Multi-Stage TR Scheme for PAPR Reduction in OFDM Signals

Dae-Woon Lim, Hyung-Suk Noh, Hyun-Bae Jeon, Jong-Seon No, and Dong-Joon Shin

***Abstract*—In the tone reservation (TR) scheme of the orthogonal frequency division multiplexing (OFDM) systems, there exists a trade-off between the peak to average power ratio (PAPR) reduction performance and the peak reduction tone (PRT) set size. In this paper, we propose a multi-stage TR scheme for PAPR reduction, which adaptively selects one of several PRT sets according to the PAPR of OFDM signal while the PRT set is fixed for the conventional TR scheme. It is shown that the PAPR reduction performance of the proposed scheme is better than that of the conventional TR scheme when the tone reservation rate (TRR) is the same.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), peak reduction tone (PRT), peak to average power ratio (PAPR), tone reservation (TR), tone reservation rate (TRR).**

I. Introduction

An orthogonal frequency division multiplexing (OFDM) system has been spotlighted as a standard for the next generation broadcasting and wireless communication systems due to its high data transmission capability in the multi-path fading environment. It was shown that the multiple carrier system shows better performance than the single carrier system in the frequency selective fading channel [1]. However, the OFDM system suffers from high peak to average power ratio (PAPR) of time domain signal obtained by inverse fast Fourier transform (IFFT). If an OFDM signal has high PAPR, it will cause significant signal distortion such as in-band distortion and out-of-band radiation in a nonlinear high power amplifier (HPA).

There are many techniques to reduce PAPR of OFDM signals. Clipping [2], clipping and filtering [3], and companding [4]–[6] schemes modify the time-domain signals but induce signal distortion. Selected mapping (SLM) [7] and partial transmit sequence (PTS) [8] are probabilistic schemes which generate several candidate signals and select the one with the minimum PAPR for transmission. Many techniques for PTS and SLM have been proposed to enhance PAPR reduction performance and reduce the computational complexity [9]–[14]. Tone injection (TI) [15] and active constellation extension (ACE) [16] schemes are to change the original constellation points to reduce PAPR but increase the power of transmit signal.

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D.-W. Lim is with the Department of Information Communication Engineering, Dongguk University, Seoul 100-715, Korea (e-mail: dwlim01@dongguk.edu).

H.-S. Noh, H.-B. Jeon, and J.-S. No are with the Department of Electrical Engineering and Computer Science, Seoul National University, Seoul 151-744, Korea (e-mail: imeanu@ccl.snu.ac.kr; lucidream@ccl.snu.ac.kr; jsno@snu.ac.kr).

D.-J. Shin is with the Department of Electronics and Computer Engineering, Hanyang University, Seoul 133-791, Korea (e-mail: djshin@hanyang.ac.kr).

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Tone reservation (TR) scheme has been proposed to reduce the PAPR in discrete multitone (DMT) systems [15]. TR scheme requires a sacrifice in data transmission efficiency because some of subcarriers in an OFDM symbol should be reserved as peak reduction tones (PRTs) which are used only for reducing PAPR without carrying data. The size of PRT set plays a critical role in TR scheme. To achieve lower PAPR, more subcarriers should be reserved as PRTs which reduces the data transmission efficiency. In this paper, a multi-stage TR scheme is proposed, which can reduce the PAPR of OFDM signal without increasing the tone reservation rate (TRR) [17] which indicates the portion of PRTs in subcarriers.

The rest of this paper is organized as follows. In Section II, OFDM system and TR scheme are reviewed. A multi-stage TR scheme is proposed in Section III and the simulation results are shown in Section IV. Finally, conclusions are given in Section V.

II. OFDM System and Tone Reservation

In an OFDM system, an input data symbol vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]$ in the frequency domain is modulated by N orthogonal subcarriers to generate a discrete time OFDM signal x_k. In other words, a discrete time OFDM signal x_k is obtained by performing IFFT on $\mathbf{X}$ as

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi \frac{n}{N} k}, 0 \le k \le N-1 \quad (1)$$

where N is the number of subcarriers and k is the discrete time index. The PAPR of an OFDM signal $\mathbf{x} = [x_0, x_1, \ldots, x_{N-1}]$ is defined as

$$\text{PAPR}(\mathbf{x}) = \frac{\max\limits_{0 \le k \le N-1} |x_k|^2}{\mathbf{E}\left[\|\mathbf{x}\|_2^2\right]} \quad (2)$$

where $\text{E}[\cdot]$ denotes the expectation operation and $\|\mathbf{x}\|_2$ is the second norm of a vector $\mathbf{x}$.

In order to reduce the PAPR of OFDM signal using TR scheme, some subcarriers are reserved as a PRT set which is used to generate the peak canceling signal. Let $\mathbf{R} = \{i_0, i_1, \ldots, i_{W-1}\}$ denote the ordered set of indices of PRTs and $\mathbf{R}^C$ be the complement set of $\mathbf{R}$ in $\mathbf{N} = \{0, 1, \ldots, N-1\}$, where W is the size of PRT set. Also, the TRR is defined as

$$\text{TRR} = \frac{W}{N} \times 100[\%]. \quad (3)$$

Note that TRR is closely related with the data transmission efficiency (or throughput).

When TR scheme is used, an input symbol A_n in the frequency domain can be written as

$$A_n = X_n + C_n = \begin{cases} C_n, & n \in \mathbf{R} \\ X_n, & n \in \mathbf{R}^C \end{cases} \quad (4)$$

where X_n is an input data symbol and C_n is a symbol assigned to PRT. Here, we assume that $X_n = 0$ for $n \in \mathbf{R}$ and $C_n = 0$ for $n \in \mathbf{R}^{\mathrm{C}}$. Then the discrete time OFDM signal a_k can be rewritten as

$$\begin{aligned} a_k &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} (X_n + C_n) e^{j2\pi \frac{n}{N} k} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi \frac{n}{N} k} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n e^{j2\pi \frac{n}{N} k} \\ &= x_k + c_k \end{aligned} \tag{5}$$

where $0 \le k \le N-1$ and $\mathbf{c} = [c_0, c_1, \ldots, c_{N-1}]$ is called the peak canceling signal. The peak canceling signal $\mathbf{c}$ should be designed to reduce the peak of OFDM signal $\mathbf{x}$ efficiently. Several algorithms have been proposed to construct peak canceling signal [15], [18] and the following iterative algorithm is used in this paper.

Let $\mathbf{p} = [p_0, p_1, \ldots, p_{N-1}]$ be the time domain kernel obtained by performing IFFT on the frequency domain kernel $\mathbf{P} = [P_0, P_1, \ldots, P_{N-1}]$ which has 1's for PRTs and 0's for the remaining tones [19]. The peak canceling signal $\mathbf{c}$ can be iteratively obtained by shifting and scaling the time domain kernel $\mathbf{p}$. The peak canceling signal $\mathbf{c}^l$ at the l-th iteration is obtained as

$$\mathbf{c}^l = \mathbf{c}^{l-1} + \alpha_l \mathbf{P}_{((\tau_l))} = \sum_{i=1}^{l} \alpha_i \mathbf{p}_{((\tau_i))} \tag{6}$$

where $\mathbf{p}_{((\tau_i))}$ denotes a circular shift of $\mathbf{p}$ by τ_i and α_i is a complex valued scaling factor which is computed to reduce the peak value at the l-th iteration to the desired threshold level γ. The circular shift size τ_i and the scaling factor α_i are obtained by

$$\tau_i = \underset{0 \le k \le N-1}{\operatorname{argmax}} \left| x_k + c_k^{i-1} \right| \tag{7}$$

$$\alpha_i = -\left(|x_{\tau_i}| - \gamma \right) e^{j\theta_{\tau_i}} \tag{8}$$

where $\theta_{\tau_i} = \arg(x_{\tau_i})$. Since $\mathbf{p}$ is not a delta function, its sidelobe may cause magnitude regrowth of the samples over the PAPR threshold level. Thus, we choose the PAPR threshold level γ a little bit lower than the target PAPR level for the output OFDM signal. Due to the linear property of Fourier transform, the modification in (6) made on the peak canceling signal $\mathbf{c}^l$ does not distort the data bearing signal $\mathbf{x}$.

The PAPR of an OFDM signal $\mathbf{a} = [a_0, a_1, \ldots, a_{N-1}]$ with TR scheme is redefined [15] as

$$\mathrm{PAPR}(\mathbf{a}) = \frac{\max\limits_{0 \le k \le N-1} |x_k + c_k|^2}{\mathbf{E}\left[\|\mathbf{x}\|_2^2 \right]}. \tag{9}$$

The PAPR reduction performance of TR scheme mainly depends on the size of PRT set, the maximum number of iterations, and the selection of PRTs [19]. Fig. 1 shows the complementary cumulative distribution function (CCDF) of the PAPR for the conventional TR scheme with various TRRs. Here, the PAPR threshold value γ is 6.5 dB, which is 0.5 dB smaller than the target PAPR level 7.0 dB, N is 1024, and 16-QAM is used.

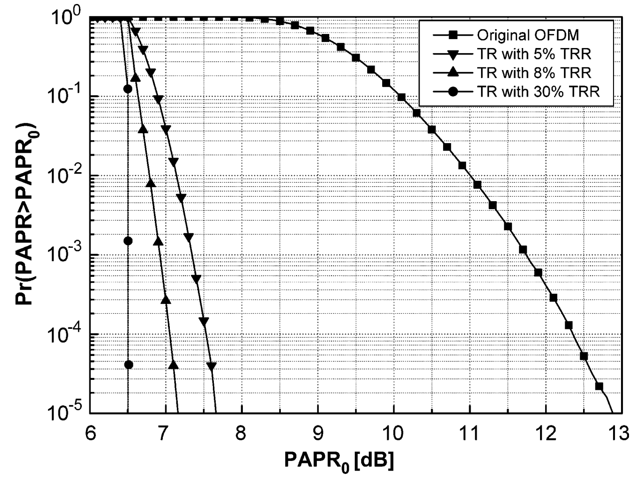

Fig. 1. CCDF of the PAPR for OFDM signals with the conventional TR scheme for various TRRs ($\gamma = 6.5$ dB, $N = 1024$, and 16-QAM is used).

All PRT sets are randomly selected and OFDM signals are four times oversampled to generate an approximate analog signal [19], [20]. The OFDM signal with 30% TRR does not exceed the PAPR level 7.0 dB while the OFDM signals with 5% and 8% TRRs exceed that level with the probabilities of 4% and 0.02%, respectively.

III. Multi-Stage TR Scheme

In this section, we propose a multi-stage TR scheme having better PAPR reduction performance than the conventional TR scheme, without reducing the average transmission efficiency.

A. *Structure of Multi-Stage TR Scheme*

In order to achieve low PAPR of OFDM signal using low TRR, we propose a new multi-stage TR scheme as shown in Fig. 2. The multi-stage TR scheme utilizes the conventional TR schemes in a sequential manner. Since it is straightforward to construct multi-stage TR scheme with more than 2 stages, we will only focus on two-stage TR scheme shown in Fig. 2. The first TR block TR_1 is the conventional TR scheme using PRT_1 and γ_1 as its PRT set and threshold level while the second TR block TR_2 uses PRT_2 and γ_2. In two-stage TR scheme, the peak of an OFDM signal is initially reduced by TR_1 using the threshold level γ_1. After processed by TR_1, the OFDM signal is transmitted if the PAPR of the processed OFDM signal is lower than the target PAPR threshold level γ_2. Otherwise, the OFDM signal should be processed by TR_2 for further reduction of PAPR. For two-stage TR scheme, an additional 1-bit side information should be transmitted to indicate which TR block was used.

For two PRT sets PRT_1 and PRT_2, the ordered sets $\mathbf{R}_1$ and $\mathbf{R}_2$ of indices of PRTs are used, respectively. The size of PRT_2 is larger than that of PRT_1 and $\mathbf{R}_1 \subset \mathbf{R}_2$. Let W_1 and W_2 denote the size of $\mathbf{R}_1$ and $\mathbf{R}_2$, respectively. The frequency domain kernel $\mathbf{P}_m$ is constructed by assigning 1's to the tones in $\mathbf{R}_m$, where $m = 1, 2$. Since W_2 is bigger than W_1, sidelobes of $\mathbf{p}_2$ are much lower than those of $\mathbf{p}_1$ and thus $\mathbf{p}_2$ can reduce PAPR more effectively [19].

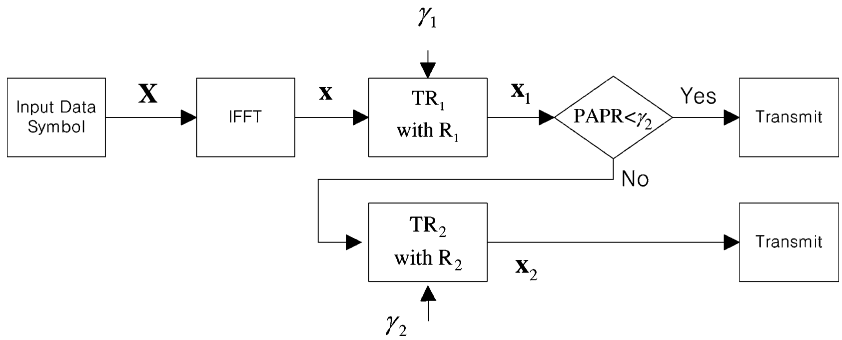

Fig. 2. A block diagram of multi-stage TR scheme (two-stage TR scheme).

B. Average TRR of Multi-Stage TR Scheme

TRR of the conventional TR scheme is already defined in (3). The average TRR ρ_{avg} of two-stage TR scheme is defined as

$$\rho_{avg} = \Pr(\mathrm{PAPR}(\mathbf{x}_1) < \gamma_2) \times \rho_1 + \{1 - \Pr(\mathrm{PAPR}(\mathbf{x}_1) < \gamma_2)\} \times \rho_2 \quad (10)$$

where ρ_1 and ρ_2 are the TRR values of TR_1 and TR_2, respectively, and $\mathrm{PAPR}_{\mathbf{x}_1}$ is the PAPR of the OFDM signal $\mathbf{x}_1$ after TR_1 is applied. Since $\rho_2 > \rho_1$, to minimize ρ_{avg}, it is desirable to select the threshold level γ_1 such that $\Pr(\mathrm{PAPR}_{\mathbf{x}_1} < \gamma_2)$ is quite high (say, higher than 0.9). Then, two-stage TR scheme can reduce the PAPR level of OFDM signals below the target threshold level γ_2 while achieving the average TRR close to ρ_1.

For the general N-stage TR scheme, the average TRR can be similarly defined as

$$\begin{aligned}\rho_{avg} =& \Pr(\mathrm{PAPR}(\mathbf{x}_1) < \gamma_N) \times \rho_1 \\ &+ \sum_{k=2}^{N-1} \prod_{i=1}^{k-1} (1 - \Pr(\mathrm{PAPR}(\mathbf{x}_i) < \gamma_N)) \\ &\times \Pr(\mathrm{PAPR}(\mathbf{x}_k) < \gamma_N) \times \rho_k \\ &+ \prod_{i=1}^{N-1} (1 - \Pr(\mathrm{PAPR}(\mathbf{x}_i) < \gamma_N)) \times \rho_N \end{aligned} \quad (11)$$

where ρ_k is the TRR value at the k-th stage and γ_N is the threshold level in the N-th stage which is also the target threshold level of the N-stage TR scheme.

C. Selection of Threshold Levels

It is important to select γ_1 and γ_2 appropriately to achieve low average TRR as well as good PAPR reduction performance. Usually, we set the threshold level for TR_2 as γ_2 which is also the target PAPR level, and the threshold level for TR_1 as γ_1 which is lower than γ_2. These two threshold values can be empirically determined as follows.

Suppose that the target PAPR is 7.0 dB. Then, set $\gamma_2 = 7.0\,\mathrm{dB}$. Fig. 3 shows the relationship between the threshold level γ and the PAPR_0 satisfying $\Pr(\mathrm{PAPR} > \mathrm{PAPR}_0) = 10^{-2}$ in the conventional TR scheme [21] for $N = 1024$ and 2048 with 16-QAM. The probability 10^{-2} is chosen because it is good enough to make ρ_{avg} close to ρ_1. Fig. 3 shows that the optimal threshold value γ_1 is 7.0 dB, 6.8 dB, and 6.5 dB for 3%, 4%, and 5% TRRs with $N = 1024$ and 6.8 dB, 6.5 dB, and 6.4 dB for 4%, 6%, and 7% TRRs with $N = 2048$, respectively, which are lower than or equal to γ_2.

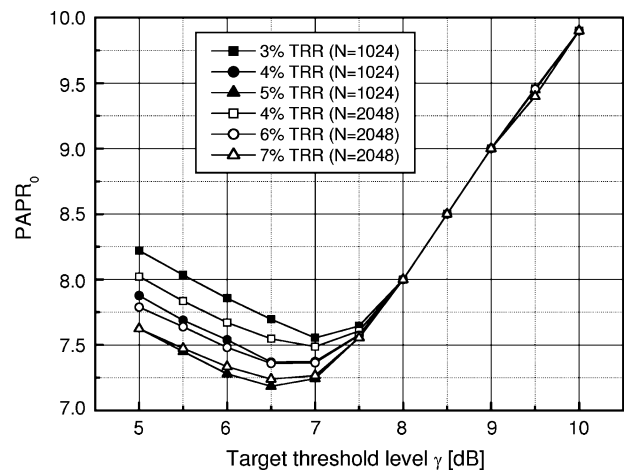

Fig. 3. A relationship between PAPR_0 and the target threshold level of the conventional TR scheme for $N = 1024$ and 2048 with 16-QAM when $\Pr(\mathrm{PAPR} > \mathrm{PAPR}_0) = 10^{-2}$.

D. Data Allocation

Since different PRT sets are used for two TR blocks, the number of data subcarriers in OFDM symbol is $N - W_1$ or $N - W_2$ if no guardbands and pilot symbols are considered. If the PAPR of the OFDM signal $\mathbf{x}_1$ in TR_1 satisfies the target PAPR γ_2, then the data transmission efficiency is $1 - \rho_1$. If $\mathbf{x}_1$ does not satisfy the target PAPR, it should go through TR_2 and the data transmission efficiency becomes $1 - \rho_2$ which is lower than $1 - \rho_1$. Once the OFDM signal $\mathbf{x}_1$ is passed to TR_2, the input data symbols already assigned to the set $\mathbf{E} = \mathbf{R}_2 \backslash \mathbf{R}_1$ should be overwritten by the peak canceling signal generated by PRT_2 and those input data symbols in $\mathbf{E}$ should be transmitted in the next OFDM symbol. Therefore, for two-stage TR scheme, the first $N - W_2$ input data symbols should be allocated to $\mathbf{R}_2^{\mathcal{C}} = \mathbf{N} \backslash \mathbf{R}_2$ and the next $|\mathbf{E}| = W_2 - W_1$ input data symbols should be allocated to $\mathbf{E}$. As an example, the input data allocation for two-stage TR scheme with $N = 16$, $W_1 = 2$, and $W_2 = 6$ is given in Fig. 4, where $\mathbf{R}_1 = \{0, 7\}$, $\mathbf{R}_2 = \{0, 1, 5, 7, 10, 13\}$, and $\mathbf{E} = \{1, 5, 10, 13\}$. The first 10 input data symbols from $X[0]$ to $X[9]$ are assigned to $\mathbf{R}_2^{\mathcal{C}}$ and the next 4 input data symbols from $X[10]$ to $X[13]$ are assigned to $\mathbf{E}$. If TR_2 is activated, then the input data symbols assigned

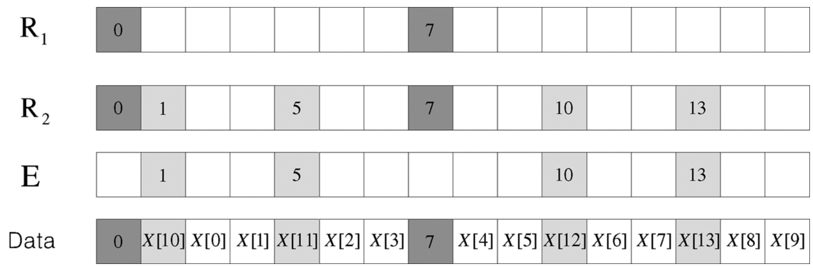

Fig. 4. Input data allocation for two-stage TR scheme.

to $\mathbf{E}$ are overwritten by the peak canceling signal generated by PRT_2 and those input data symbols in $\mathbf{E}$ should be transmitted in the next OFDM symbol.

E. Signal Selection in the Second TR Block

The OFDM signal $\mathbf{x}_1$ from TR_1 is directly passed to TR_2 if $\mathbf{x}_1$ does not satisfy the target PAPR, where the peak canceling signal generated by PRT_2 is applied to $\mathbf{x}_1$. Clearly, the input data symbols in $\mathbf{E}$ are overwritten by the peak canceling signal. This method takes advantage of the fact that TR_1 already reduces the peak values of its input OFDM signal and does not require an additional IFFT operation. Since the threshold level γ_1 in TR_1 is lower than γ_2 in TR_2, the magnitude of most samples in $\mathbf{x}_1$ are lower than γ_2 and generally a few iterations are required in TR_2 to achieve the target PAPR. Since we do not remove the input data symbols in $\mathbf{E}$, total transmit power may increase a little bit, but it is negligible.

IV. Simulation Results

Numerical analysis is performed for the OFDM systems for $N = 1024$ and 2048 respectively when 16-QAM is used, where no guardbands and pilot symbols are considered. The target PAPR level is 7 dB and all OFDM signals are four times oversampled. Table I depicts TRRs of two-stage TR scheme and conventional TR scheme for $N = 1024$ where 5%, 8%, and 10% of conventional TR scheme correspond to its PRT sets with $|\mathbf{R}| = 51$, 81, and 102, respectively. For the various two-stage TR schemes, $|\mathbf{R}_1| = 31$, 41, 51, and $|\mathbf{R}_2| = 306$ (which are referred as (3% 30%), (4% 30%), (5% 30%) TRRs, respectively), and the threshold levels $\gamma_1 = 7.0$ dB, 6.8 dB, 6.5 dB, respectively, and $\gamma_2 = 7.0$ dB are used. Table II also lists TRRs for $N = 2048$ where 8%, 10%, and 12% of conventional TR scheme correspond to its PRT sets with $|\mathbf{R}_1| = 163$, 204, 246, respectively. (4% 20%), (6% 20%), and (7% 20%) stand for two-stage TR schemes with $|\mathbf{R}_1| = 82$, 122, and 143, respectively and $|\mathbf{R}_2| = 410$. Threshold levels for two-stage TR scheme are $\gamma_1 = 6.8$ dB, 6.5 dB, 6.4 dB, respectively, and $\gamma_2 = 7.0$ dB. The maximum number of iterations in each TR block is set to 40 and 100000 OFDM symbols are generated. Table I and Fig. 5 show that two-stage TR scheme with (5% 30%) TRR has better performance than those with (3% 30%) and (4% 30%) TRRs in terms of the data transmission efficiency, the average number of iterations, and PAPR reduction because two-stage TR schemes with (3% 30%) and (4% 30%) TRRs pass more OFDM signals to TR_2 and, therefore, need more iterations in TR_2. Among 100000 OFDM symbols, 95984 OFDM symbols (which is about 96%) for two-stage TR scheme with (5% 30%) TRR satisfy the target PAPR 7.0 dB only by using TR_1 and the remaining 4016 OFDM symbols (which is about 4%) go through TR_2. Fig. 5 shows that PAPR of the OFDM signals with two-stage TR scheme with (5% 30%) TRR does not exceed 7.0 dB at the probability of 10^{-5} while the OFDM signal

TABLE I
Data Transmission Efficiency and Average Number of Iterations of Two-Stage TR Scheme and Conventional TR Scheme With $N = 1024$ and 16-QAM

	Two-stage TR scheme			Conventional TR scheme		
TRR	(3%, 30%)	(4%, 30%)	(5%, 30%)	5%	8%	10%
Success probability at 1st stage	0.01	0.82	0.96	-	-	-
Data transmission efficiency	0.703	0.916	0.940	0.95	0.92	0.90
Average number of iterations	76.0	43.8	40.7	40.0	40.0	39.9

TABLE II
Data Transmission Efficiency and Average Number of Iterations of Two-Stage TR Scheme and Conventional TR Scheme With $N = 2048$ and 16-QAM

	Two-stage TR scheme			Conventional TR scheme		
TRR	(4%, 20%)	(6%, 20%)	(7%, 20%)	8%	10%	12%
Success probability at 1st stage	0.36	0.83	0.93	-	-	-
Data transmission efficiency	0.857	0.916	0.922	0.92	0.90	0.88
Average number of iterations	58.45	43.26	41.0	40.0	40.0	40.0

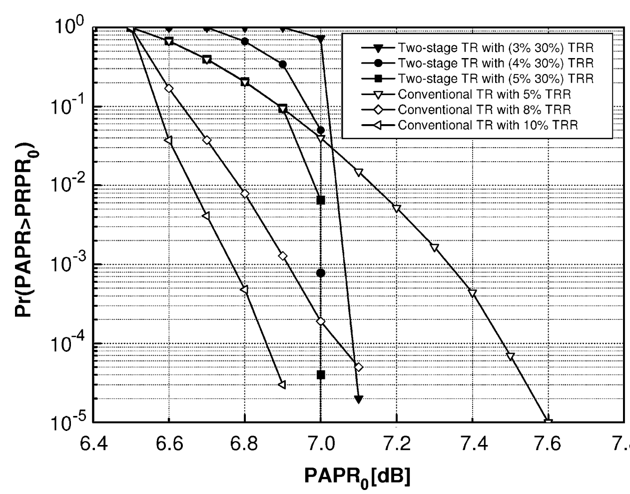

Fig. 5. PAPR reduction performance of the conventional TR scheme with 5%, 8%, and 10% TRRs and the two-stage TR scheme with (3%, 30%), (4%, 30%), and (5%, 30%) TRRs when $N = 1024$ and 16-QAM are used.

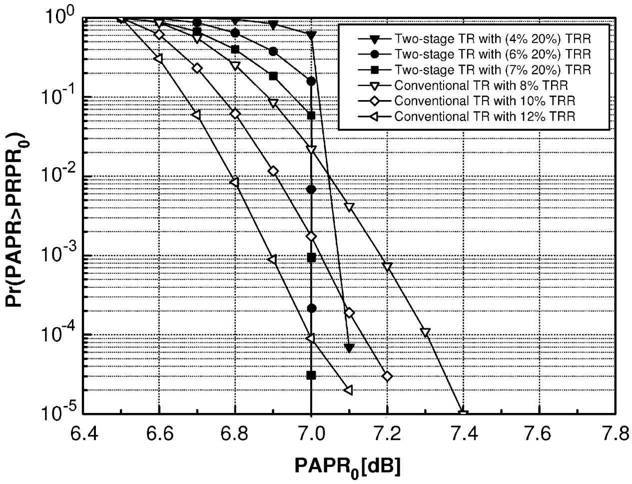

Fig. 6. PAPR reduction performance of the conventional TR scheme with 8%, 10%, and 12% TRRs and the two-stage TR scheme with (4%, 20%), (6%, 20%), and (7%, 20%) TRRs when $N = 2048$ and 16-QAM are used.

with the conventional TR schemes with 5% and 8% TRRs can have PAPR more than 7.0 dB at the same probability. The data transmission efficiency $1 - \rho_{avg}$ is 0.95 for the conventional TR scheme with 5% TRR and 0.94 for two-stage TR scheme with (5% 30%) TRR. But the PAPR reduction performance of two-stage TR scheme with (5% 30%) TRR is much better than that of the conventional TR scheme with 5% TRR. Similarly, Fig. 6 shows that two-stage TR scheme with (7% 20%) TRR has better PAPR reduction performance than the conventional TR schemes with 8%, 10%, and 12% TRRs. Also, the data transmission efficiency of two-stage TR scheme is better than that of the conventional TR schemes.

V. Conclusions

In this paper, a multi-stage TR scheme has been proposed, which adaptively uses TR schemes of different PRT sets according to the PAPR level of the OFDM signals. The main idea to construct two-stage TR scheme is to select proper threshold level in the first TR block to minimize the probability of activation of the second TR block to increase the data transmission efficiency without degrading the PAPR reduction performance.

The simulation results confirm that two-stage TR scheme significantly improves PAPR reduction performance while keeping its TRR the same as the conventional TR scheme. Also, the configuration for two-stage TR scheme can be easily extended to the multi-stage TR scheme with multiple TR blocks under the same system requirements.

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