Near Optimal PRT Set Selection Algorithm for Tone Reservation in OFDM Systems

Dae-Woon Lim, Hyung-Suk Noh, Jong-Seon No, *Member, IEEE*, and Dong-Joon Shin, *Member, IEEE*

***Abstract*—In the tone reservation (TR) scheme, it is known that finding the optimal peak reduction tone (PRT) set is equivalent to solving the secondary peak minimization problem. However, this problem cannot be solved for the practical number of tones because it is nondeterministic polynomial-time (NP)-hard. In this paper, two efficient methods for selecting a near optimal PRT set are proposed. The first method is a random search algorithm with reduced computational complexity based on the observation that the secondary peak value of the time domain kernel, which is obtained by inverse fast Fourier transforming the characteristic sequence of the PRT set, statistically tends to decrease as the variance of the time domain kernel decreases. The second method is a deterministic selection algorithm using the cyclic difference set. The near optimality of these methods is confirmed through the numerical analysis.**

***Index Terms*—Cyclic difference sets, orthogonal frequency division multiplexing (OFDM), peak reduction tone (PRT), peak to average power ratio (PAPR), tone reservation (TR).**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) has become a promising solution for the next generation wireless communication systems which require various high data rate services. Multiplexing a serial data stream into a large number of orthogonal tones makes the bandwidth-efficient OFDM signals [1]. It has been shown that the performance of OFDM system over frequency selective fading channels is better than that of single-carrier system [2].

However, one of the major drawbacks of OFDM system is high peak to average power ratio (PAPR). Since the OFDM signal is the superposition of a large number of modulated subchannel signals, it may have a high instantaneous signal peak with respect to the average signal level. Usually efficient high power amplifier (HPA) in the transmitter is operating in the nonlinear region. This nonlinearity added to the signal with high PAPR causes intermodulation product, which results in inter-carrier interference, high out-of-band harmonic distortion power, and the bit error rate performance degradation. Recently, lots of works have been done in developing schemes to reduce the PAPR of OFDM signals [3]–[10]. There are many factors that should be considered to design the PAPR reduction schemes. These factors include PAPR reduction capability, power increase in transmit signal, BER increase at the receiver, loss in data transmission rate, computational complexity increase, and so on [15]. A simple and widely used scheme is clipping the OFDM signal to limit the PAPR below a threshold level, but it causes both in-band distortion and out-of-band radiation [5]. Block coding [6], encoding input data into codewords with low PAPR, is another well-known PAPR reduction scheme, but it incurs the loss of transmission rate. Selected mapping (SLM) [18], partial transmit sequence (PTS) [17], interleaving [16], and scrambling [19] are based on probabilistic approach. These schemes statistically improve the characteristic of the PAPR distribution of the OFDM signals without signal distortion [20]. In the SLM scheme, a set of alternative symbol sequence is generated from a given input symbol sequence by being multiplied with the phase sequences. Then the one with the lowest PAPR in the set is selected for transmission.

Tellado proposed the tone reservation (TR) scheme which reserves a small number of peak reduction tones (PRTs) to reduce the PAPR of OFDM signals [7]. The PAPR reduction performance of the TR scheme increases in proportion to the number of reserved tones (loss in data transmission rate) [8], which is usually between 5\% and 15\%. Although the data transmission rate loss of TR scheme is larger than that of probabilistic approaches, the PAPR reduction capability of TR scheme is greater than that of probabilistic approaches.

It is known that a randomly generated PRT set performs better than the contiguous PRT set and the interleaved PRT set. It is also known that finding the optimal PRT set is equivalent to finding the time domain kernel with the minimum secondary peak, where the time domain kernel is obtained by inverse fast Fourier transforming (IFFT-ing) the characteristic sequence of the PRT set [8]. The secondary peak minimization problem is known to be nondeterministic polynomial-time (NP)-hard, which cannot be solved for the practical number of tones. In this paper, two efficient methods for selecting a near optimal PRT set are proposed. The first method is a random search algorithm with reduced computational complexity and the second method is a deterministic selection algorithm using the cyclic difference set.

The rest of this paper is organized as follows. In Section II, the TR scheme is described and two methods for selecting a near optimal PRT set are explained in Section III. Finally, conclusion is given in Section IV.

II. Tone Reservation Scheme

In the TR scheme, some tones are reserved to generate PAPR reduction signals and they are not used for data transmission

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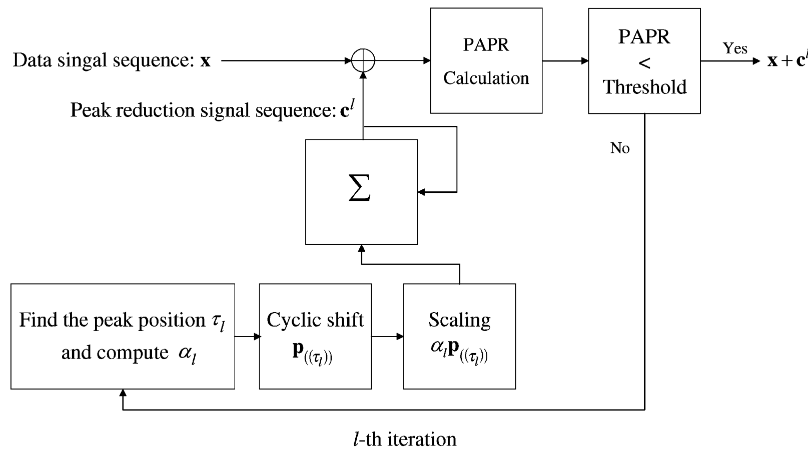


Fig. 1. Block diagram of TR scheme.

[10]. Let $\mathcal{R} = \{i_0, i_1, \cdots, i_{W-1}\}$ denote the ordered set of the positions of the reserved tones and $\mathcal{R}^C$ be the complement set of $\mathcal{R}$ in $\mathcal{N} = \{0, 1, \cdots, N-1\}$, where N is the number of tones of the OFDM signal and W is the number of reserved tones for PAPR reduction. $\mathcal{R}$ is also called a PRT set. The input symbol A_k of the TR scheme is expressed as

$$A_k = X_k + C_k = \begin{cases} C_k, & k \in \mathcal{R} \\ X_k, & k \in \mathcal{R}^C \end{cases}$$

where X_k is a data symbol and C_k a PAPR reduction symbol. Using the IFFT matrix $\mathbf{Q}$, the OFDM signal sequence in the time domain $\mathbf{a} = [a_0 a_1 \cdots a_{N-1}]^T$ is obtained as

$$\mathbf{a} = \mathbf{Q}(\mathbf{X} + \mathbf{C}) \tag{1}$$

where $\mathbf{X} = [X_0 X_1 \cdots X_{N-1}]^T$ and $\mathbf{C} = [C_0 C_1 \cdots C_{N-1}]^T$.

Let the data signal sequence $\mathbf{x}$ and the PAPR reduction signal sequence $\mathbf{c}$ be defined as $\mathbf{x} = [x_0 x_1 \cdots x_{N-1}]^T = \mathbf{QX}$ and $\mathbf{c} = [c_0 c_1 \cdots c_{N-1}]^T = \mathbf{QC}$, respectively. Since the Fourier transform is a linear operation, the OFDM signal sequence is the sum of the data signal sequence and the PAPR reduction signal sequence, that is, $\mathbf{a} = \mathbf{x} + \mathbf{c}$.

In [7], the PAPR of the OFDM signal sequence $\mathbf{a}$ is defined as

$$\mathrm{PAPR}(\mathbf{a}) = \frac{\max\limits_{0 \le t \le N-1} |x_t + c_t|^2}{\mathrm{E}\left[|x_t|^2\right]} \tag{2}$$

where $\mathrm{E}[\cdot]$ is the expectation operator. The PAPR reduction signal sequence $\mathbf{c}$ can be iteratively obtained as follows. Let $\mathbf{p} = [p_0 p_1 \cdots p_{N-1}]^T$ be the time domain kernel defined as

$$\mathbf{p} = \mathbf{QP}$$

where $\mathbf{P} = [P_0 P_1 \cdots P_{N-1}]^T$ is called a frequency domain kernel with binary $\{0, 1\}$ elements. Thus, the maximum peak of $\mathbf{p}$ is always p_0. The time domain kernel $\mathbf{p}$ is used to synthesize the PAPR reduction signal sequence $\mathbf{c}$, iteratively [8]. That is, the PAPR reduction signal sequence $\mathbf{c}^l$ at the l-th iteration is obtained as

$$\mathbf{c}^l = \sum_{i=1}^{l} \alpha_i \mathbf{p}_{((\tau_i))} \tag{3}$$

where $\mathbf{p}_{((\tau_i))}$ denotes a circular shift of $\mathbf{p}$ to the right by τ_i and α_i is a complex scaling factor.

For simplicity, we assume that only one maximum peak of OFDM signal is reduced at each iteration in (3). Let $\mathbf{u}^l = [u_0^l u_1^l \cdots u_{N-1}^l]$ be the signal sequence after l-th iteration such as $\mathbf{u}^l = \mathbf{x} + \mathbf{c}^l$. Then, the input signal of l-th iteration is $\mathbf{u}^{l-1}$. At l-th iteration, we first find the location of the maximum peak value of $\mathbf{u}^{l-1}$, which is the circular shift τ_l and expressed as

$$\tau_l = \arg\max_{0 \le t \le N-1} \left| x_t + c_t^{l-1} \right|.$$

Then, the maximum peak value of $\mathbf{u}^{l-1}$ is $u_{\tau_l}^{l-1} = x_{\tau_l} + c_{\tau_l}^{l-1}$. After finding the location of the maximum peak value, the time domain kernel $\mathbf{p}$ is shifted by τ_l to align the peaks of $\mathbf{p}$ and $\mathbf{u}^{l-1}$ since $\mathbf{p}$ and $\mathbf{u}^{l-1}$ have the peak at 0 and τ_l, respectively. Finally, the shifted time domain kernel $\mathbf{p}_{((\tau_l))}$ is scaled by α_l and added to $\mathbf{u}^{l-1}$. The complex scaling factor α_l is set for the magnitude of the summed signal at τ_l to be the threshold level ζ such as

$$\left| u_{\tau_l}^{l-1} + \alpha_l p_0 \right| = \zeta$$

where the threshold level affects the PAPR reduction performance, which is usually determined from the simulation results. If the maximum number of iterations is reached or the threshold level (desired level) is obtained, the iteration stops. From the shift property of the Fourier transform, it is guaranteed that $\mathbf{Q}^{-1}\mathbf{p}_{((\tau_i))}$ always has zero value in $\mathcal{R}^C$. Fig. 1 shows the block diagram of the TR scheme.

The PAPR reduction performance depends on the time domain kernel $\mathbf{p}$ and the best performance can be achieved when the time domain kernel $\mathbf{p}$ is a discrete impulse because the maximum peak can be cancelled without affecting other signal samples at each iteration. But, in order for the time domain kernel $\mathbf{p}$ to be a discrete impulse, all the tones should be allocated to the PRT set. As the number of reserved tones becomes larger, the PAPR reduction performance is improved but the data transmission rate decreases. In general, the number of reserved tones is usually less than 15 percentage of N.

When the PRT set $\mathcal{R}$ is given, it is known that the optimal choice of $\mathbf{P}$ corresponds to minimizing the secondary peaks of

$\mathbf{p}$. In this situation, it is shown in [8] that the optimal frequency domain kernel $\mathbf{P}$ is obtained as

$$P_k = \begin{cases} 1, & k \in \mathcal{R} \\ 0, & k \in \mathcal{R}^{\mathrm{C}}. \end{cases} \tag{4}$$

Then, the optimal frequency domain kernel $\mathbf{P}$ corresponds to the characteristic sequence of the PRT set $\mathcal{R}$ and the maximum peak of $\mathbf{p}$ is always $p_0 = |\mathcal{R}|$ because $\mathbf{P}$ is a $\{0, 1\}$ sequence.

Clearly, the PAPR reduction performance of the TR scheme depends on the selection of the PRT set $\mathcal{R}$. Let $\mathcal{R}^{opt}$ be the optimal PRT set given by [8]

$$\mathcal{R}^{opt} = \mathop{\arg\min}_{\mathcal{R}=\{i_o, i_1, \cdots, i_{W-1}\}} \| [|p_1||p_2| \cdots |p_{N-1}|] \|_\infty$$

where $\|\cdot\|_v$ denotes the v-norm and ∞-norm refers to the maximum value. From now on, we will assume that $\mathbf{p}$ is the time domain kernel obtained by IFFT-ing the characteristic sequence $\mathbf{P}$ of the PRT set $\mathcal{R}$.

It is known that this problem is NP-hard because the time domain kernel $\mathbf{p}$ must be optimized over all possible discrete sets $\mathcal{R}$ [8]. Thus, it cannot be solved for the practical values of N and W. Since the computational complexity for evaluating the secondary peak of the time domain kernel $\mathbf{p}$ by IFFT-ing the characteristic sequence of the PRT set is very high, searching over all possible PRT sets takes too much time for large N and practical value W. For example with $N = 128$ and $W = 10$, the number of candidate PRT sets is $\binom{128}{10} \simeq 2.27 \times 10^{14}$. This motivates us to find an efficient method to find a near optimal PRT set.

III. Two Efficient Methods of Selecting Near Optimal PRT Set

In this section, we investigate the relationship between the secondary peak and variance of the time domain kernel $\mathbf{p}$. Then, two efficient methods are proposed to select a near optimal PRT set. The first method is a random search algorithm based on minimizing the variance of the time domain kernel $\mathbf{p}$. The second method selects a near optimal PRT set using the cyclic difference set because the characteristic sequence of the cyclic difference set has a good aperiodic autocorrelation property.

A. A Method Using Variance Minimization

Let $\mathbf{y} = [y_0 y_1 y_2 \cdots y_{N-1}]^T$, $\sum_{t=0}^{N-1} y_t = \gamma$, and $0 \le y_t \le y_0$, $1 \le t \le N-1$. Suppose that y_0 is a fixed value and y_t is a variable, $1 \le t \le N-1$. Then, we have

$$\max_{1 \le t \le N-1} y_t \ge \frac{1}{N-1} \sum_{t=1}^{N-1} y_t = \frac{\gamma - y_0}{N-1}.$$

In this case, it is clear that $\max_{1 \le t \le N-1} y_t$ is minimized when $y_1 = y_2 = \cdots = y_{N-1}$. The variance $\sigma_{\mathbf{y}}^2$ of $\mathbf{y}$ can be given as

$$\begin{aligned} \sigma_{\mathbf{y}}^2 &= \frac{1}{N} \sum_{t=0}^{N-1} y_t^2 - \left(\frac{1}{N} \sum_{t=0}^{N-1} y_t \right)^2 \\ &\ge \frac{1}{N(N-1)} \left(\sum_{t=1}^{N-1} y_t \right)^2 + \frac{1}{N} y_0^2 - \left(\frac{\gamma}{N} \right)^2 \end{aligned}$$

where the inequality holds from the Cauchy-Schwartz inequality.

Thus, the variance $\sigma_{\mathbf{y}}^2$ can also be minimized when $y_1 = y_2 = \cdots = y_{N-1}$. Let $y_0 = |p_0|^2$ and $y_t = |p_t|^2$, $1 \le t \le N-1$, where $\mathbf{p} = [p_o p_1 \cdots p_{N-1}]$ is the time domain kernel defined in Section II. Since y_t cannot have the same value, the minimization of the secondary peak is not identical to the minimization of $\sigma_{\mathbf{y}}^2$. Thus, by doing numerical analysis, we investigate the relationship between the secondary peak and the variance of the time domain kernel $\mathbf{p}$, which is IFFT-ed signal of the characteristic sequence of PRT set. In Figs. 2 and 3, the variances and the secondary peaks of the time domain kernels $\mathbf{p}$ are shown for all the possible PRT sets when $N = 32$ and $W = 8$ and for 10^7 randomly selected PRT sets when $N = 1024$ and $W = 102$, where the first peak power is normalized to 1. The numerical results show that the time domain kernels $\mathbf{p}$ with the same variance can have various secondary peaks and the secondary peak value statistically tends to decrease as the variance decreases. It is worth mentioning that the time domain kernels

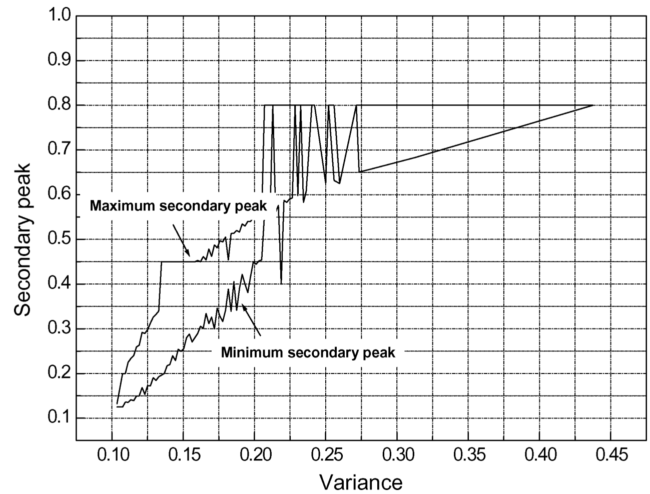


Fig. 2. Relationship between the secondary peak and the variance of the time domain kernels when $N = 32$ and $W = 8$. The time domain kernels of all the possible PRT sets are used.

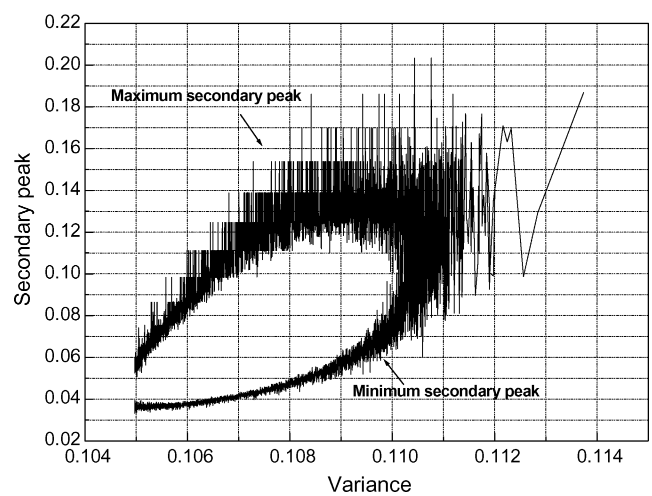


Fig. 3. Relationship between the secondary peak and the variance of the time domain kernels when $N = 1024$ and $W = 102$. The time domain kernels of the randomly generated 10^7 PRT sets are used.

TABLE I
COMPUTATIONAL COMPLEXITIES OF VARIANCE CALCULATION AND SECONDARY PEAK SEARCH

Type of operation	Variance calculation	Secondary peak search
AND	$\frac{(N-1)(N-2)}{2}$	$-$
Integer addition	$\frac{(N-1)(N-2)}{2}+2(N-2)$	$-$
Integer multiplication	$2(N-2)$	$-$
Complex addition	$-$	$N\log N+N$
Complex multiplication	$-$	$\frac{N}{2}\log N+2N$

with the minimum secondary peak are contained in the time domain kernels with the minimum or near minimum variance although the time domain kernel with the minimum variance does not guarantee the minimum secondary peak.

Now, we are going to evaluate the variance $\sigma_{\mathbf{p}}^2$ of $\mathbf{p}$ and show that $\sigma_{\mathbf{p}}^2$ can be calculated only by using integer operations without doing IFFT of $\mathbf{P}$. The power spectrum of p_t [11] can be expressed as

$$|p_t|^2=\frac{1}{N}R_0+\frac{1}{N}\sum_{\tau=1}^{N-1}R_\tau\cos\left(j2\pi\frac{\tau}{N}t\right) \tag{5}$$

where R_τ denotes the aperiodic autocorrelation of $\mathbf{P}$ defined by

$$R_\tau=\sum_{k=0}^{N-1-\tau}P_kP_{k+\tau}^*=\sum_{k=0}^{N-1-\tau}P_kP_{k+\tau}. \tag{6}$$

The time average μ of $|p_t|^2$ is given as

$$\mu=\frac{1}{N}\sum_{t=0}^{N-1}|p_t|^2=\frac{1}{N}R_0. \tag{7}$$

Using (5) and (7), the variance σ^2 of $|p_t|^2$ is obtained as

$$\sigma_{\mathbf{p}}^2=\frac{1}{N}\sum_{t=0}^{N-1}\left(|p_t|^2-\mu\right)^2=\frac{2}{N^2}\sum_{\tau=1}^{N-1}\left(R_\tau^2+R_\tau R_{N-\tau}\right). \tag{8}$$

As well as $P_kP_{k+\tau}$ in (6) can be implemented with AND logic, the additions can be parallelized. The multiplication in (8) can be done using a look-up table that contains $R_{\tau_1}^2$ and $R_{\tau_1}R_{\tau_2}$ for $0\le R_{\tau_1},R_{\tau_2}\le W$.

From the observation that the time domain kernels with the minimum secondary peak are contained in the time domain kernels with the minimum or near minimum variance, we propose a new method with reduced computational complexity to find a near optimal PRT set or the time domain kernel $\mathbf{p}$, which consists of the following steps.

- Step 1 (Variance calculation): Generate sufficiently large number of PRT sets and calculate the variances of the time domain kernels of these sets. Select a small percentage (usually, 0.01 percentage) of the PRT sets with the variance values as small as possible.
- Step 2 (Secondary peak search): Generate the time domain kernels by IFFT-ing the characteristic sequences of the selected PRT sets in Step 1. Choose the time domain kernel with the minimum secondary peak.

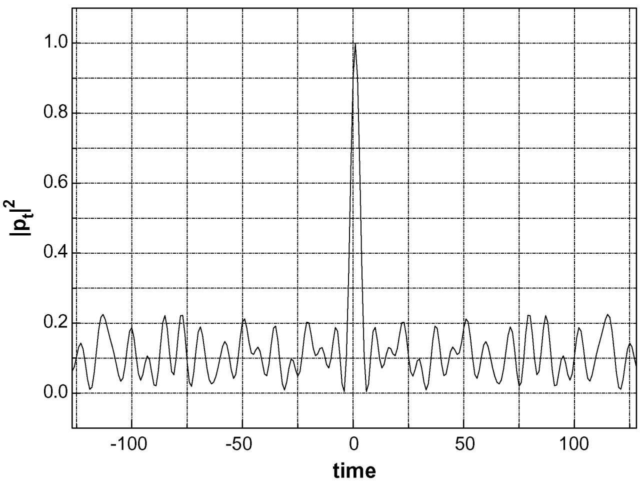
The computational complexities of the variance calculation and the secondary peak search are compared in Table I. While only integer operations are used in Step 1 to compute the variance of the time domain kernel by using the aperiodic autocorrelation of the frequency domain kernel, the powers of the time domain kernels should be calculated using the complex multiplications and additions and they should be compared to select the secondary peak of the time domain kernel in Step 2. Thus, the computational complexity of the proposed method for selecting a near optimal PRT set via two steps can be reduced if the variance calculation in Step 1 is implemented using the dedicated hardware. For the practical values of N and W, the proposed scheme shows lower computational complexity than the secondary peak minimization scheme.

Simulations are performed for the 64-tone OFDM system specified in *IEEE* 802.11 standard for wireless local area network (WLAN) [12]. The number of used tones is 52 and the remaining 12 tones are set to zero to shape the power spectral density of the transmit signal. Among 52 tones, 8 tones are reserved for PRT set and the remaining 44 tones are used to transmit data symbols. The input symbols are modulated using 16-QAM constellation. The 10^8 input data symbol sequences for IFFT are randomly generated to obtain the statistics of the PAPR distribution. The complex baseband OFDM signal is oversampled by a factor of four which is sufficient to represent the analog signal [13].

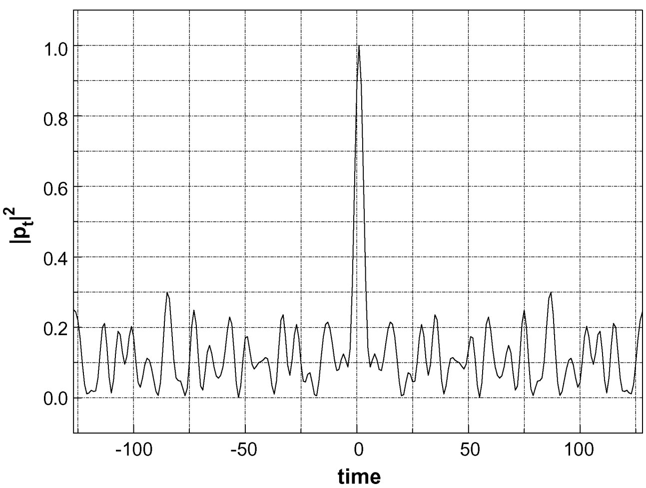
The 10^7 PRT sets are randomly generated and the variance of the corresponding time domain kernels are computed. From these 10^7 PRT sets, the 10^3 PRT sets are selected with the variance values as small as possible. The 10^3 time domain kernels are generated by IFFT-ing the selected PRT sets and their secondary peaks are searched. The time domain kernel with the minimum secondary peak is chosen as the time domain kernel for the TR scheme. For the purpose of the comparison, the time domain kernels of the above 10^7 PRT sets are also generated and their secondary peaks are compared. The time domain kernel with the minimum secondary peak is chosen as the time domain kernel for the TR scheme. As a result, it is shown that the time domain kernel by the secondary peak minimization and the time domain kernel by the proposed variance minimization scheme are identical.

In the simulation, the PAPR reduction performance of various PRT sets, namely the proposed PRT set, the same variance PRT set, the minimum secondary peak PRT set, and the contiguous PRT set is compared, where the same variance PRT set is the one with the same variance as that of the time domain kernel of the proposed PRT set but the secondary peak is larger than that of the time domain kernel of the proposed PRT set.

The proposed PRT set and the minimum secondary peak PRT set are {11, 12, 14, 25, 32, 37, 41, 47}, the same variance PRT set {9, 23, 27, 39, 44, 45, 47, 54}, and the contiguous PRT set



(a)



(b)

Fig. 4. $|p_t|^2$ of the time domain kernel when $N = 64$ and $W = 8$. (a) The PRT set selected by the proposed variance minimization scheme; (b) The PRT set with the same variance as that of the set selected by the proposed variance minimization scheme.

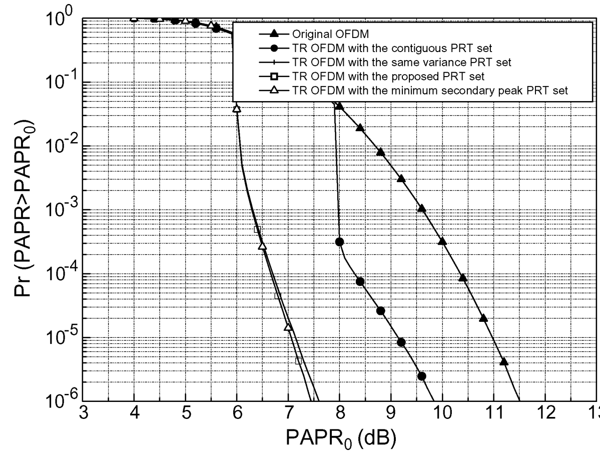


Fig. 5. CCDF of the PAPR for the TR scheme with various PRT sets when $N = 64$ and $W = 8$.

TABLE II
DIFFERENCE TABLE OF CYCLIC (13, 4, 1) DIFFERENCE SET $\mathcal{D} = \{2, 3, 5, 11\}$

mod 13	2	3	5	11
2	0	1	3	9
3	12	0	2	8
5	10	11	0	6
11	4	5	7	0

{6, 7, 8, 9, 10, 11, 12, 13}. The power $|p_t|^2$ of the time domain kernels of the proposed PRT set and the same variance PRT set are shown in Fig. 4 where the first peak power is normalized to 1. The secondary peak powers of the time domain kernels of the proposed PRT set and the same variance PRT set are 0.23 and 0.30, respectively.

The maximum number of iterations for the TR scheme is 10. If the desired peak power is obtained at the i-th iteration, the iteration stops. Otherwise, the peak power at the i-th iteration is compared with the minimum peak value obtained until the previous iteration. If the peak power at the i-th iteration is smaller than the minimum peak value obtained until the previous iteration, the signal sequence at the i-th iteration is updated as the minimum peak signal sequence. If the peak power at the i-th iteration is larger than the minimum peak value obtained until the previous iteration, the saved minimum peak signal sequence is not changed. If a desired peak power is not obtained even at the last iteration, the saved minimum peak signal sequence is transmitted. The threshold level for the proposed PRT set, the minimum secondary peak PRT set, and the same variance PRT set is 3.98 (6.0 dB) and the threshold level for the contiguous set is 6.31 (8.0 dB).

Fig. 5 illustrates the probability that the PAPR of the OFDM signal exceeds the given value PAPR_0. The PAPR of the original OFDM signal without the PAPR reduction scheme is about 11.5 dB at 10^{-6}. The TR scheme with the proposed PRT set reduces the PAPR by 4.1 dB at 10^{-6} while the TR scheme with the adjacent PRT set reduces the PAPR only by 1.7 dB at 10^{-6}. The TR scheme with the same variance PRT set degrades the PAPR reduction performance by 0.15 dB compared to the TR scheme with the proposed PRT set.

B. A Method Using Cyclic Difference Sets

A cyclic difference set, also called a cyclic (v, q, λ) difference set, is a set $\mathcal{D} = \{d_0, d_1, = \ldots, d_{q-1}\}$ of q integers modulo v such that the congruence $d_i - d_j \equiv t \pmod{v}$ has exactly λ solution pairs (d_i, d_j) in $\mathcal{D}$ for each integer $t, 1 \le t \le v-1$ [14]. Since there are $q(q-1)$ choices of $d_i \ne d_j$ from $\mathcal{D}$, giving $v-1$ values of t exactly λ times, we have $q(q-1) = (v-1)\lambda$ as a necessary condition for the existence of a cyclic (v, q, λ) difference set. For example, $\mathcal{D} = \{2, 3, 5, 11\}$ is a cyclic (13, 4, 1) difference set as seen in the Table II. If a constant g is added to each element of a cyclic (v, q, λ) difference set $\mathcal{D}$, the new set $\mathcal{D}' = \mathcal{D} + g = \{d_0 + g, d_1 + g, \ldots, d_{q-1} + g\}$ is again a cyclic (v, q, λ) difference set, with the same difference table, because $(d_i + g) - (d_j + g) \equiv d_i - d_j \pmod{v}$. In this case, $\mathcal{D}$ and $\mathcal{D}'$ are often considered the same cyclic difference set.

It is clear that $m\mathcal{D} = \{md_0, md_1, \ldots, md_{q-1}\}$ becomes again a cyclic (v, q, λ) difference set if m is relative prime to v. Combining these two operations, from a cyclic (v, q, λ) difference set $\mathcal{D}$, $v\phi(v)$ cyclic difference sets with the same parameters can be generated in the form $m\mathcal{D} + g$, where the Euler phi-function $\phi(v)$ is the number of integers modulo v, which are relatively prime to v.

If $m\mathcal{D} = \mathcal{D} + g$ for some g, then m is said to be a multiplier. If m is a multiplier of the cyclic difference set $\mathcal{D}$, there is a translate $\mathcal{D}' = \mathcal{D} + a$ of $\mathcal{D}$ such that $m\mathcal{D}' = \mathcal{D}'$, provided that $\gcd(v, m-1) = 1$. It is proved that if $\mathcal{D}$ is a cyclic (v, q, λ) difference set and p is a prime divisor of $k - \lambda$ with $p > \lambda$, then p is a multiplier of $\mathcal{D}$.

Since the characteristic sequence of the cyclic difference set has a good aperiodic Hamming autocorrelation property, the cyclic difference set could be a good PRT set. Although the PRT set generated by using the cyclic difference set shows good PAPR reduction performance, unfortunately, there are restrictions on using the cyclic difference sets because they cannot be constructed for the arbitrary (v, q, λ).

Simulations are performed to show that the cyclic difference sets can be used as the PRT sets for the TR scheme, where the number of tones is 1024 and the number of reserved tones is 32. Since there is no cyclic (1024, 32, 1) difference set, it is needed to modify the existing cyclic (1057,33,1) difference set which is {0, 1, 31, 35, 40, 149, 155,171, 191, 369, 396, 425, 450, 508, 521, 558, 613, 627, 651, 674, 700, 715, 717, 774, 777, 785, 795, 884, 912, 960, 1006, 1013, 1025}. The last element of the cyclic (1057,33,1) difference set is removed to become a PRT set for the above TR scheme.

$|p_t|^2$ of the time domain kernels of the PRT set chosen from the randomly generated 10^7 sets and the PRT set generated by using the cyclic difference set is shown in Fig. 6. The secondary peak of the time domain kernel of the PRT set generated by using the cyclic difference set is smaller than that of the time domain kernel of the PRT set chosen from the randomly generated 10^7 sets.

In Fig. 7, the PAPR reduction performance of the PRT set generated by using the cyclic difference set is compared with that of the PRT set chosen from the randomly generated 10^7 sets in the TR scheme. The PAPR of the original OFDM signal is 12.7 dB at 10^{-5}. The two PRT sets reduce PAPR by 2 dB at 10^{-5} and the PAPR reduction performance of the PRT set generated by using the cyclic difference set is slightly better than that of the PRT set chosen from the randomly generated 10^7 sets.

IV. Conclusion

It is shown that the variance of the time domain kernel obtained by IFFT-ing the characteristic sequence of the PRT set is expressed as the sum of the aperiodic autocorrelation values of the frequency domain kernel. Then, the relationship between the secondary peak and the variance of the time domain kernel is investigated by the numerical analysis. The results show that the time domain kernels with the same variance can have various secondary peaks and the secondary peak value statistically tends to decrease as the variance decreases. It is also shown that the time domain kernels with the minimum secondary peak are contained in the time domain kernels with the minimum or near minimum variance.

Two efficient methods are proposed to select a near optimal PRT set. The first method is a random search algorithm which is composed of two steps. The first step is to calculate the variances of the characteristic sequences of PRT sets and the second step to search the minimum secondary peak. Since only integer operations are needed to compute the variance in Step 1, the computational complexity of the variance calculation can be much

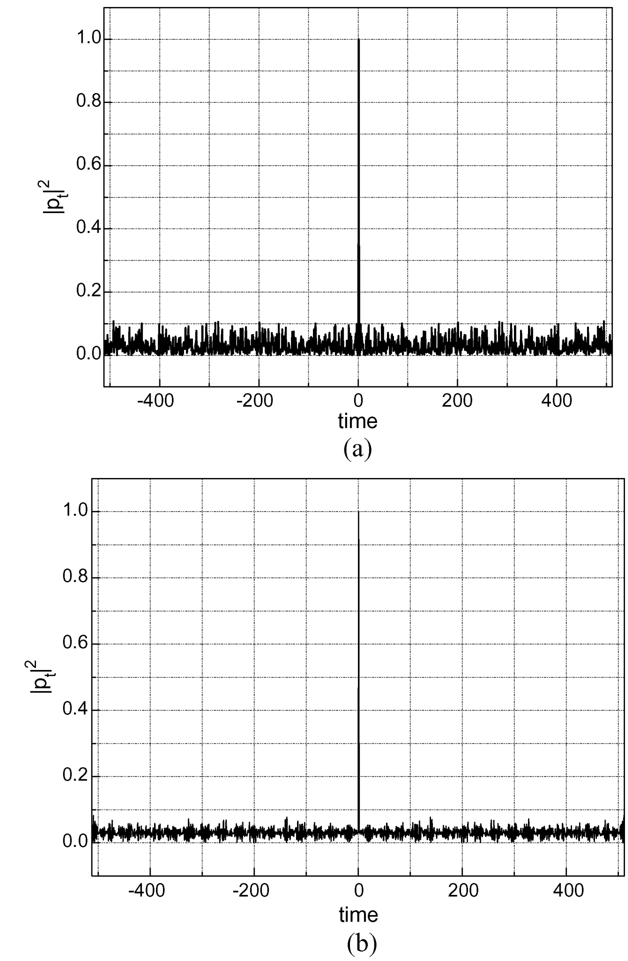


Fig. 6. $|p_t|^2$ of the time domain kernel when $N = 1024$ and $W = 32$. (a) The PRT set chosen from the randomly generated 10^7 sets; (b) the PRT set generated by using the cyclic (1057, 33, 1) difference set.

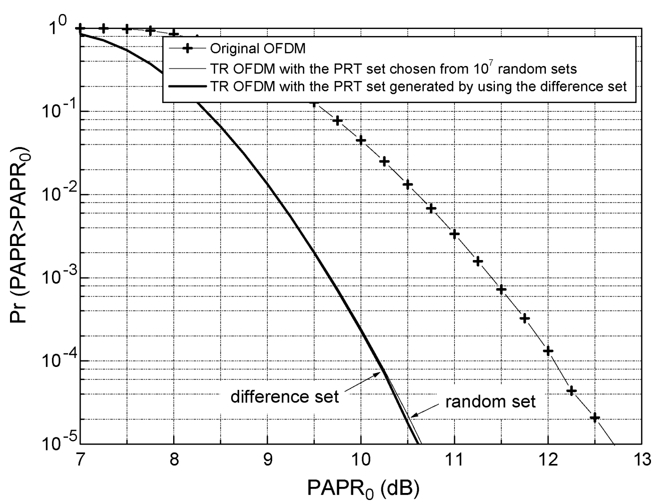


Fig. 7. CCDF of the PAPR for the TR scheme with two PRT sets when $N = 1024$ and $W = 32$.

smaller than the secondary peak search if the variance calculation in Step 1 is implemented using the dedicated hardware. The second method selects a near optimal PRT set using the cyclic difference set because the characteristic sequence of the cyclic difference set has a good aperiodic autocorrelation property.

REFERENCES

[1] W. Y. Zou and Y. Wu, "COFDM: An overview," *IEEE Trans. Broadcast.*, vol. 41, no. 1, pp. 1–8, Mar. 1995.

[2] H. Steendam and M. Moeneclaey, "Analysis and optimization of the performance of OFDM on frequency-selective time-selective fading channels," *IEEE Trans. Commun.*, vol. 47, no. 12, pp. 1811–1819, Dec. 1999.

[3] H. Nikookar and K. S. Lidsheim, "Random phase updating algorithm for OFDM transmission with low PAPR," *IEEE Trans. Broadcast.*, vol. 48, no. 2, pp. 123–128, Jun. 2002.

[4] T. Jiang, Y. Yang, and Y. Song, "Exponential companding transform for PAPR reduction in OFDM systems," *IEEE Trans. Broadcast.*, vol. 51, no. 2, pp. 244–248, Jun. 2005.

[5] H. Ochiai and H. Imai, "Performance of the deliberate clipping with adaptive symbol selection for strictly band-limited OFDM systems," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 11, pp. 2270–2277, Nov. 2000.

[6] J. A. Davis and J. Jedwab, "Peak-to-mean power control in OFDM, Golay complementary sequences, and Reed-Muller codes," *IEEE Trans. Inf. Theory*, vol. 45, no. 7, pp. 2397–2417, Nov. 1999.

[7] J. Tellado and J. Cioffi, "PAR reduction in multicarrier transmission systems," ANSI Document, T1E1.4 Technical Subcommittee, Dec. 8, 1997, pp. 1–14, no. 97-367.

[8] J. Tellado, *Multicarrier Modulation with Low PAPR*. Massachusetts: Kluwer, 2000.

[9] B. S. Krongold and D. L. Jones, "PAR reduction in OFDM via active constellation extension," *IEEE Trans. Broadcast.*, vol. 49, no. 3, pp. 258–268, Sep. 2003.

[10] B. S. Krongold and D. L. Jones, "An active-set approach for OFDM PAR reduction via tone reservation," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 495–509, Feb. 2004.

[11] N. Y. Ermolova and P. Vainikainen, "On the relationship between peak factor of a multicarrier signal and aperiodic autocorrelation of the generating sequence," *IEEE Commun. Lett.*, vol. 7, no. 3, pp. 107–108, Mar. 2003.

[12] *IEEE Standard for Wireless Local Area Network Medium Access Control and Physical Layer Specification*, IEEE 802.11, Nov. 1997.

[13] M. Sharif, M. Gharavi-Alkhansari, and B. H. Khalaj, "On the peak-to-average power of OFDM signals based on oversampling," *IEEE Trans. Commun.*, vol. 51, no. 1, pp. 72–78, Jan. 2003.

[14] H. A. Lin, "From Cyclic Hadamard Difference Sets to Perfectly Balanced Sequences," Ph.D. dissertation, University of Southern California, CA, 1998.

[15] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction technique for multicarrier transmission," *IEEE Wireless Commun.*, vol. 12, no. 2, pp. 56–65, May 2005.

[16] H. Ochiai and H. Imai, "Performance of the deliberate clipping with adaptive symbol selection for strictly band-limited OFDM systems," *IEEE J. Select. Areas Commun.*, vol. 18, no. 11, pp. 2270–2277, Nov. 2000.

[17] D. W. Lim, S. J. Heo, J. S. No, and H. Chung, "A new PTS OFDM scheme with low complexity for PAPR reduction," *IEEE Trans. Broadcast.*, vol. 52, no. 1, pp. 77–82, Mar. 2006.

[18] S. J. Heo, H. S. Noh, J. S. No, and D. J. Shin, "A modified SLM scheme with low complexity for PAPR reduction of OFDM systems," *IEEE Trans. Broadcast.*, vol. 53, no. 4, pp. 804–808, Dec. 2007.

[19] M. Breiling, S. H. Müller, and J. B. Huber, "SLM peak power reduction without explicit side information," *IEEE Commun. Lett.*, vol. 5, no. 6, pp. 239–241, Jun. 2001.

[20] R. J. Baxley and G. T. Zhou, "Comparing selected mapping and partial transmit sequence for PAR reduction," *IEEE Trans. Broadcast.*, vol. 53, no. 4, pp. 797–803, Dec. 2007.