Bit-Based SLM Schemes for PAPR Reduction in QAM Modulated OFDM Signals

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***Abstract*—In this paper, we propose two bit-based selected mapping (SLM) schemes for reducing peak to average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals with quadrature amplitude modulation (QAM), called bitwise SLM (BSLM) and partial bit inverted SLM (PBISLM). Contrary to the conventional SLM which rotates the phases of QAM symbols in the frequency domain, the proposed schemes change the magnitudes as well as the phases of QAM symbols by applying binary phase sequences to the binary data sequence before mapped to QAM symbols. Simulation results show that the proposed schemes have better PAPR reduction performance with shaping gain than the conventional SLM scheme for the QAM modulated OFDM signals, especially for the small number of subcarriers.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), peak to average power ratio (PAPR), quadrature amplitude modulation (QAM), selected mapping (SLM), shaping gain.**

I. Introduction

SINCE orthogonal frequency division multiplexing (OFDM) can support high data rate and provide high reliability in voice, data, and multimedia communications, it has been adopted as a standard technique in many wireless communication systems. One of main advantages of OFDM is the robustness against frequency selective fading or narrowband interference. However, a critical drawback of OFDM is high peak to average power ratio (PAPR) which results in significant inter-modulation and undesirable out-of-band radiation when an OFDM signal passes through nonlinear devices such as high power amplifier (HPA) [1].

Several techniques have been proposed to mitigate the PAPR of OFDM signals. Clipping is used to reduce the peak power by clipping the OFDM signals to the threshold level [2] but it causes inband distortion and out-of-band radiation. Companding schemes scale the time-domain signals nonlinearly such that the signals with large amplitude are suppressed and the signals with small amplitude are expanded, which also distorts the signals unavoidably [3]–[5]. Tone reservation (TR), tone injection (TI) [6], and active constellation extension (ACE) [7] utilize some subcarriers only to reduce PAPR. The drawbacks of these methods are data rate loss or transmission power increment. Selected mapping (SLM) [8] and partial transmit sequence (PTS) [9] select the signal with the minimum PAPR among several candidate signals generated by multiplying phase sequences to the data sequence before or after inverse fast Fourier transform (IFFT). To overcome the high computational complexity that these schemes accompany inherently, several SLM and PTS schemes have been proposed to improve the PAPR reduction performance and reduce the computational complexity [10]–[13]. Also, some SLM schemes such as bit interleaving or scrambling [14], [15] modify binary data sequence before applying quadrature amplitude modulation (QAM) to generate alternative symbol sequences. In these cases, alternative symbol sequences undergo the change of both amplitude (or power) and phase.

In this paper, we reconsider the conditions on the phase sequences for good SLM scheme and analyze the relation between the independency of alternative signal sequences and the covariance of the average symbol powers of them. Based on these results, we propose two bit-based SLM schemes which vary not only the phase but also the power of data symbols as follows. Bitwise SLM (BSLM) generates the alternative symbol sequences through multiplying a binary data sequence by binary phase sequences with the same length as that of the binary data sequence. Partial bit inverted SLM (PBISLM) generates the alternative symbol sequences by inverting data bits at predetermined bit positions or not according to the binary phase sequences. The PAPR reduction performance of the proposed SLM schemes is better than that of the conventional SLM scheme for the QAM modulated OFDM signals, especially for the small number of subcarriers. Moreover, they can reduce the average transmission power compared to the conventional SLM, which is called the shaping gain.

The rest of this paper is organized as follows. In Section II, OFDM system and conventional SLM scheme are reviewed. The conditions on the phase sequences for good SLM scheme are derived in Section III and two bit-based SLM schemes are proposed in Section IV. The numerical analysis is provided in Section V and the conclusions are given in Section VI.

II. Conventional SLM Scheme

An OFDM signal sequence $\mathbf{a} = [a_0 a_1 \cdots a_{N-1}]$ using N subcarriers can be expressed as

$$a_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j2\pi \frac{k}{N} n}, \quad 0 \le n < N \tag{1}$$

where $\mathbf{A} = [A_0 A_1 \cdots A_{N-1}]$ is an input symbol sequence usually modulated by using phase shift keying (PSK) or QAM and n

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stands for a discrete time index. The PAPR of the OFDM signal sequence $\mathbf{a}$ in the discrete time domain can be defined as

$$\mathrm{PAPR}(\mathbf{a}) \doteq \frac{\max_{0\le n<N} |a_n|^2}{\mathrm{E}\left[|a_n|^2\right]} \tag{2}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator.

In the conventional SLM scheme, a transmitter generates U distinct alternative symbol sequences, all representing the same input symbol sequence, and selects the one with the minimum PAPR for transmission. To generate U alternative symbol sequences, an input symbol sequence $\mathbf{A}$ is multiplied by U different phase sequences of length N, $\mathbf{P}^{(u)} = [P_0^{(u)} P_1^{(u)} \cdots P_{N-1}^{(u)}]$, $0 \le u < U$, $P_i^{(u)} = e^{j\phi_i^{(u)}}$. The first phase sequence $\mathbf{P}^{(0)}$ is usually the all-1 sequence. Then, the alternative symbol sequences $\mathbf{A}^{(u)} = [A_0^{(u)} A_1^{(u)} \cdots A_{N-1}^{(u)}]$, $0 \le u < U$, are generated by $A_i^{(u)} = A_i P_i^{(u)}$. After U alternative symbol sequences are transformed by IFFT, the alternative OFDM signal sequence $\mathbf{a}^{(\tilde{u})} = \mathrm{IFFT}(\mathbf{A}^{(\tilde{u})})$ with the smallest PAPR is selected for transmission.

If we assume that the alternative OFDM signal sequences $\mathbf{a}^{(u)} = [a_0^{(u)} a_1^{(u)} \cdots a_{N-1}^{(u)}]$, $0 \le u < U$, are mutually independent, the complementary cumulative distribution function (CCDF) of PAPR for the SLM scheme can be given [9] as

$$\Pr\left(\mathrm{PAPR}\left(\mathbf{a}^{(\tilde{u})}\right) > \mathrm{PAPR}_0\right) = \left(1 - (1 - e^{-\mathrm{PAPR}_0})^N\right)^U. \tag{3}$$

There are some design criteria for good phase sequences of the conventional SLM scheme. In [16], two criteria are suggested, one is the orthogonality between phase sequences and the other is the aperiodicity of the phase sequences. It is shown in [17] that the alternative OFDM signal sequences are asymptotically mutually independent if the phases of symbols in each phase sequence are independent and identically distributed (i.i.d.) with zero expectation value $E[e^{j\phi_i}] = 0$, and the SLM scheme satisfying this condition can have the optimal PAPR reduction performance.

III. Conditions for Mutually Independent OFDM Signals

OFDM signal sequences obtained after IFFT can be assumed to be complex Gaussian distributed for large N by the central limit theorem. Thus zero covariance of two alternative OFDM signals implies that they are mutually independent [17]. However, this assumption does not hold for small N because OFDM signal sequence for small N may not be approximated as complex Gaussian random vector. It is known that the CCDF of PAPR for the conventional SLM scheme and the theoretical CCDF are almost identical for QPSK, but they become different for 16-QAM even though the phase sequences satisfy the optimal conditions in [16], [16]. Also, this difference becomes larger as U increases or N decreases. These observations lead us to consider new SLM schemes for OFDM symbol with non-constant envelope modulation.

If OFDM signal sequences are not complex Gaussian distributed, zero covariance of two alternative OFDM signals does not guarantee the mutual independency between them. In this case, instead of covariance, we consider the property of joint cumulants of alternative OFDM signals such that two alternative OFDM signal sequences are mutually independent if the joint cumulants of all orders are equal to zero [19]. Since it is not easy to calculate high order joint cumulants, we will only consider joint cumulants up to the fourth order to investigate the independency of alternative OFDM signal sequences. In general, the kth symbol of the uth alternative symbol sequence $\mathbf{X}^{(u)}$ can be expressed as

$$X_k^{(u)} = A_k \Lambda_k^{(u)} e^{j\phi_k^{(u)}}, \quad 0 \le k < N \quad \text{and} \quad 0 \le u < U \tag{4}$$

where $\Lambda_k^{(u)}$ and $\phi_k^{(u)}$ are the amplitude gain and phase rotation of the kth symbol in the uth alternative symbol sequence, respectively. Note that in the conventional SLM, $\Lambda_k^{(u)} = 1$ for all k and u. Then the uth alternative OFDM signal sequence is given as

$$x_n^{(u)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k \Lambda_k^{(u)} e^{j\phi_k^{(u)}} e^{j2\pi \frac{k}{N} n}, \quad 0 \le n < N. \tag{5}$$

It can be easily shown that the second and third order joint cumulants are zero regardless of Λ_k if $\phi_k^{(u)}$ is i.i.d. with $E[e^{j\phi_k^{(u)}}] = (1/N)\sum_{k=0}^{N-1} e^{j\phi_k^{(u)}} = 0$ for $u = 0, 1, \ldots, U-1$. Thus we will propose new SLM schemes which make the fourth order joint cumulants of any pair of alternative OFDM signals close to zero. Through numerical analysis, it will be shown that the PAPR reduction performance improves as the fourth order joint cumulant between alternative OFDM signal sequences decreases.

If the phase sequences satisfy the i.i.d. and mean zero conditions for the phase $\phi_k^{(u)}$, the fourth order joint cumulant of two alternative OFDM signal sequences can be given as

$$\begin{aligned}
&\mathrm{cum}\left(x_n^{(l)}, x_n^{(l)*}, x_n^{(m)}, x_n^{(m)*}\right) \\
&= E\left[x_n^{(l)} x_n^{(l)*} x_n^{(m)} x_n^{(m)*}\right] - E\left[x_n^{(l)} x_n^{(l)*}\right] E\left[x_n^{(m)} x_n^{(m)*}\right] \\
&= \frac{1}{N^2}\left\{E\left[\left(\sum_{k=0}^{N-1}\left|A_k\Lambda_k^{(l)}\right|^2\right)\left(\sum_{k=0}^{N-1}\left|A_k\Lambda_k^{(m)}\right|^2\right)\right]\right. \\
&\qquad \left. - E\left[\sum_{k=0}^{N-1}\left|A_k\Lambda_k^{(l)}\right|^2\right] E\left[\sum_{k=0}^{N-1}\left|A_k\Lambda_k^{(m)}\right|^2\right]\right\}.
\end{aligned} \tag{6}$$

It can be easily shown that the fourth order joint cumulant in (6) is equivalent to the covariance of average symbol powers of alternative symbol sequences given by

$$\begin{aligned}
\mathrm{cov}\left(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)}\right) &= E\left[\left(\bar{\mathrm{P}}^{(l)} - E\left[\bar{\mathrm{P}}^{(l)}\right]\right)\left(\bar{\mathrm{P}}^{(m)} - E\left[\bar{\mathrm{P}}^{(m)}\right]\right)\right] \\
&= E\left[\bar{\mathrm{P}}^{(l)}\bar{\mathrm{P}}^{(m)}\right] - E\left[\bar{\mathrm{P}}^{(l)}\right] E\left[\bar{\mathrm{P}}^{(m)}\right]
\end{aligned} \tag{7}$$

where

$$\bar{\mathrm{P}}^{(u)} = \frac{1}{N}\sum_{k=0}^{N-1}\left|A_k\Lambda_k^{(u)}\right|^2. \tag{8}$$

If $E[\bar{\mathrm{P}}^{(u)}]$ is normalized to one, (7) can be rewritten as

$$\begin{aligned}
&\mathrm{cov}\left(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)}\right) \quad (9)\\
&= E\left[\bar{\mathrm{P}}^{(l)}\bar{\mathrm{P}}^{(m)}\right] - E\left[\bar{\mathrm{P}}^{(l)}\right]E\left[\bar{\mathrm{P}}^{(m)}\right]\\
&= \frac{1}{N^2}\left\{E\left[N|A_k|^4\left|\Lambda_k^{(l)}\right|^2\left|\Lambda_k^{(m)}\right|^2\right]\right.\\
&\qquad \left.+E\left[N(N-1)\left|A_i\Lambda_i^{(l)}\right|^2\left|A_k\Lambda_k^{(m)}\right|^2\right] - N^2\right\}\\
&= \frac{1}{N^2}\left(NE\left[|A_k|^4\left|\Lambda_k^{(l)}\right|^2\left|\Lambda_k^{(m)}\right|^2\right] + N(N-1) - N^2\right)\\
&= \frac{1}{N}\left(E\left[|A_k|^4\left|\Lambda_k^{(l)}\right|^2\left|\Lambda_k^{(m)}\right|^2\right] - 1\right). \quad (10)
\end{aligned}$$

For simplicity, we assume that the average symbol power of input symbol sequence $\mathbf{A}$ modulated with M-QAM is normalized to one hereafter.

If $\mathbf{X}^{(l)}$ and $\mathbf{X}^{(m)}$ are mutually independent, that is, two alternative symbol sequences are generated independently for a given modulation, the covariance of average symbol powers of them is zero because (9) becomes zero. However, the covariance is not zero in the conventional SLM even if the phase sequences satisfy the optimality conditions because $E[|A_k|^4|]$ in (10) is not one, which means that mutually independent alternative OFDM signal sequences cannot be generated by using the conventional SLM scheme for QAM modulation. Therefore, we have to design the phase sequences which change the amplitude gain $\Lambda_k^{(u)}$ to make (9) close to zero and equivalently, the fourth order joint cumulant in (6) close to zero.

IV. Bit-Based SLM Schemes

In this section, two new bit-based SLM schemes are proposed, which change the magnitudes as well as the phases of the symbols of input symbol sequence. Also, it is shown that the covariance of average symbol powers of alternative symbol sequences in the frequency domain can be made close to zero by the proposed schemes.

A. Bitwise SLM Scheme

An input symbol sequence $\mathbf{A}$ of length N with M-QAM can be expressed as the following binary sequence of length $N\log_2 M$

$$\mathbf{A}_B = [A_{0,0}\ldots A_{0,\log_2 M-1}\cdots A_{N-1,0}\ldots A_{N-1,\log_2 M-1}] \quad (11)$$

where $A_{k,l} \in \{\pm 1\}$ means the lth bit of the kth M-QAM symbol. If a phase sequence $\mathbf{P}$ is a binary sequence composed of $\{\pm 1\}$ with length $N\log_2 M$, the alternative binary sequence is generated by multiplying the input symbol sequence in the binary form by the binary phase sequence before mapped to M-QAM symbols as

$$\mathbf{X}_B^{(u)} = \Big[A_{0,0}P_0^{(u)}\ldots A_{0,\log_2 M-1}P_{\log_2 M-1}^{(u)}\cdots A_{N-1,\log_2 M-1}P_{N\log_2 M-1}^{(u)}\Big]. \quad (12)$$

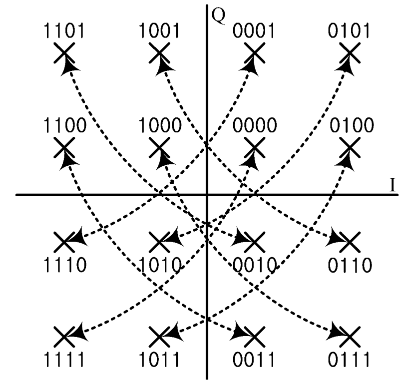

Fig. 1. An example of partial bit inversion of Gray mapped 16-QAM constellation for PBISLM.

$\mathbf{X}_B^{(u)}$'s are mapped to M-QAM symbols to generate the alternative symbol sequences $\mathbf{X}^{(u)}$, $\mathbf{X}^{(u)}$'s are IFFTed, and the OFDM signal sequence $\mathbf{x}^{(\tilde{u})} = \mathrm{IFFT}(\mathbf{X}^{(\tilde{u})})$ with the minimum PAPR is selected for transmission, where $\tilde{u} = \arg\min_{0\le u<U}\mathrm{PAPR}(\mathbf{x}^{(u)})$. This SLM scheme is called bitwise SLM.

$X_k^{(u)}$ is selected from M-QAM symbols according to $\{P_{k\log_2 \mathrm{M}}^{(u)}\cdots P_{(k+1)\log_2 \mathrm{M}-1}^{(u)}\}$. If the binary phase sequences are randomly generated, the probability that $X_k^{(u)}$ is any M-QAM symbol is $1/M$, which is the same as the case of independently generated symbol sequences. Thus, we can expect that the covariance of average symbol powers of two alternative symbol sequences in the BSLM is zero.

B. Partial Bit Inverted SLM Scheme

In this subsection, we propose another new SLM scheme called PBISLM, where the alternative symbol sequences are generated by multiplying some preselected bits of each M-QAM symbol $A_k^{(u)}$ by $P_k^{(u)}$ in the binary phase sequence $\mathbf{P}^{(u)} = [P_0^{(u)}P_1^{(u)}\cdots P_{N-1}^{(u)}]$, $0 \le u < U$, $P_k^{(u)} \in \{\pm 1\}$. Let $\mathbf{S} = \{i_0, i_1, \ldots, i_{W-1}\}$ denote a subset of bit indices $\mathbf{L} = \{0, 1, \ldots, \log_2 M - 1\}$ for M-QAM symbol and $\mathbf{S}^{\mathrm{C}}$ be the complement set of $\mathbf{S}$ in $\mathbf{L}$. The lth bit $X_{k,l}^{(u)}$ of the kth symbol in the binary form of the uth alternative symbol sequence can be written as

$$X_{k,l}^{(u)} = \begin{cases} A_{k,l}P_k^{(u)}, & l \in \mathbf{S}\\ A_{k,l}, & l \in \mathbf{S}^{\mathrm{C}}. \end{cases} \quad (13)$$

If $P_k^{(u)}$ is -1, the bits of A_k corresponding to $\mathbf{S}$ are inverted and thus $A_k^{(u)}$ is mapped to other M-QAM symbol $X_k^{(u)}$. After the alternative symbol sequences $\mathbf{X}^{(u)}$ are IFFTed, the OFDM signal sequence $\mathbf{x}^{(\tilde{u})} = \mathrm{IFFT}(\mathbf{X}^{(\tilde{u})})$ with the minimum PAPR is selected for transmission.

In the PBISLM scheme, the average power $\bar{\mathrm{P}}^{(u)}$ of $\mathbf{X}^{(u)}$ is different from the average power of $\mathbf{A}$ if M-QAM is used, depending on the selection of the set $\mathbf{S}$ for the given constellation mapping. From (10), we have to make $E[|X_k^{(l)}|^2|X_k^{(m)}|^2]$ as close to one as possible to have very small covariance $\mathrm{cov}(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)})$. For some M-QAM symbol mappings, we analyze the covariance of average symbol powers of alternative symbol sequences in (10) in PBISLM as follows. Fig. 1 shows a Gray mapping for 16-QAM. If we use $\mathbf{S} = \{0, 1, 2, 3\}$ and $\mathbf{S}^{\mathrm{C}} = \emptyset$, all bits for the kth input symbol are inverted when $P_k^{(u)} = -1$ as in Fig. 1. Assuming that $E[|A_k|^2] = 1$, the input symbols A_k are classified into three subsets E_1, E_2, and E_3 according to their powers such that $E_1 = \{0000, 1000, 1010, 0010\}$ with symbol power $P_1 = 0.2$, $E_2 = \{0100, 0001, 1001, 1100, 1110, 1011, 0011, 0110\}$ with symbol power $P_2 = 1.0$, and $E_3 = \{0101, 1101, 1111, 0111\}$ with symbol power $P_3 = 1.8$. Then the amplitude gain of the symbol in alternative symbol sequence generated by PBISLM is

$$\Lambda_k^{(u)} = \begin{cases} 1, & A_k \in E_2 \text{ or } P_k^{(u)} = 1 \\ \sqrt{P_1/P_3}, & A_k \in E_3 \text{ and } P_k^{(u)} = -1 \\ \sqrt{P_3/P_1}, & A_k \in E_1 \text{ and } P_k^{(u)} = -1. \end{cases} \tag{14}$$

If the phase sequences $\mathbf{P}^{(u)}$ are randomly generated and balanced in terms of the number of $+1$'s and -1's, $E[e^{j\phi_k^{(u)}}]$ is also zero. Then the covariance of average symbol powers of two alternative symbol sequences in the PBISLM is calculated as

$$\begin{aligned} \mathrm{cov}\left(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)}\right) &= \frac{1}{N}\left(E\left[|A_k|^4 \left|\Lambda_k^{(l)}\right|^2 \left|\Lambda_k^{(m)}\right|^2\right] - 1\right) \\ &= \frac{1}{N}\left\{\frac{1}{2}\left(\frac{1}{4}(P_1)^2 \frac{P_3}{P_1} + \frac{1}{2} + \frac{1}{4}(P_3)^2 \frac{P_1}{P_3}\right)\right. \\ &\qquad \left. + \frac{1}{2}\left(\frac{1}{4}(P_1)^2 + \frac{1}{2} + \frac{1}{4}(P_3)^2\right) - 1\right\} \\ &= 0. \end{aligned} \tag{15}$$

Therefore, the average symbol powers of alternative symbol sequences in the PBISLM are uncorrelated as the case of independently generated symbol sequences.

Now, we consider PBISLM for the Gray mapped 64-QAM constellation given in Fig. 2. In order to compare the covariance of average symbol powers of two alternative symbol sequences, we introduce two types of PBISLM according to the selection of $\mathbf{S}$. For Type-I, $\mathbf{S} = \{0, 2, 3, 5\}$ and $\mathbf{S}^{\mathrm{C}} = \{1, 4\}$, and for Type-II, $\mathbf{S} = \{0, 1, 2, 3, 4, 5\}$ and $\mathbf{S}^{\mathrm{C}} = \emptyset$. The covariance of average symbol powers of two alternative symbol sequences in Type-I is smaller than that in Type-II, e.g., 2.83×10^{-4} for Type-I and 1.13×10^{-3} for Type-II when $N = 64$, which are calculated similarly to (15). The PAPR reduction performance of these two types of PBISLM is compared through numerical analysis in the next section.

The covariance of average symbol powers of two alternative symbol sequences can be made closer to zero by using different $\mathbf{S}$ for each constellation point, but this does not guarantee $E[e^{j\phi_k^{(u)}}] = 0$ and the distance property of Gray mapping at the receiver. In general, $\mathbf{S}$ is selected such that the constellation points with the smallest power in each quadrant are mapped to the constellation points with the largest power in the opposite quadrant by bit inverting as Fig. 1 of 16-QAM and Type-I of 64-QAM. Then, $E[|A_k|^4|\Lambda_k^{(l)}|^2|\Lambda_k^{(m)}|^2]$ in (10) can be made closest to one while keeping $E[e^{j\phi_k^{(u)}}] = 0$ and the distance property of Gray mapping at the receiver.

To generalize (9) for PBISLM with M-QAM, we only consider the constellation points $(i, j), 1 \le i \le K = \sqrt{M}/2, 1 \le j \le i$, in the lower triangle part of the first quadrant in M-QAM constellation to evaluate the symbol power. Input symbols A_k from M-QAM constellation with equal probability are classified into R subsets $E_1, \ldots, E_R$ according to their powers where the total number of groups R is computed as

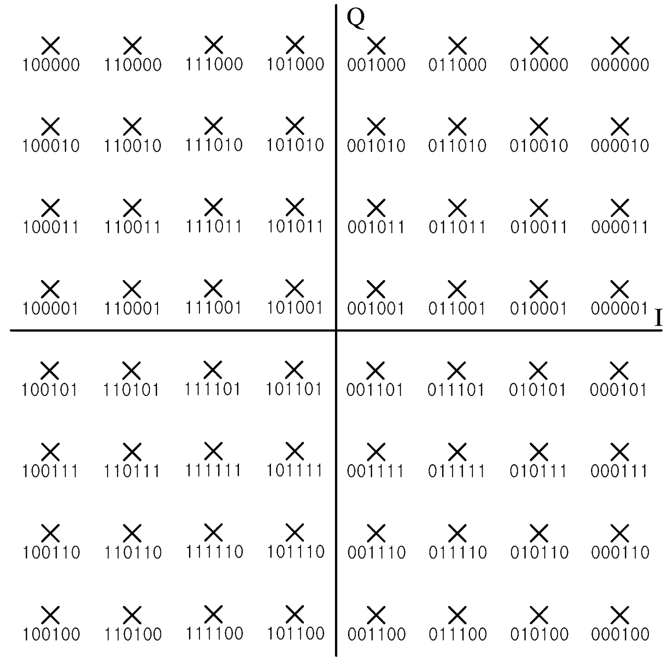

Fig. 2. An example of Gray mapped 64-QAM constellation.

$$R = 1 + 2 + \cdots + K = \frac{K(K+1)}{2}. \tag{16}$$

The subset index $r \in \{1, 2, \ldots, R\}$ in E_r can be replaced with $(i(i-1)/2) + j$, where (i, j) is the constellation point in E_r. Then, the symbol power $P(r)$ for E_r is given as

$$P(r) = P\left(\frac{i(i-1)}{2} + j\right) = \left(\left(i - \frac{1}{2}\right)d\right)^2 + \left(\left(j - \frac{1}{2}\right)d\right)^2 \tag{17}$$

where d is the distance between two closest constellation points. The probability that a symbol is in E_r is

$$p(r) = p\left(\frac{i(i-1)}{2} + j\right) = \begin{cases} \frac{1}{K^2}, & i = j \\ \frac{2}{K^2} & i \neq j. \end{cases} \tag{18}$$

For an alternative symbol sequence generated by PBISLM, the amplitude gain of the symbol is

$$\Lambda_k^{(u)} = \begin{cases} 1, & P_k^{(u)} = 1 \\ \sqrt{g(r)}, & A_k \in E_r \text{ and } P_k^{(u)} = -1 \end{cases} \tag{19}$$

where the power gain $g(r)$ is

$$\begin{aligned} g(r) &= g\left(\frac{i(i-1)}{2} + j\right) \\ &= \frac{\left(K - i + \frac{1}{2}\right)^2 + \left(K - j + \frac{1}{2}\right)^2}{\left(i - \frac{1}{2}\right)^2 + \left(j - \frac{1}{2}\right)^2}. \end{aligned} \tag{20}$$

If the phase sequences $\mathbf{P}^{(u)}$ are orthogonal to each other and balanced in terms of the number of $+1$'s and -1's, the covari-

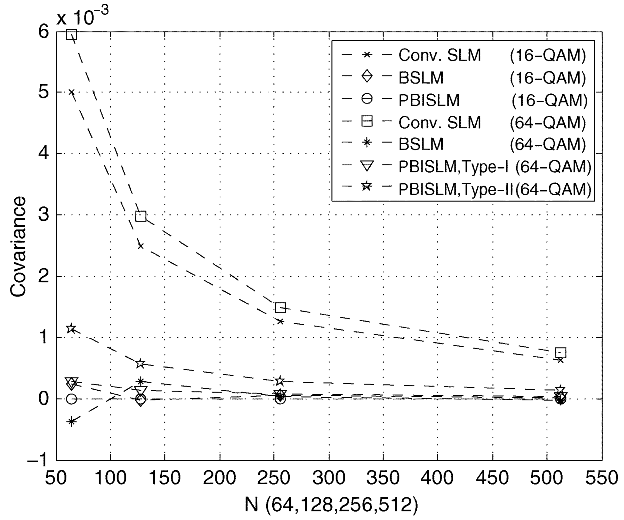

Fig. 3. Comparison of the covariance of average symbol powers of two alternative symbol sequences for various SLM schemes.

ance of average symbol powers of two alternative symbol sequences is given as

$$\begin{aligned}\mathrm{cov}\left(\bar{\mathrm{P}}^{(l)}, \bar{\mathrm{P}}^{(m)}\right) &= \frac{1}{N}\left(E\left[|A_k|^4\left|\Lambda_k^{(l)}\right|^2\left|\Lambda_k^{(m)}\right|^2\right] - 1\right)\\ &= \frac{1}{N}\left(\frac{\sum_{r=1}^{R} N(r)^2 p(r)\left(g(r)+1\right)}{2} - 1\right)\end{aligned} \tag{21}$$

where the normalized symbol power $N(r)$ is

$$N(r) = \frac{P(r)}{\sum_{r=1}^{R} p(r)P(r)}. \tag{22}$$

The alternative symbol sequences are generated by multiplying an input symbol sequence with a phase sequence in the conventional SLM. But in practice, this multiplication is performed by changing the constellation points of an input symbol sequence to another constellation points according to a phase sequence. BSLM and PBISLM also change the constellation points of an input symbol sequence to different constellation points according to a binary phase sequence. The rest of the procedures in the proposed schemes are the same as those in the conventional SLM. Therefore, the complexity of the proposed scheme is also the same as that of the conventional SLM. In BSLM, we may need more memories or registers to store the phase sequences. But the elements of phase sequences are binary values and the amount of the increased memory or register is negligible.

It is clear that BSLM and PBISLM also preserve the distance property of Gray mapping after removing the binary phase sequence $\mathbf{P}^{(\tilde{u})}$ at the receiver similarly as the conventional SLM scheme. Therefore, there is no BER degradation in both SLM schemes assuming that the receiver knows perfect side information.

In the proposed schemes, side information should be sent along with the data similarly to the conventional SLM scheme and the size of side information in BSLM and PBISLM is the same as that in the conventional SLM for the same U.

C. Shaping Gain of the Proposed Schemes and Their PAPR

Since the proposed schemes modify the average power of alternative symbol sequences, the average power of the OFDM signal sequence selected for transmission can be different from that of the input symbol sequence. By using the proposed schemes, the average power of the OFDM signal sequence can be reduced, which is regarded as a power shaping scheme [20]. The shaping gain γ is the gain due to the average power reduction defined as

$$\gamma \doteq \frac{\bar{\mathrm{P}}_{av}}{\bar{\mathrm{P}}_{av,shape}} = \frac{E[\bar{\mathrm{P}}]}{E[\bar{\mathrm{P}}^{(\tilde{u})}]} = \frac{E\left[|a_n|^2\right]}{E\left[\left|x_n^{(\tilde{u})}\right|^2\right]} \tag{23}$$

where $\bar{\mathrm{P}} = (1/N)\sum_{k=0}^{N-1}|A_k|^2$ and $\bar{\mathrm{P}}^{(u)} = (1/N)\sum_{k=0}^{N-1}|A_k\Lambda_k^{(\tilde{u})}|^2$. In [21] and [22], PAPR reduction schemes using trellis shaping or MMSE are proposed, which achieve shaping gain as well as PAPR reduction. In [21], the SNR for AWGN channel is derived by considering the shaping gain as

$$10\log_{10}\frac{E_s}{N_0} = 10\log_{10} m\frac{E_b}{N_0} + 10\log_{10}(1-r) + 10\log_{10}\gamma[\mathrm{dB}] \tag{24}$$

where $m = \log_2 M$ for M-QAM, E_b is the transmit energy per information bit, and N_0 is the one-sided power spectral density of an AWGN process. r in the second term corresponds to the rate of loss due to SLM scheme, that is, the amount of side information which is the same for the proposed SLM schemes and the conventional SLM scheme. γ in the third term is the shaping gain defined in (23) which is one for the conventional SLM and more than one for the proposed SLM schemes.

In the next section, it will be shown that two proposed schemes also have the shaping gain in addition to PAPR reduction through numerical analysis. Thus PAPR in (2) should be normalized by power shaping gain as

$$\mathrm{PAPR}_{shape}(\mathbf{a}) \doteq \frac{\max_{0\le n<N}|a_n|^2}{\bar{\mathrm{P}}_{av,shape}}. \tag{25}$$

V. Simulation Results

In this section, we compare the covariance of average symbol powers of two alternative symbol sequences for the conventional SLM and the proposed schemes and also compare the PAPR reduction performance of them for $U = 8$. The rows of cyclic Hadamard matrix are used for phase sequences [16] and the all-1 sequence is used for $u = 0$ to include the original input symbol sequence among the alternative symbol sequences.

Fig. 3 compares the covariance of average symbol powers of two alternative symbol sequences for $N = 64$, 128, 256 and 512 when 16-QAM and 64-QAM are used. The conventional SLM scheme has the largest covariance and the covariance for 64-QAM is larger than that for 16-QAM. In the case of PBISLM

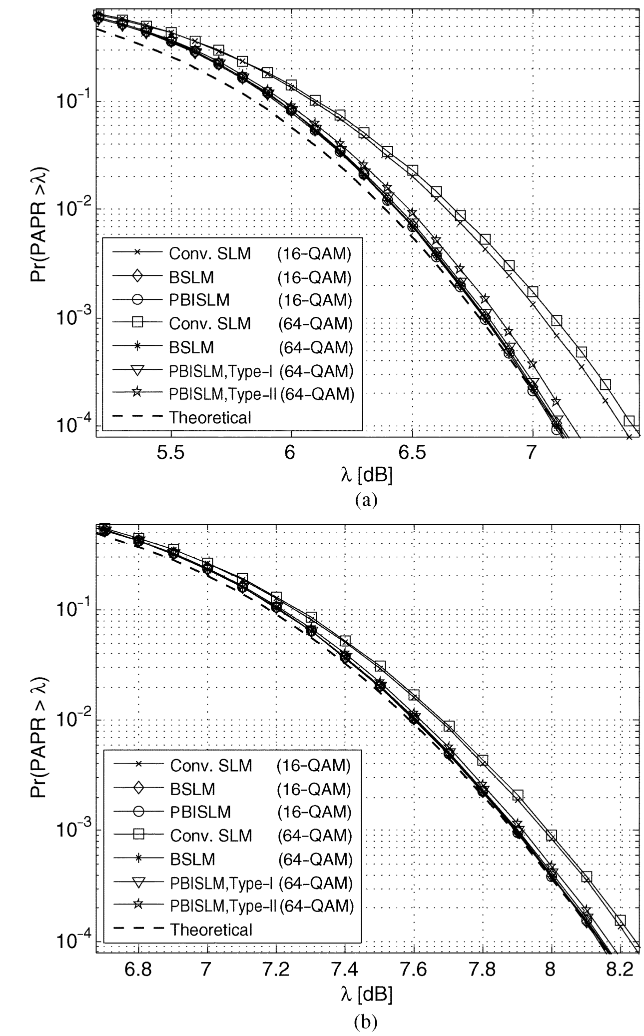

Fig. 4. Comparison of PAPR reduction performance of the conventional SLM, proposed schemes, and theoretical CCDF of (3) for $U = 8$: (a) $N = 64$; (b) $N = 256$.

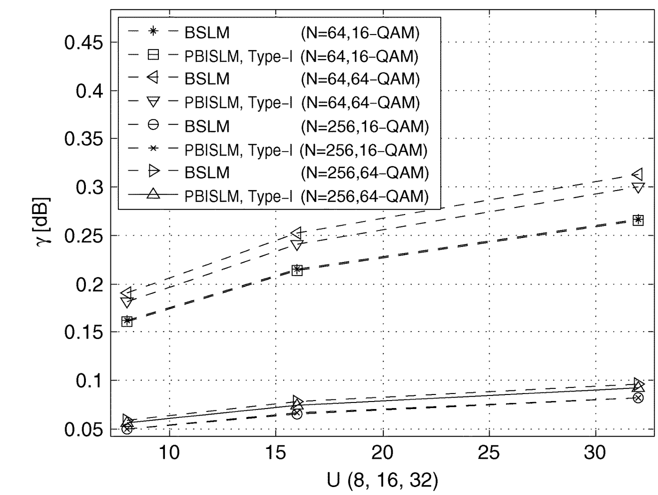

Fig. 5. Shaping gain of BSLM and PBISLM with $N = 64$ and 256 for $U = 8, 16$, and 32.

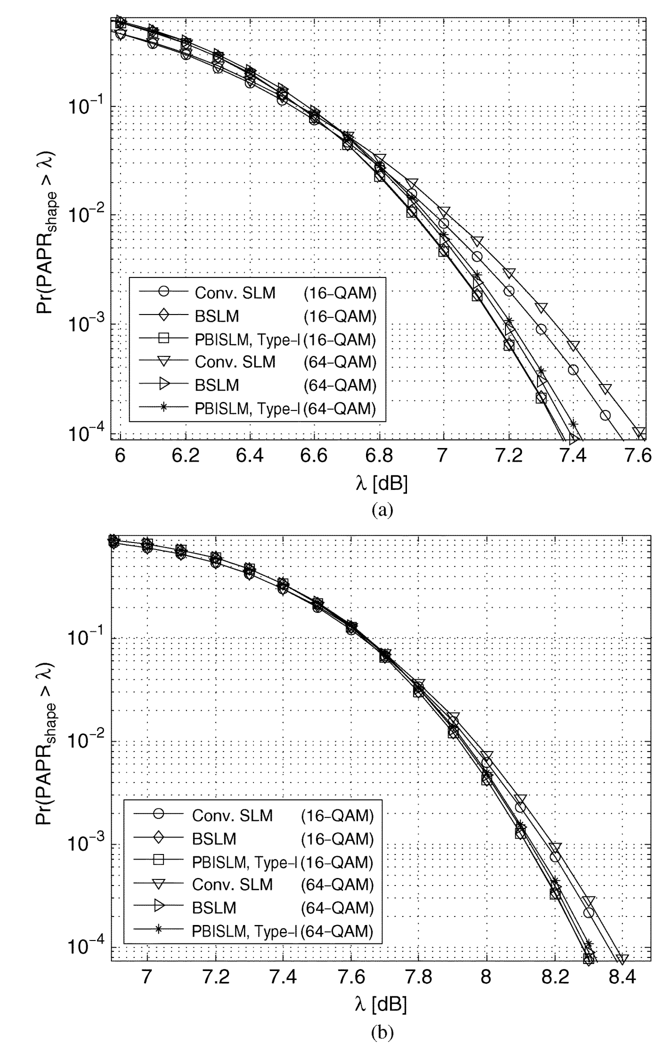

Fig. 6. Comparison of PAPR_{shape} reduction performance of the conventional SLM and proposed schemes for $U = 16$: (a) $N = 64$; (b) $N = 256$.

for 64-QAM, the covariance in Type-I is smaller than that in Type-II. Although the covariance does not become zero for the proposed schemes for smaller N, it is clear the covariance in the proposed schemes is closer to zero than that of the conventional SLM for any N and M.

Fig. 4 compares the PAPR reduction performance of various SLM schemes for $N = 64$ and 256. Note that oversampling is not performed when we compare the results with the theoretical CCDF in (3). The conventional SLM scheme shows the worst performance, especially for 64-QAM, but the performance gap with other schemes decreases as N increases. The PAPR reduction performance of PBISLM with Type-II is slightly worse than that of PBISLM with Type-I for $N = 64$. The PAPR reduction performance of BSLM and PBISLM with Type-I is almost identical to the theoretical CCDF curves in (3). Note that the PAPR reduction performance shows similar tendency as the covariance given in Fig. 3.

Now, the shaping gain is considered to evaluate the PAPR reduction performance. Fig. 5 shows the shaping gain of BSLM and PBISLM (Type-I for 64-QAM) through the numerical analysis. The shaping gain increases as U increases, N decreases, or the modulation order M increases. Two proposed schemes have the same shaping gain for 16-QAM but BSLM has slightly bigger shaping gain than PBISLM for 64-QAM. Since the required power is reduced to transmit the same amount of information, SNR gain can be obtained from the shaping gain as in (24) and therefore BER performance is improved assuming that the distance property of QAM at the receiver is preserved with perfect side information.

Fig. 6 compares the PAPR_{shape} reduction performance in (25) of BSLM, PBISLM (Type-I for 64-QAM), and conventional SLM for $N = 64$ and 256 when 16-QAM and 64-QAM, $U = 16$, and four times oversampling are used. It is clear that the PAPR_{shape} reduction performance of the proposed schemes is still better than that of the conventional SLM.

VI. Conclusion

We proposed two new bit-based SLM schemes for PAPR reduction in QAM modulated OFDM signals, called BSLM and PBISLM. The proposed schemes modify the magnitude as well as the phase by applying binary phase sequence to the input symbol sequence in the binary form to generate alternative OFDM signal sequences. The proposed schemes do not increase the computational complexity and the amount of side information compared to the conventional SLM. The improvement in the PAPR reduction performance of two proposed schemes increases as U increases and N decreases for QAM modulated OFDM signals. Simulation results show that the proposed schemes have better PAPR reduction performance than the conventional SLM and converge to the theoretical CCDF. Furthermore, the proposed schemes have shaping gain which can improve BER performance. Especially, BSLM does not depend on the constellation mapping scheme and has better PAPR reduction performance and more shaping gain than other SLM schemes.

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