$ subblocks, we propose a subblock partitioning sequence $\mathbf{S} = [S_0 S_1 \cdots S_{N-1}]^T$ given by

$$S_k = \begin{cases} 0, & k = 0 \\ \sum_{j=0}^{u-1} 2^j M_{k-1+j}, & 1 \le k \le N-1 \end{cases} \tag{4}$$

where the subscript of M is computed modulo $N-1$. Certainly from the run property of an m-sequence, the frequency of each symbol v, $0 \le v \le V-1$ in $\mathbf{S}$ is exactly 2^{n-u}. For example, with $N = 8$ and $V = 2^2$, an m-sequence $\mathbf{M}$ of length 7 is given as $\mathbf{M} = [1001011]$. Then the subblock partitioning sequence $\mathbf{S}$ in (4) is

$$\mathbf{S} = [01021233].$$

Although not proven, this sequence is believed to have a good PAPR reduction performance due to the pseudo-random properties of an m-sequence. In fact, the numerical analysis shows that the sequence has the comparable performance as that offered by a random partitioning method.

C. Computational Complexity

When the number of subcarriers is $N = 2^n$, the numbers of complex multiplication n_{mul} and complex addition n_{add} of the conventional PTS OFDM scheme are given by $n_{mul} = 2^{n-1}nV$ and $n_{add} = 2^n nV$ where V is the number of subblocks.

When the intermediate signal sequence is partitioned after the l-th stage of IFFT, it is clear that the numbers of complex computations of the new PTS OFDM scheme are given by $n_{mul} = 2^{n-1}n + 2^{n-l}(n-l)(V-1)$ and $n_{add} = 2^n n + 2^n(n-l)(V-1)$.

TABLE I
COMPUTATIONAL COMPLEXITY REDUCTION RATIO

$n-l$	CCRR(%)								
	$N=256$ $(n=8)$			$N=2048$ $(n=11)$			$N=8192$ $(n=13)$		
	$V=2$	$V=4$	$V=8$	$V=2$	$V=4$	$V=8$	$V=2$	$V=4$	$V=8$
3	28.6	42.9	50.0	36.4	54.5	63.6	38.5	57.7	67.3
4	21.4	32.1	37.5	31.8	47.7	55.7	34.6	51.9	60.6
5	14.3	21.4	25.0	27.3	40.9	47.7	30.8	46.2	53.8
6	7.1	10.7	12.5	22.7	34.1	39.8	26.9	40.4	47.1

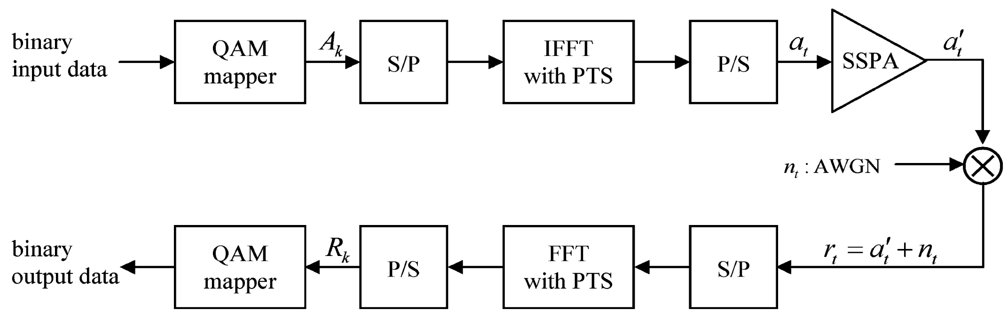

Fig. 2. A new PTS OFDM system model with SSPA and AWGN channel.

Thus, the computational complexity reduction ratio (CCRR) of the new PTS OFDM scheme over the conventional PTS OFDM scheme is defined as

$$\begin{aligned}\mathrm{CCRR} &= \left(1-\frac{\text{Complexity of new PTS}}{\text{Complexity of conventional PTS}}\right)\times 100\\ &= \left(1-\frac{1}{V}\right)\frac{l}{n}\times 100(\%).\end{aligned}$$

Table I gives CCRR of the new PTS OFDM scheme over the conventional PTS OFDM scheme with typical values of V, l, and n.

D. System Performance

Considering a system with a real RF transmitting amplifier, the nonlinear distortions introduced by HPA degrade system performance. One method to avoid the problem is the operation of the HPA in its linear region. The operating point of the amplifier is usually given by the output back off (OBO) of the HPA with

$$\mathrm{OBO} = 10\log\frac{P_{\max}}{P_{\mathrm{avr}}} \tag{5}$$

where $P_{\max}$ is the maximum output power (saturating power) of the HPA and P_{avr} is the mean output power. There is a trade-off between the efficiency and OBO such that the efficiency of the HPA is very small for large OBO.

One of the nonlinear HPA models is a solid state high power amplifier (SSPA) which has a more linear behavior in the small signal region than a traveling wave tube amplifier (TWTA). The AM/PM conversion of the SSPA is usually assumed to be small enough, so that it can be neglected. The AM/AM conversion function is $f(r)$ expressed as

$$f(r) = \frac{vr}{\left(1+\left(\frac{v}{A_0}r\right)^{2p}\right)^{\frac{1}{2p}}},\quad p>0, A_0>0, \text{ and } v>0 \tag{6}$$

where r is the magnitude of an OFDM signal a_t, A_0 is the limiting output amplitude, v is the small signal gain, and p determines the smoothness of the transition from the linear region to the limiting region. From (5) the OBO of SSPA model is given as

$$\mathrm{OBO}_{\mathrm{SSPA}} = \frac{A_0^2}{\int_0^\infty f^2(r)p(r)dr} \tag{7}$$

where $p(r)$ is the probability density function approximated as a Rayleigh distribution function for the original OFDM signal.

Fig. 2 shows the block diagram of the new PTS OFDM system model to evaluate the system performance. The input binary data are randomly generated and mapped into QAM symbols. Then, the symbols are IFFT-ed using the new PTS scheme. The OFDM signals are amplified with the nonlinear SSPA and transmitted into additive white Gaussian noise (AWGN) channel.

IV. SIMULATION RESULTS

Simulations are performed for the OFDM system of the IEEE standard 802.16 for mobile wireless metropolitan area network (WMAN). The OFDM system specified in IEEE 802.16 has 2048 subcarriers with QPSK, 16-QAM, and 64-QAM constellations. The number of used subcarriers is 1702. Among the remaining 346 subcarriers, 345 subcarriers are set to zero to shape the power spectrum of the transmit signal and one subcarrier is used for DC. The 100 000 input symbol sequences are generated randomly with uniform distribution. The OFDM signal is oversampled by a factor of four which is sufficient to represent the analog signal [14]. The symbols of the rotating factors are chosen from $\{\pm 1, \pm j\}$ for $V = 2, 4$ and from $\{\pm 1\}$ for $V = 8$. Figs. 3 and 4 illustrate the probability that the PAPR of the OFDM signal exceeds the given threshold. Fig. 3 shows the simulation results as the stage of block partition is varied for $n-l = 2, 3, 5$. The new PTS scheme with 2048 subcarriers has almost the same performance compared to the conventional one when $n-l$ is 5. From the simulation results, we

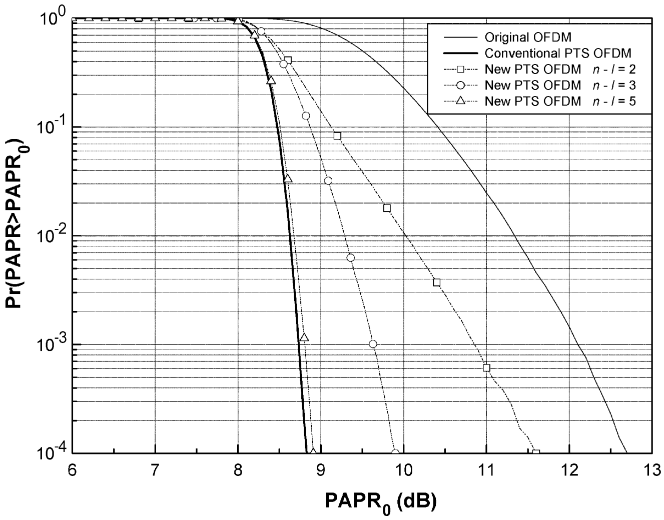

Fig. 3. CCDF of the PAPR of new and conventional PTS OFDM scheme for various stages of multiplication when $N = 2048$, $V = 8$, 16-QAM constellation, and four times oversampling are used.

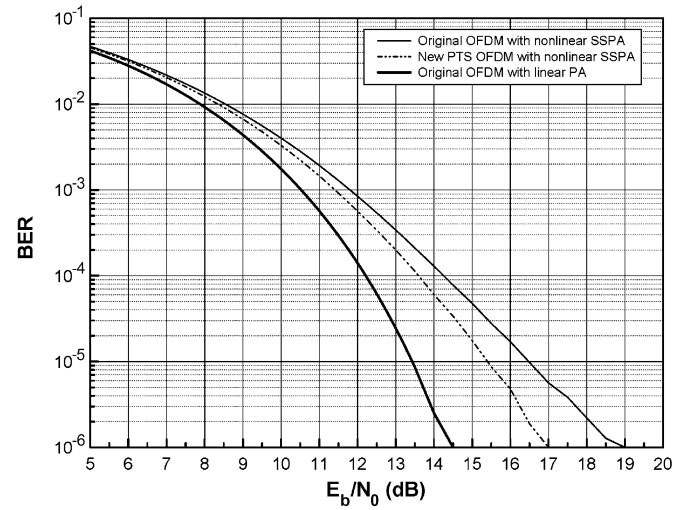

Fig. 5. AWGN channel BER performance of the original OFDM scheme and the new PTS OFDM scheme with $N = 2048$, $n - l = 5$, 16-QAM constellation, and four times oversampling when the nonlinear SSPA with $p = 10$ are operated at the OBO of 5 dB.

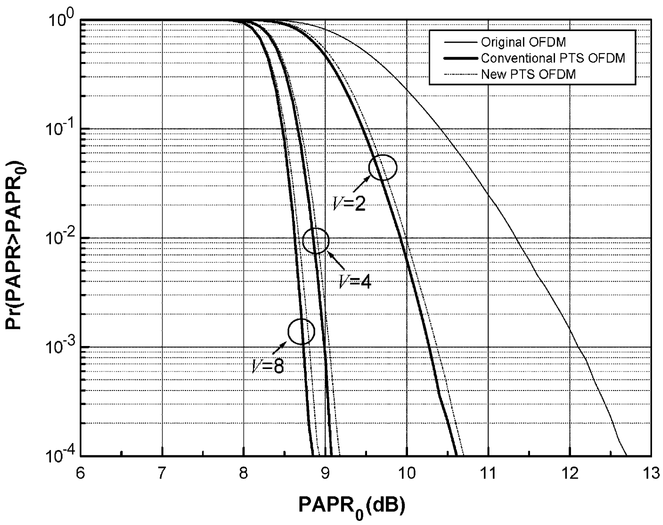

Fig. 4. PAPR reduction performance comparison of the conventional PTS OFDM scheme and the new PTS OFDM scheme when $N = 2048, n - l = 5$ 16-QAM constellation, and four times oversampling are used.

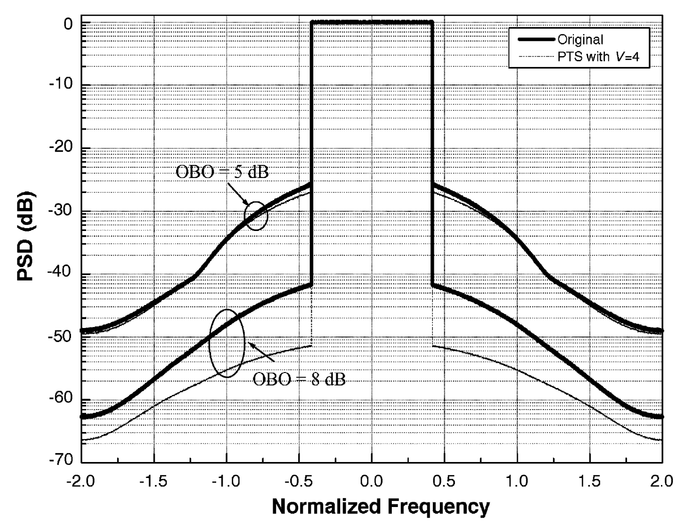

Fig. 6. PSD of the original OFDM scheme and the new PTS OFDM scheme when $N = 2048, n - l = 5$, 16-QAM constellation, four times oversampling, and SSPA with $p = 10$ are used.

can say that the optimal value for $n - l$ does not depend on the number of subcarriers and it is around 5 when the number of subcarriers is between 256 and 8192. Fig. 4 shows a comparison of the PAPR reduction performance between the conventional PTS OFDM scheme and the new PTS OFDM scheme with $n - l = 5$, 16-QAM constellation and four times oversampling. As one can see, the new scheme has almost the same PAPR reduction performance as that of the conventional one. In the case of $N = 2048$ and $n - l = 5$, the new scheme reduces the computational complexity by 27%–48% as the number of blocks increases from 2 to 8.

The nonlinear SSPA with $p = 10$ and AWGN channel are assumed to evaluate the BER performance and the power spectral density of the new PTS OFDM scheme. In the simulation, the input and output power of the nonlinear SSPA is set to have unity to preserve the transmitted power. Then, the OBO of the nonlinear SSPA becomes $20 \log A_0$. The small signal gain v of the nonlinear SSPA in (6) is adjusted to keep the same OBO of the nonlinear SSPA since the distribution of the amplifier input is changed when the new PTS scheme is applied. Fig. 5 shows the BER performance over AWGN channel when the nonlinear SSPA with $p = 10$ is operated at $\mathrm{OBO} = 5$ dB. The new PTS OFDM scheme improves E_b/N_0 by 1.7 dB at $\mathrm{BER} = 10^{-6}$.

Fig. 6 shows the power spectral density (PSD) of the distorted OFDM signals by a nonlinear SSPA with $p = 10$. The new PTS OFDM scheme reduces the out of band radiation comparing to the original OFDM scheme. The amount of reduction of $\mathrm{OBO} = 8$ dB is much larger than that of $\mathrm{OBO} = 5$ dB. The out of band radiation of the new PTS OFDM signal with $V = 4$ is below −50 dB when the nonlinear SSPA is operated at $\mathrm{OBO} = 8$ dB.

V. Conclusions

There is a trade-off between the computational complexity and performance in the PAPR reduction method. A new PTS OFDM scheme has been proposed and its performance is analyzed in reference to the standard of IEEE 802.16 for WMAN. The numerical analysis shows that the new PTS OFDM scheme with 2048 subcarriers reduces the computational complexity by 48% with the performance degradation under 0.2 dB at 10^{-4} when an intermediate signal sequence is partitioned into 8 subblocks at the stage $l = 6$. Since the computational complexity reduction ratio increases as the number of subcarriers increases, the proposed scheme becomes more suitable for the high data rate OFDM systems such as a digital multimedia broadcasting system.

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