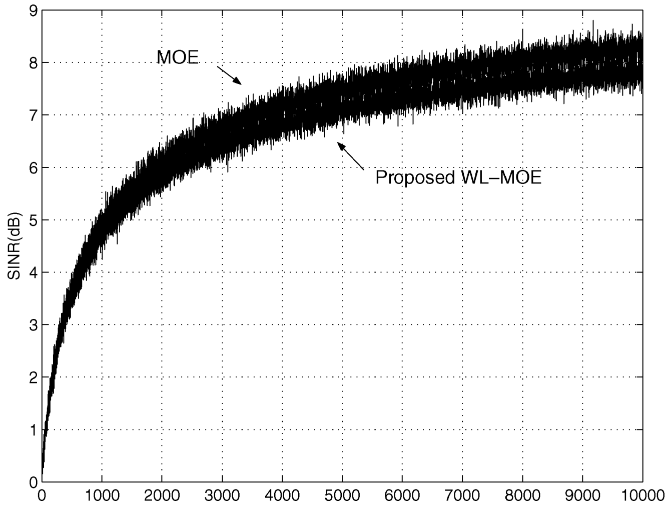

Fig. 4. SINR comparison based on QPSK (proper signal and interference).

is fairly small, so it can be concluded that the proposed WL-MOE algorithm can be profitably employed for any combination of real or complex modulation.

VI. Conclusion

In this correspondence, a new blind widely linear minimum output energy (WL-MOE) algorithm has been proposed for reducing multiple access interference in multiuser communication systems such as code-division multiple-access (CDMA) cellular systems. The proposed algorithm is applicable to the complex-valued modulation types needed for high-speed wireless data systems, whereas prior work was applicable only to real-valued (one-dimensional) modulation. In addition to flexibly handling both complex and real modulation types, it was proven mathematically that the proposed algorithm has better convergence speed than conventional algorithms when the received signal is improper. Through SINR comparison, it was also shown that the proposed algorithm outperforms the conventional MOE in terms of both SINR and convergence speed when a nonnegligible portion of the interference is improper while its SINR performance cannot be better than that of the conventional WL-MOE due to the more severe constraint. Numerical results have further confirmed those results. Therefore, the proposed algorithm should be used for complex-valued modulation systems, while the conventional WL-MOE algorithm should be used in systems exclusively employing real-valued modulation.

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On the Phase Sequence Set of SLM OFDM Scheme for a Crest Factor Reduction

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***Abstract*—The crest factor distribution of orthogonal-frequency-division-multiplexing (OFDM) symbol sequences is evaluated, and it is shown that OFDM symbol sequences with a short period are expected to have a high crest factor. The crest factor relationship between two input symbol sequences, Hamming distance D apart, is also derived. Using these two results, two criteria are proposed for a phase sequence set of the selected mapping (SLM) OFDM scheme and suggest the rows of the cyclic Hadamard matrix constructed from an m-sequence as a near-optimal phase sequence set of the SLM OFDM scheme.**

***Index Terms*—Crest factor, cyclic Hadamard matrix, orthogonal-frequency-division multiplexing (OFDM), phase sequence, selected mapping (SLM).**

I. Introduction

Orthogonal-frequency-division-multiplexing (OFDM) systems are one of the strong candidates for the standard of the next-generation mobile radio communication system. It has been accepted as a standard for the wireless local area networks (WLANs) and mobile wireless metropolitan area networks (WMANs). In the OFDM system, parallel data symbols are transmitted using the orthogonal subcarriers. It is known that the OFDM system is efficient with respect to spectral bandwidth, and its performance over frequency-selective fading channels is better than that of a single-carrier modulation. One of the major drawbacks of an OFDM system is that it has a high crest factor. Due to the nonlinearity of a high-power amplifier (HPA), the high crest factor

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brings on signal distortion in the nonlinear HPA, which induces the degradation of bit error rate (BER).

Numerous research [1]–[3], [5], [7]–[10], [12], [15], [16] has been devoted to reducing the crest factor of the OFDM signals. The methods for the crest factor reduction can be classified into two categories. First, there are deterministic methods that limit the crest factor of the OFDM signals below a threshold level. Clipping and block coding belong to this category. The second category is based on probabilistic approach. These methods statistically improve the characteristic of the crest factor distribution of the OFDM signals without signal distortion. Selected mapping (SLM) and partial transmit sequence (PTS) [1], [3], [7], [10] are included in this category. In the SLM OFDM scheme, a set of alternative symbol sequences is generated from a given input symbol sequence by being multiplied by the phase sequences. Then, the one with the lowest crest factor in the set is selected for transmission.

In [10], Ochiai and Imai proposed the use of pseudorandom interleaver followed by an encoder and quaternary phase-shift keying (QPSK) mapper to generate statistically independent alternative QPSK symbol sequences. Since the pseudorandom interleaving preserves the Hamming weight of binary input data, the correlation between the crest factors of the sequences after interleaving could be higher in probabilistic sense than that in the SLM scheme. The pseudorandom interleaving approach called *adaptive symbol selection* has the same computational complexity of inverse fast Fourier transform (IFFT) as that of the SLM OFDM scheme.

In [1], the scrambling scheme, which abstains from explicit transmission of side information, is proposed. After a label is inserted, an input symbol sequence is fed into a scrambler consisting of a shift-registers with a feedback branch only. The labels determine the stages of the scrambler so that each of the alternative symbol sequence is obtained from each of the distinct labels. When an input symbol sequence is all one sequence, the alternative symbol sequences have the period $2^r - 1$, where r is the number of shift-registers. Thus, the minimum number of shift-registers to avoid periodicity of alternative symbol sequences is expressed as $r = \lceil \log_2(N \log_2 M + 1) \rceil$, where M is the order of modulation, N the number of subcarriers, and $\lceil x \rceil$ the smallest integer exceeding or equal to x.

It is known from the simulation results in various literature [12], [13] that the randomly generated phase sequence set outperforms any other candidates in SLM OFDM scheme. In [13], Ohkubo and Ohtsuki have proposed design criteria for a phase sequence set of the SLM OFDM scheme. They claimed that the sequence set having a low average and a large variance of the crest factors of its members is good as a phase sequence set of the SLM OFDM scheme. However, their criteria cannot be considered sufficient. Moreover, it is difficult to design a phase sequence set satisfying their criteria in a systematic manner. This motivates us to approach this problem from a different viewpoint. In this correspondence, we propose two criteria for a phase sequence set of the SLM OFDM scheme. First, the members of a phase sequence set need to be orthogonal to each other. Second, the componentwise product of any two members of a phase sequence set should not be periodic or similar to periodic sequences.

The correspondence is organized as follows. In Section II, the SLM OFDM scheme and the definition of the crest factor are described. The crest factor distribution of periodic input symbol sequences is evaluated in Section III, and the crest factor relationship between two alternative symbol sequences, Hamming distance D apart, is derived in Section IV. In Section V, we propose two criteria for a phase sequence set and suggest the rows of the cyclic Hadamard matrix constructed from an m-sequence as a near optimal phase sequence set of the SLM OFDM scheme. Finally, concluding remarks are given in Section VI.

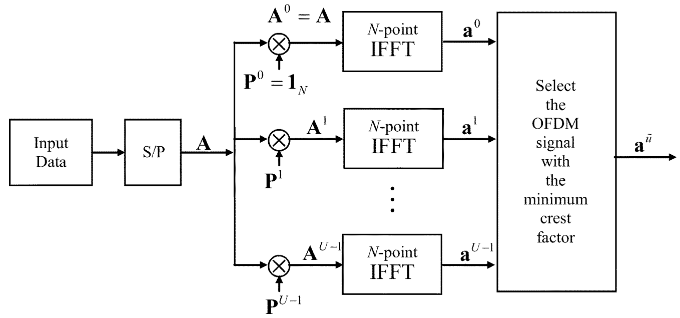

Fig. 1. Block diagram of the SLM OFDM scheme.

II. SLM OFDM Scheme

The discrete-time-domain OFDM signal sequence $\mathbf{a} = [a_0 a_1 \cdots a_{N-1}]$ of N subcarriers can be expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi \frac{n}{N} t}, \quad 0 \le t \le N-1$$

where $\mathbf{A} = [A_0 A_1 \cdots A_{N-1}]$ is an input symbol sequence and t is a discrete-time index. A measure of the envelope variation of the OFDM signal sequence is the crest factor ζ, which is defined as the ratio of the maximum value to the root-mean-square value of the signal envelope as follows:

$$\zeta_{\mathbf{a}} = \frac{\underset{0 \le t \le N-1}{\text{Max}} |a_t|}{\sqrt{\mathrm{E}\left[|a_t|^2\right]}}$$

where $\mathrm{E}[\cdot]$ denotes the expected value.

In the SLM OFDM scheme [7] shown in Fig. 1, alternative symbol sequences $\mathbf{A}^u = [A_0^u A_1^u \cdots A_{N-1}^u]$, $0 \le u \le U-1$, are generated by multiplying the phase sequences $\mathbf{P}^u = [P_0^u P_1^u \cdots P_{N-1}^u]$, $0 \le u \le U-1$, to the input symbol sequence $\mathbf{A}$. We use the expression $\mathbf{A}^u = \mathbf{A} \otimes \mathbf{P}^u$ to represent the componentwise multiplication, i.e., $A_n^u = A_n P_n^u$, $0 \le n \le N-1$. Each symbol of the phase sequences should have unit magnitude to preserve the power, and it is desirable to use $P_n^u \in \{\pm 1\}$ for maintaining the original constellation of an input symbol sequence. The first phase sequence $\mathbf{P}^0$ is usually the all-one sequence $\mathbf{1}_N$. After alternative symbol sequences are inverse fast Fourier transformed ("IFFTed") individually, the OFDM signal sequence $\mathbf{a}^{\tilde{u}} = \mathrm{IFFT}(\mathbf{A}^{\tilde{u}})$ with the lowest crest factor is selected for transmission, where $\tilde{u}$ is expressed as

$$\tilde{u} = \underset{0 \le u \le U-1}{\arg\min} \left(\zeta_{\mathbf{a}^u}\right).$$

The information on the phase sequence used for the transmitted signal must be conveyed to the receiver in the SLM OFDM scheme. In the conventional SLM OFDM scheme, this information, represented as an index symbol sequence, is augmented to the data symbol sequence to form the input symbol sequence. Usually, the index information is encoded for error detection and correction due to its importance [10]. For example, in M-QAM signaling, when the encoder code rate is R and the number of phase sequences is U, the number of index symbols to transmit is $\lceil \log_M U/R \rceil$.

The computational complexity of the SLM OFDM scheme is increased in proportion to the number of phase sequences. If the crest factors of U alternative symbol sequences are independent and identically distributed, then the probability that the crest factor ζ of an SLM OFDM signal sequence exceeds a threshold value ζ_0 can be written as

$$\Pr_{\mathrm{SLM-OFDM}}(\zeta > \zeta_0) = \left(\Pr_{\mathrm{Original-OFDM}}(\zeta > \zeta_0)\right)^U. \quad (1)$$

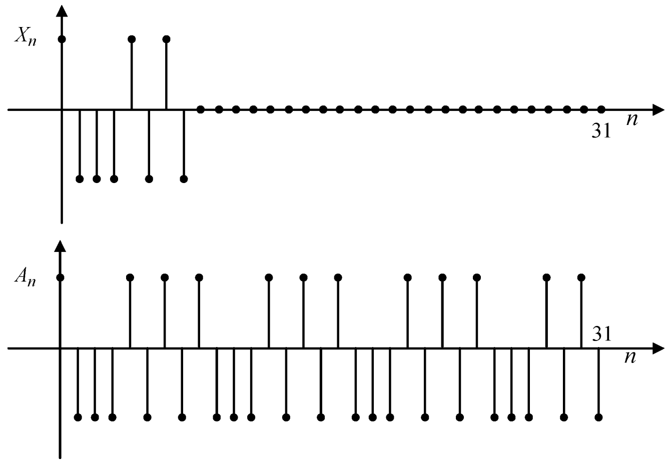

Fig. 2. Example of $\mathbf{X}$ and $\mathbf{A}$ with $N = 32$ and $V = 4$.

Since the alternative symbol sequences are generated from a given input symbol sequence, assuming the statistical independence between the alternative symbol sequences, and further between their crest factors, seems a little absurd. Thus, the complementary cumulative distribution function (CCDF) of the crest factor of the SLM OFDM scheme could not be expressed as (1) in general. However, by the simulation results, the crest factor reduction performance of a randomly generated sequence set, which has been considered "the best" is slightly worse than the value given in (1). Although (1) is not the exact theoretical limit of the crest factor reduction performance in the SLM OFDM scheme, it still can be used as a reference for the performance comparison when we propose criteria for a phase sequence set.

III. Crest Factor Distribution of Periodic Input Symbol Sequences

In this section, we will show that periodic input symbol sequences are more likely to have a high crest factor. Let $\mathbf{X}$ be a symbol sequence of length N, which has nonzero complex values in the interval, $0 \le n \le N/V - 1$ and 0 otherwise, where V is a divisor of N. An input symbol sequence $\mathbf{A}$ of length N is generated by repeating the nonzero value of $\mathbf{X}$ V times. Then, A_n is expressed as

$$A_n = \sum_{v=0}^{V-1} X_{n-\frac{vN}{V}}, \quad 0 \le n \le N-1$$

where subscript of X is computed modulo N. Fig. 2 shows an example of $\mathbf{X}$ and $\mathbf{A}$ with $N = 32$ and $V = 4$. Let $\mathbf{a}$ and $\mathbf{x}$ denote the OFDM signal sequences of $\mathbf{A}$ and $\mathbf{X}$, respectively. According to the shift property of Fourier transform, a_t is given as

$$a_t = x_t \sum_{v=0}^{V-1} e^{j2\pi \frac{v}{V} t} = \begin{cases} V x_t, & t = 0 \mod V \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The crest factor distributions of the band-limited OFDM signals and the discrete time OFDM signals with $V = 1$ are evaluated in [11] and [7], respectively. We will derive the crest factor distribution of the discrete-time OFDM signals for $V \ge 1$. Before evaluating the crest factor distribution of $\mathbf{a}$, we calculate the distribution of the magnitude of x_t. As $\mathbf{X}$ has nonzero value only within the interval $0 \le n \le N/V - 1$, x_t is given as

$$x_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{\frac{N}{V}-1} X_n e^{j2\pi \frac{n}{N} t}.$$

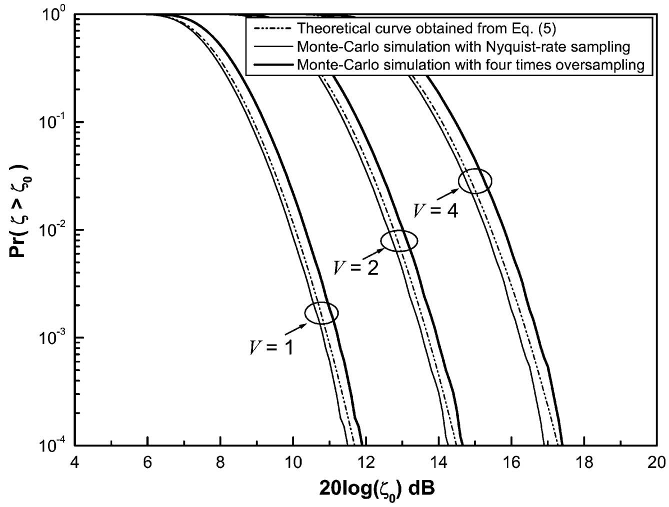

Fig. 3. CCDF of the crest factor of an original OFDM scheme with $V \in \{1, 2, 4\}$ when 256 subcarriers and 16-QAM constellation are used.

For symmetric constellation such as M-PSK or M-QAM, it is legitimate to consider $X_n, 0 \le n \le N/V - 1$ as an independent zero-mean random variable with variance σ_X^2. Using Parseval's theorem, the average power σ_x^2 of the OFDM signal x_t is calculated as $\sigma_x^2 = \sigma_X^2/V$, and the average power σ_a^2 of the OFDM signal a_t as $\sigma_x^2 = \sigma_X^2$. From the central limit theorem for large N/V, x_t can be approximated as a zero-mean complex Gaussian random variable with variance σ_x^2. Let u be the magnitude of a_t at $t = 0 \mod V$. Then, u has a Rayleigh distribution with a probability density function

$$p_u = \frac{2u}{V\sigma_a^2} e^{-\frac{u^2}{V\sigma_a^2}}, \quad u \ge 0. \tag{3}$$

Using (3), the probability that u does not exceed γ is given by

$$\Pr(u \le \gamma) = \int_0^{\gamma} p_u du = 1 - e^{-\frac{\gamma^2}{V\sigma_a^2}}. \tag{4}$$

Suppose that OFDM signal samples are independent and identically distributed [7], [11]. Using (2) and (4), the probability $\mathrm{P}_\zeta(\zeta_0)$ that the crest factor ζ exceeds the threshold value $\zeta_0 = \gamma/\sigma_a$ can be written as

$$\begin{aligned}\mathrm{P}_\zeta(\zeta_0) &= \Pr\left(\operatorname*{Max}_{0 \le t \le N-1} |a_t| > \gamma\right) \\ &= 1 - (\Pr(u \le \gamma))^{\frac{N}{V}} = 1 - \left(1 - e^{-\frac{\zeta_0^2}{V}}\right)^{\frac{N}{V}}. \end{aligned} \tag{5}$$

Fig. 3 shows the CCDF of the crest factor obtained from (5) and Monte Carlo simulation with $V \in \{1, 2, 4\}$, 256 subcarriers, and 16-QAM constellation. In the simulation, the OFDM signals are Nyquist-sampled and oversampled by a factor of four, which is sufficient to represent the analog signal [14]. As one can see, the crest factor becomes large as V increases. This means that an input symbol sequence with a short period is expected to have a high crest factor.

In the SLM OFDM scheme, even if an input symbol sequence $\mathbf{A}$ is periodic, the alternative symbol sequence obtained by being multiplied by an aperiodic phase sequence could be aperiodic. Thus, the members of the phase sequence set need to be aperiodic except the all-one sequence. Also from the fact that the phase sequence set $\{\mathbf{1}_N, \mathbf{P}^1, \mathbf{P}^2, \cdots, \mathbf{P}^{U-1}\}$ has exactly the same performance as the set $\{\mathbf{P}^i, \mathbf{P}^1 \otimes \mathbf{P}^i, \mathbf{P}^2 \otimes \mathbf{P}^i, \cdots, \mathbf{P}^{U-1} \otimes \mathbf{P}^i\}$ over the ensemble of input symbol sequences, we can say that the componentwise product $\mathbf{P}^j \otimes \mathbf{P}^i$ also needs to be aperiodic.

IV. CREST FACTOR RELATIONSHIP BETWEEN TWO ALTERNATIVE SYMBOL SEQUENCES

In this section, it will be shown that the upper bound of the difference between the crest factors of two alternative symbol sequences could be statistically maximized when the phase sequences are orthogonal. Let $\mathbf{A}$ be an input symbol sequence of length N with some constellation such as M-PSK or M-QAM, and let $\mathbf{P}^i$ and $\mathbf{P}^j$ be the ± 1 phase sequences of length N with Hamming distance $D = D_1 + D_2$, where D_1 is the number of index n_{d1} such that $(P^i_{n_{d1}}, P^j_{n_{d1}}) = (1, -1)$ and D_2 the number of index n_{d2} such that $(P^i_{n_{d2}}, P^j_{n_{d2}}) = (-1, 1)$. Let $\mathbf{A}^i$ and $\mathbf{A}^j$ be the alternative symbol sequences given by $\mathbf{A}^i = \mathbf{A} \otimes \mathbf{P}^i$ and $\mathbf{A}^j = \mathbf{A} \otimes \mathbf{P}^j$. Then, $A^i_n - A^j_n$ is given as

$$\begin{aligned} A^i_n - A^j_n &= A_n\left(P^i_n - P^j_n\right) \\ &= 2A_n\left(\sum_{d1=0}^{D_1-1}\delta_{n-n_{d1}} - \sum_{d2=0}^{D_2-1}\delta_{n-n_{d2}}\right). \end{aligned} \quad (6)$$

Let $\mathbf{a}^i$ and $\mathbf{a}^j$ denote the OFDM signal sequences of $\mathbf{A}^i$ and $\mathbf{A}^j$, respectively. From (6), we have

$$\begin{aligned} \left|a^j_t\right| - \left|a^i_t\right| &\le \left|a^j_t - a^i_t\right| \\ &= \frac{2}{\sqrt{N}}\left|\sum_{d1=0}^{D_1-1} A_{n_{d1}} e^{j2\pi\frac{n_{d1}}{N}t} - \sum_{d2=0}^{D_2-1} A_{n_{d2}} e^{j2\pi\frac{n_{d2}}{N}t}\right| \\ &\le \frac{2}{\sqrt{N}}\sum_{d=0}^{D-1}\left|A_{n_d}\right| \end{aligned} \quad (7)$$

where n_d denotes the index such that $P^i_{n_d} + P^j_{n_d} = 0$. Equation (7) can be rewritten in the form of the upper and lower bound of $|a^j_t|$ as

$$\left|a^i_t\right| - \frac{2}{\sqrt{N}}\sum_{d=0}^{D-1}|A_{n_d}| \le \left|a^j_t\right| \le \left|a^i_t\right| + \frac{2}{\sqrt{N}}\sum_{d=0}^{D-1}|A_{n_d}|. \quad (8)$$

From (8), the crest factor $\zeta_{\mathbf{a}^j}$ is bounded as

$$\zeta_{\mathbf{a}^i} - \frac{2}{\sigma\sqrt{N}}\sum_{d=0}^{D-1}|A_{n_d}| \le \zeta_{\mathbf{a}^j} \le \zeta_{\mathbf{a}^i} + \frac{2}{\sigma\sqrt{N}}\sum_{d=0}^{D-1}|A_{n_d}| \quad (9)$$

where $\sigma^2 = \mathrm{E}[|A_n|^2] = \mathrm{E}[|A^i_n|^2] = \mathrm{E}[|A^j_n|^2] = \mathrm{E}[|a_t|^2] = \mathrm{E}[|a^i_t|^2] = \mathrm{E}[|a^j_t|^2]$. When A_n is an M-PSK modulated symbol with an average power of unity, (9) is simplified as

$$\zeta_{\mathbf{a}^i} - \frac{2D}{\sqrt{N}} \le \zeta_{\mathbf{a}^j} \le \zeta_{\mathbf{a}^i} + \frac{2D}{\sqrt{N}}. \quad (10)$$

Since D is Hamming distance of two sequences $\mathbf{A}^i$ and $\mathbf{A}^j$, the Hamming distance of two sequences $-\mathbf{A}^i$ and $\mathbf{A}^j$ becomes $N - D$. Then, the crest factor $\zeta_{\mathbf{a}^j}$ is bounded as

$$\zeta_{-\mathbf{a}^i} - \frac{2(N-D)}{\sqrt{N}} \le \zeta_{\mathbf{a}^j} \le \zeta_{-\mathbf{a}^i} + \frac{2(N-D)}{\sqrt{N}}. \quad (11)$$

Since the OFDM signal sequences $\mathbf{a}^i$ and $-\mathbf{a}^i$ have the same crest factor, (11) can be rewritten as

$$\zeta_{\mathbf{a}^i} - \frac{2(N-D)}{\sqrt{N}} \le \zeta_{\mathbf{a}^j} \le \zeta_{\mathbf{a}^i} + \frac{2(N-D)}{\sqrt{N}}. \quad (12)$$

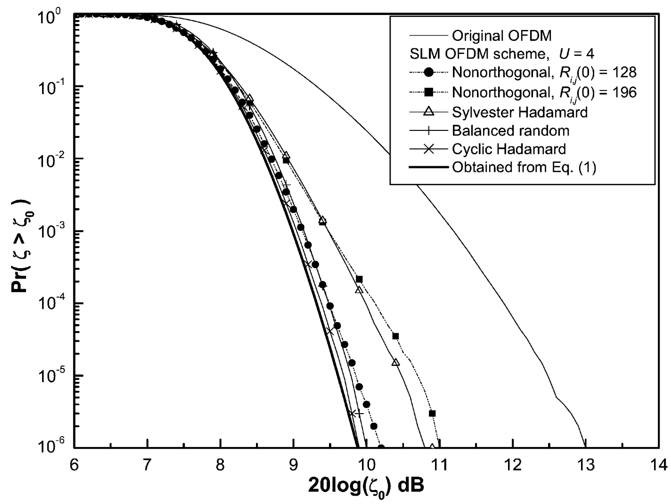

Fig. 4. CCDF of the crest factor of the SLM OFDM scheme for various phase sequence sets when 256 subcarriers and 16-QAM constellation are used with four times oversampling.

Since the one with the minimum crest factor is selected from U alternative OFDM signal sequences, it certainly is better to maximize this upper bound, especially when the crest factor of the original OFDM signal sequence is high. Then probabilistically, the minimum crest factor in the U alternative OFDM signal sequences can be reduced. From (10) and (12), the crest factor $\zeta_{\mathbf{a}^j}$ could be maximally apart from $\zeta_{\mathbf{a}^i}$ when Hamming distance D is $N/2$, which means that the phase sequences $\mathbf{P}^i$ and $\mathbf{P}^j$ are orthogonal. Considering that the first phase sequence is all-one sequence, this condition ensures the balancedness of other members of the phase sequence set.

When A_n is an M-QAM symbol, $|A_{n_d}|$ can be further upper bounded by the maximum magnitude $\mathrm{A}_{\max}$. Thus, (10) and (12) can be written as

$$\zeta_{\mathbf{a}^i} - \frac{2\mathrm{A}_{\max}D}{\sigma\sqrt{N}} \le \zeta_{\mathbf{a}^j} \le \zeta_{\mathbf{a}^i} + \frac{2\mathrm{A}_{\max}D}{\sigma\sqrt{N}}$$

$$\zeta_{\mathbf{a}^i} - \frac{2\mathrm{A}_{\max}(N-D)}{\sigma\sqrt{N}} \le \zeta_{\mathbf{a}^j} \le \zeta_{\mathbf{a}^i} + \frac{2\mathrm{A}_{\max}(N-D)}{\sigma\sqrt{N}}$$

respectively. Therefore, the same conclusion can be drawn for M-QAM.

V. PHASE SEQUENCE SET OF THE SLM OFDM SCHEME

In the SLM OFDM scheme, the crest factor reduction performance depends on how to design a phase sequence set $\{\mathbf{P}^u | 0 \le u \le U - 1\}$. Considering the fact that the true objective of the SLM OFDM scheme is to reduce the probability of the crest factor exceeding some threshold level rather than to reduce the crest factor of each alternative symbol sequence itself, we may say in general that a phase sequence set that makes as many crest factors of alternative symbol sequences look statistically independent as possible can perform well.

In the previous sections, we have seen that the componentwise product of any two phase sequences should not be periodic and the upper bound of the difference between the crest factors of two alternative symbol sequences is maximized when their phase sequences are orthogonal to each other. Although we cannot show how the bound maximization leads toward the aforementioned independence of the crest factors, the following simulation results support the previous sections.

Fig. 4 shows the CCDF of the crest factor of the SLM OFDM scheme with 256 subcarriers and 16-QAM constellation for five different phase sequence sets, namely the balanced random sequence set, two nonorthogonal sequence sets, the cyclic Hadamard sequence set (rows of the cyclic Hadamard matrix constructed from an m-sequence) [6], and the Sylvester Hadamard sequence set (rows of the Sylvester Hadamard matrix). In the simulation, the OFDM signals are oversampled by a factor of four. For each set, we used four phase sequences, and in the balanced random sequence set, the phase sequences are generated randomly with a uniform distribution. The nonorthogonal sequence sets are generated randomly to have $R_{i,j}(0) = 128$ and 196 for $i \neq j$, where $R_{i,j}(\tau)$ denotes the cross-correlation value between two phase sequences.

In the cases of the cyclic Hadamard sequence set and the Sylvester Hadamard sequence set with $U = 4$, the first row (the all-one sequence) and the other three rows of 256 × 256 Hadamard matrix are selected as a phase sequence set. There are $\binom{255}{3}$ cases to choose three rows from 255 rows. The selection of rows does not affect much the crest factor reduction performance in the case of the cyclic Hadamard sequence set while the crest factor reduction performance of the Sylvester Hadamard sequence set is affected by the selection of rows. In the simulation, the first four rows are selected from the cyclic Hadamard matrix and the first, seventeenth, thirty-third, and forty-ninth rows are selected from the Sylvester Hadamard matrix. Note that the periods of the seventeenth, thirty-third, and forty-ninth rows are 32, 64, and 64, respectively.

In Fig. 4, we can see that the crest factor reduction performance of nonorthogonal sequence set becomes worse as $R_{i,j}(0)$ becomes larger. The crest factor reduction performance of the cyclic Hadamard sequence set approximately achieves that obtained from (1) with $U = 4$, where the CCDF of the original OFDM scheme is given from the simulation with four times oversampling. However, the crest factor reduction performance of the Sylvester Hadamard sequence set is not as good as that of the balanced random sequence set or the cyclic Hadamard sequence set, even though the members of the Sylvester Hadamard sequence set are orthogonal. In Section III, we have shown that the periodicity might increase the probability of a high crest factor. When all the members of a phase sequence set are periodic as in the Sylvester Hadamard sequence set case, all the alternative symbol sequences from a periodic input symbol sequence are likely to be periodic. This is the reason why the Sylvester Hadamard sequence set does not show a good crest factor reduction performance.

If we generate sequences randomly with an uniform distribution, the sequences may attain the near orthogonality between themselves. This might explain why a random sequence set as a phase sequence set shows a good crest factor reduction performance.

A Hadamard matrix $\mathbf{H}_N$ of order N is an $N \times N$ square matrix with elements $+1$ and -1 satisfying $\mathbf{H}_N \mathbf{H}_N^T = N\mathbf{I}_N$, where $\mathbf{I}_N$ is the identity matrix of order N. A cyclic Hadamard matrix is a Hadamard matrix with an additional property that in the standard form, removing the top row and the left-most column, the rows are cyclic shifts of each other. The rows of the cyclic Hadamard matrix are orthogonal and the componentwise product of any two rows is aperiodic. Thus, the rows of the cyclic Hadamard matrix could be a good phase sequence set of the SLM OFDM scheme although the above two criteria might not be sufficient for the optimal phase sequence set.

VI. Conclusion

We have proposed two criteria for a phase sequence set of the SLM OFDM scheme. First, the members of the phase sequence set need to be orthogonal to each other. Second, the componentwise product of any two members of the phase sequence set should not be periodic or similar to periodic sequences. The rows of the cyclic Hadamard matrix constructed from an m-sequence meet the above two criteria. Although the two criteria cannot be said to be sufficient conditions for the optimal phase sequence set, it is shown from the simulation results that the cyclic Hadamard sequence set approaches to the performance curve obtained from (1) within 0.1 dB. This might provide some ground for the cyclic Hadamard sequence set to serve as the near-optimal phase sequence set of the SLM OFDM scheme.

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