# New Construction of Quaternary Low Correlation Zone Sequence Sets from Binary Low Correlation Zone Sequence Sets

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*Abstract:* **In this paper, using binary $(N, M, L, \epsilon)$ low correlation zone (LCZ) sequence sets, we construct new quaternary LCZ sequence sets with parameters $(2N, 2M, L, 2\epsilon)$. Binary LCZ sequences for the construction should have period $N \equiv 3 \bmod 4$, $L|N$, and the balance property. The proposed method corresponds to a quaternary extension of the extended construction of binary LCZ sequence sets proposed by Kim, Jang, No, and Chung [1].**

*Index Terms:* **Correlation, low correlation zone (LCZ), pseudorandom, quasi-synchronous code division multiple access (QS-CDMA), sequences.**

## I. INTRODUCTION

In microcellular networks such as femtocell networks, the delay among the signals of multiple users can be maintained within a few chips. Quasi-synchronous code division multiple access (QS-CDMA) system is the system devised for such environment [2]. In order to suppress the inter-user interference from quasi-synchronized signals, low correlation zone (LCZ) sequences have been used as signature sequences in the QS-CDMA systems [2]–[4]. Let $\mathcal{S}$ be a set of $M$ sequences of period $N$. If the magnitude of correlation function between any two sequences in $\mathcal{S}$ takes the values less than or equal to $\epsilon$ within the range $|\tau| < L$ for the offset $\tau$, then the sequence set is called LCZ sequence set with parameters $(N, M, L, \epsilon)$.

LCZ sequences sets were first constructed using GMW sequences for binary case [3] and for $p$-ary case [5]. Kim, Jang, No, and Chung introduced a construction method of quaternary LCZ sequences [6] from binary sequences with ideal autocorrelation property and the construction yields the first optimal LCZ sequence set with respect to Tang-Fan-Matsufuji bound [7]. Jang, No, Chung, and Tang also constructed an optimal $p$-ary LCZ sequence set [8].

In [1], Kim, Jang, No, and Chung proposed several design methods of LCZ sequence sets by manipulating sequences of the same alphabet. Using a similar mapping to the binary case of the construction, we construct quaternary LCZ sequence sets with parameters $(2N, 2M, L, 2\epsilon)$ from binary $(N, M, L, \epsilon)$ LCZ sequence sets, where the binary LCZ sequences have period $N \equiv 3 \mod 4$, $L|N$, and balance property. Using an optimal binary LCZ sequence set with parameters $(N, M, L, 1)$, we can construct a quaternary LCZ sequence set with parameters $(2N, 2M, L, 2)$, which is optimal with respect to Tang-Fan-Matsufuji bound [7].

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## II. PRELIMINARIES

Let $a(t)$ and $b(t)$ be $q$-ary sequences of period $N$. Then their correlation function is defined as

$$R_{a,b}(\tau) = \sum_{t=0}^{N-1} \omega_q^{a(t)-b(t+\tau)}$$

where $\omega_q$ is the complex primitive $q$th root of unity. The $R_{a,b}(\tau)$ is called the autocorrelation function of $a(t)$ if $a(t) = b(t)$ and the cross-correlation function between $a(t)$ and $b(t)$, otherwise.

Let $N$ be a positive integer such that $N = 3 \mod 4$ and $P = 2N$. Let $Z_P$ be the set of integers modulo $P$, i.e., $Z_P = \{0, 1, \cdots, P-1\}$. Let $a_i(t)$ be a binary sequence of period $N$ with balance property. Let $D_u^i$ be the characteristic set of $a_i(t-u)$, i.e.,

$$D_u^i = \{t \mid a_i(t-u) = 1,\ 0 \le t \le N-1\} = D_0^i + u$$

where $u \in Z_N$, $D_0^i + u = \{d + u \mid d \in D_0^i\}$, and "+" denotes addition modulo $N$. Binary sequences are said to be *balanced* if the occurrences of one in a period is the same as or once more than those of zero. From the balance of $a_i(t)$, it is clear that

$$|D_u^i| = \frac{N+1}{2},\ |\overline{D}_u^i| = \frac{N-1}{2}$$

where $\overline{D}_u^i = Z_N \backslash D_u^i$.

Let $u$ and $v$ be positive integers and $\sigma$ be the correlation value between $a_i(t-u)$ and $a_k(t-v)$ such that $|\sigma| \le \epsilon$ except for the inphase autocorrelation. Then it is easy to check

$$\begin{aligned} |D_u^i \cap D_v^k| &= \frac{N+\sigma}{4} + \frac{1}{2} \\ |D_u^i \cap \overline{D}_v^k| &= \frac{N-\sigma}{4} \\ |\overline{D}_u^i \cap D_v^k| &= \frac{N-\sigma}{4} \\ |\overline{D}_u^i \cap \overline{D}_v^k| &= \frac{N+\sigma}{4} - \frac{1}{2}. \end{aligned} \tag{1}$$

If $u = v$ and $i = k$, it is straightforward to check from the balance property that

$$\begin{aligned}|D_u^i \cap D_v^k| &= \frac{N+1}{2}\\ |D_u^i \cap \overline{D}_v^k| &= 0\\ |\overline{D}_u^i \cap D_v^k| &= 0\\ |\overline{D}_u^i \cap \overline{D}_v^k| &= \frac{N-1}{2}.\end{aligned}$$

By the Chinese remainder theorem, we can represent $Z_P \cong Z_2 \otimes Z_N$ under the isomorphism $\phi : \zeta \longmapsto (\zeta \bmod 2, \zeta \bmod N)$, where $\otimes$ denotes the direct product. For convenience, we use the notation $\zeta \in Z_P$ interchangeably with $(\zeta \bmod 2, \zeta \bmod N)$ throughout the paper.

## III. CONSTRUCTION OF NEW QUATERNARY LCZ SEQUENCE SETS

Using a binary $(N, M, L, \epsilon)$ LCZ sequence set with period $N \equiv 3 \mod 4$ and the balance property, we construct new quaternary LCZ sequence sets with parameters $(2N, 2M, L, 2\epsilon)$.

Let $\mathcal{L}$ be a set of binary LCZ sequences with parameters $(N, M, L, \epsilon)$ and balance property, where $N \equiv 3$ mod 4 and $L|N$. Furthermore, we assume that the correlation function $R_{ij}^B(\tau)$ between any two binary sequences $a_i(t)$ and $a_j(t)$ in $\mathcal{L}$ has the absolute value smaller or equal to $\epsilon$ except for $\tau \equiv 0 \mod L$ and $\tau \neq 0$. Let $D^i$ be the characteristic set of the LCZ sequence $a_i(t)$ in $\mathcal{L}$. Then quaternary LCZ sequence sets can be constructed as follows.

**Theorem 1:** Let $\mathcal{U}_1$ be the set of $M$ quaternary sequences of period $2N$ defined by

$$u_{1,i}(t) = \begin{cases} 0, & \text{if } t \in \{0\} \otimes \overline{D}_0^i \\ 1, & \text{if } t \in \{1\} \otimes \overline{D}_0^i + L \\ 2, & \text{if } t \in \{0\} \otimes D_0^i \\ 3, & \text{if } t \in \{1\} \otimes D_0^i + L \end{cases}$$

and $\mathcal{U}_2$ be the set of $M$ sequences of period $2N$ defined by

$$u_{2,i}(t) = \begin{cases} 0, & \text{if } t \in \{0\} \otimes \overline{D}_0^i \\ 1, & \text{if } t \in \{1\} \otimes D_0^i + L \\ 2, & \text{if } t \in \{0\} \otimes D_0^i \\ 3, & \text{if } t \in \{1\} \otimes \overline{D}_0^i + L \end{cases}$$

for $0 \leq i \leq M-1$. We define the quaternary sequence set $\mathcal{Q}$ of period $2N$ and the cardinality $2M$ as

$$s_i(t) = \begin{cases} u_{1,i}(t), & \text{for } 0 \leq i \leq M-1 \\ u_{2,i-M}(t), & \text{for } M \leq i \leq 2M-1. \end{cases}$$

Then the quaternary sequence set $\mathcal{Q} = \mathcal{U}_1 \cup \mathcal{U}_2$ is a quaternary LCZ sequence set with parameters $(2N, 2M, L, 2\epsilon)$.

*Proof:* Clearly, $\mathcal{Q}$ has $2M$ sequences and the period of the sequences is $2N$. Therefore, it remains to show that the magnitude of the correlation value between any two sequences in $\mathcal{Q}$ is less than or equal to $2\epsilon$ except for the in-phase autocorrelation. Using

$$A_0^i = \begin{cases} D_0^i + L, & \text{for } 0 \leq i \leq M-1 \\ \overline{D}_0^{i_M} + L, & \text{for } M \leq i \leq 2M-1 \end{cases}$$

we simplify the constrcution of $s_i(t)$ as

$$s_i(t) = \begin{cases} 0, & \text{if } t \in \{0\} \otimes \overline{D}_0^{i_M} \\ 1, & \text{if } t \in \{1\} \otimes \overline{A}_0^i \\ 2, & \text{if } t \in \{0\} \otimes D_0^{i_M} \\ 3, & \text{if } t \in \{1\} \otimes A_0^i \end{cases}$$

where $i_M = i \mod M$. □

Let $\tau = (\tau_1, \tau_2)$, where $\tau_1 \in Z_2$ and $\tau_2 \in Z_N$. Then the correlation between $s_i(t)$ and $s_k(t)$, $0 \leq i, k \leq 2M-1$ can be computed as

$$\begin{aligned}R_{i,k}(\tau) &= \sum_{t=0}^{2N-1} \omega_4^{s_i(t)-s_k(t+\tau)} = \sum_{t=0}^{2N-1} \omega_4^{s_i(t-\tau)-s_k(t)}\\
&= \left\{|\{\tau_1\}\otimes\overline{D}_{\tau_2}^{i_M}\cap\{0\}\otimes\overline{D}_0^{k_M}|+|\{1+\tau_1\}\otimes\overline{A}_{\tau_2}^i\cap\{1\}\otimes\overline{A}_0^k|\right\}\\
&\quad+\left\{|\{\tau_1\}\otimes D_{\tau_2}^{i_M}\cap\{0\}\otimes D_0^{k_M}|+|\{1+\tau_1\}\otimes A_{\tau_2}^i\cap\{1\}\otimes A_0^k|\right\}\\
&\quad+\omega_4\left\{|\{\tau_1\}\otimes\overline{D}_{\tau_2}^{i_M}\cap\{1\}\otimes A_0^k|+|\{1+\tau_1\}\otimes\overline{A}_{\tau_2}^i\cap\{0\}\otimes\overline{D}_0^{k_M}|\right\}\\
&\quad+\omega_4\left\{|\{\tau_1\}\otimes D_{\tau_2}^{i_M}\cap\{1\}\otimes\overline{A}_0^k|+|\{1+\tau_1\}\otimes A_{\tau_2}^i\cap\{0\}\otimes D_0^{k_M}|\right\}\\
&\quad-\left\{|\{\tau_1\}\otimes\overline{D}_{\tau_2}^{i_M}\cap\{0\}\otimes D_0^{k_M}|+|\{1+\tau_1\}\otimes\overline{A}_{\tau_2}^i\cap\{1\}\otimes A_0^k|\right\}\\
&\quad-\left\{|\{\tau_1\}\otimes D_{\tau_2}^{i_M}\cap\{0\}\otimes\overline{D}_0^{k_M}|+|\{1+\tau_1\}\otimes A_{\tau_2}^i\cap\{1\}\otimes\overline{A}_0^k|\right\}\\
&\quad-\omega_4\left\{|\{\tau_1\}\otimes\overline{D}_{\tau_2}^{i_M}\cap\{1\}\otimes\overline{A}_0^k|+|\{1+\tau_1\}\otimes\overline{A}_{\tau_2}^i\cap\{0\}\otimes D_0^{k_M}|\right\}\\
&\quad-\omega_4\left\{\{\tau_1\}\otimes D_{\tau_2}^{i_M}\cap\{1\}\otimes A_0^k|+|\{1+\tau_1\}\otimes A_{\tau_2}^i\cap\{0\}\otimes\overline{D}_0^{k_M}|\right\}\end{aligned}$$

where $\omega_4$ is the fourth complex primitive root of unity, i.e., $j = \sqrt{-1}$.

If $\tau_1 = 0$, that is, $\tau \equiv 0 \bmod 2$, then $R_{i,k}(\tau)$ can be simplified as

$$\begin{aligned}&R_{i,k}(\tau) =\\ &\left\{|\overline{D}_{\tau_2}^{i_M}\cap\overline{D}_0^{k_M}|+|\overline{A}_{\tau_2}^i\cap\overline{A}_0^k|+|D_{\tau_2}^{i_M}\cap D_0^{k_M}|+|A_{\tau_2}^i\cap A_0^k|\right\}\\ &-\left\{|\overline{D}_{\tau_2}^{i_M}\cap D_0^{k_M}|+|\overline{A}_{\tau_2}^i\cap A_0^k|+|D_{\tau_2}^{i_M}\cap\overline{D}_0^{k_M}|+|A_{\tau_2}^i\cap\overline{A}_0^k|\right\}.\end{aligned}$$

In the case of $\tau_1 = 1$, that is, $\tau \equiv 1 \bmod 2$, we have

$$\begin{aligned}&R_{i,k}(\tau) =\\ &\omega_4\left\{|\overline{D}_{\tau_2}^{i_M}\cap A_0^k|+|\overline{A}_{\tau_2}^i\cap\overline{D}_0^{k_M}|+|D_{\tau_2}^{i_M}\cap\overline{A}_0^k|+|A_{\tau_2}^i\cap D_0^{k_M}|\right\}\\ &-\omega_4\left\{|\overline{D}_{\tau_2}^{i_M}\cap\overline{A}_0^k|+|\overline{A}_{\tau_2}^i\cap D_0^{k_M}|+|D_{\tau_2}^{i_M}\cap A_0^k|+|A_{\tau_2}^i\cap\overline{D}_0^{k_M}|\right\}.\end{aligned}$$

**Case 1)** $0 \leq i, k \leq M-1$ ( i.e., $s_i(t)$, $s_k(t) \in \mathcal{U}_1$):

In this case, $A^i = D^i + L$ and $A^k = D^k + L$ and we should consider the following two sub-cases.

i) $\tau \equiv 0$ mod 2 ($\tau_1 = 0$) except for inphase autocorrelation;

Table 1. List of binary LCZ sequence sets.

| | $N$ | $M$ | $L$ | $\epsilon$ |
|---|---|---|---|---|
| Long, Zhang, and Hu [3] | $2^n-1$ | $< 2^m-1$ | $(2^n-1)/(2^m-1)$ | 1 |
| Jang, No, Chung, and Tang [8] | $2^n-1$ | $2^m-1$ | $(2^n-1)/(2^m-1)$ | 1 |
| Tang and Udaya [9] | $2^n-1$ | $2^m-m-1$ | $(2^n-1)/(2^m-1)$ | 1 |

In this sub-case, $R_{i,k}(\tau)$ can be rewritten as

$$\begin{aligned}R_{i,k}(\tau) = &|\overline{D}^i_{\tau_2} \cap \overline{D}^k_0| + |(\overline{D}^i_{\tau_2}+L) \cap (\overline{D}^k_0+L)|\\ &+|D^i_{\tau_2} \cap D^k_0| + |(D^i_{\tau_2}+L) \cap (D^k_0+L)|\\ &-|\overline{D}^i_{\tau_2} \cap D^k_0| - |(\overline{D}^i_{\tau_2}+L) \cap (D^k_0+L)|\\ &-|D^i_{\tau_2} \cap \overline{D}^k_0| - |(D^i_{\tau_2}+L) \cap (\overline{D}^k_0+L)|.\end{aligned}$$

Let $R^B_{i,k}(-\tau_2) = \sigma_1$. For $\tau_2 \not\equiv 0 \bmod L$, then $|\sigma_1| \le \epsilon$ and we have

$$\begin{aligned}|D^i_{\tau_2} \cap D^k_0| = |(D^i_{\tau_2}+L) \cap (D^k_0+L)| &= \frac{N+\sigma_1}{4}+\frac{1}{2}\\ |D^i_{\tau_2} \cap \overline{D}^k_0| = |(D^i_{\tau_2}+L) \cap (\overline{D}^k_0+L)| &= \frac{N-\sigma_1}{4}\\ |\overline{D}^i_{\tau_2} \cap D^k_0| = |(\overline{D}^i_{\tau_2}+L) \cap (D^k_0+L)| &= \frac{N-\sigma_1}{4}\\ |\overline{D}^i_{\tau_2} \cap \overline{D}^k_0| = |(\overline{D}^i_{\tau_2}+L) \cap (\overline{D}^k_0+L)| &= \frac{N+\sigma_1}{4}-\frac{1}{2}.\end{aligned}$$

Thus, $R_{i,k}(\tau)$ reduces to $2\sigma_1$ and $|R_{i,k}(\tau)| = |2\sigma_1| \le 2\epsilon$ for $0 < |\tau| < L$.

When $\tau = 0$ and $i \ne k$, $R_{i,k}(\tau)$ becomes the sum of two correlation functions at $\tau_2 = 0$ between $a_i(t)$ and $a_k(t)$, and $a_i(t+L)$ and $a_k(t+L)$. From the property of binary LCZ sequence set with parameters $(N, M, L, \epsilon)$ [1], clearly $|R_{i,k}(0)| \le 2\epsilon$.

ii) $\tau \equiv 1 \bmod 2$ ($\tau_1 = 1$);

In this sub-case, $R_{i,k}(\tau)$ can be rewritten as

$$\begin{aligned}R_{i,k}(\tau) = &\omega_4\left\{|\overline{D}^i_{\tau_2} \cap (D^k_0+L)| + |(\overline{D}^i_{\tau_2}+L) \cap \overline{D}^k_0|\right\}\\ &+\omega_4\left\{|D^i_{\tau_2} \cap (\overline{D}^k_0+L)| + |(D^i_{\tau_2}+L) \cap D^k_0|\right\}\\ &-\omega_4\left\{|\overline{D}^i_{\tau_2} \cap (\overline{D}^k_0+L)| + |(\overline{D}^i_{\tau_2}+L) \cap D^k_0|\right\}\\ &-\omega_4\left\{|D^i_{\tau_2} \cap (D^k_0+L)| + |(D^i_{\tau_2}+L) \cap \overline{D}^k_0|\right\}.\end{aligned}$$

Let $R^B_{i,k}(L-\tau_2) = \sigma_2$ and $R^B_{i,k}(-L-\tau_2) = \sigma_3$. Applying the correlation values in (1), the correlation function can be computed as

$$R_{i,k}(\tau) = \omega_4\left\{-\sigma_2+\sigma_3\right\}.$$

Clearly, $|R_{i,k}(\tau)| \le 2\epsilon$ for $\tau \equiv 1 \bmod 2$ and $0 < |\tau| < L$.

**Case 2)** $0 \le i \le M-1$ and $M \le k \le 2M-1$:

In this case, $A^i_0 = D^i_0 + L$ and $A^k_0 = \overline{D}^{k_M}_0 + L$. Similarly to case 1), we should consider the following two sub-cases.

i) $\tau \equiv 0 \mod 2$ ($\tau_1 = 0$);

In this sub-case, $R_{i,k}(\tau)$ can be rewritten as

$$\begin{aligned}R_{i,k}(\tau) = &|\overline{D}^i_{\tau_2} \cap \overline{D}^{k_M}_0| + |(\overline{D}^i_{\tau_2}+L) \cap (D^{k_M}_0+L)|\\ &+|D^i_{\tau_2} \cap D^{k_M}_0| + |(D^i_{\tau_2}+L) \cap (\overline{D}^{k_M}_0+L)|\\ &-|\overline{D}^i_{\tau_2} \cap D^{k_M}_0| - |(\overline{D}^i_{\tau_2}+L) \cap (\overline{D}^{k_M}_0+L)|\\ &-|D^i_{\tau_2} \cap \overline{D}^{k_M}_0| - |(D^i_{\tau_2}+L) \cap (D^{k_M}_0+L)|.\end{aligned}$$

It is not difficult to prove that we have

$$\begin{aligned}|(D^i_{\tau_2}+L) \cap (D^{k_M}_0+L)| &= |D^i_{\tau_2} \cap D^{k_M}_0|\\ |(D^i_{\tau_2}+L) \cap (\overline{D}^{k_M}_0+L)| &= |D^i_{\tau_2} \cap \overline{D}^{k_M}_0|\\ |(\overline{D}^i_{\tau_2}+L) \cap (D^{k_M}_0+L)| &= |\overline{D}^i_{\tau_2} \cap D^{k_M}_0|\\ |(\overline{D}^i_{\tau_2}+L) \cap (\overline{D}^{k_M}_0+L)| &= |\overline{D}^i_{\tau_2} \cap \overline{D}^{k_M}_0|\end{aligned}$$

and thus $|R_{i,k}(\tau)| = 0$ for $0 \le |\tau| < L$.

ii) $\tau \equiv 1 \bmod 2$ ($\tau_1 = 1$);

In this sub-case, $R_{i,k}(\tau)$ can be rewritten as

$$\begin{aligned}R_{i,k}(\tau) = &\omega_4\left\{|\overline{D}^i_{\tau_2} \cap (\overline{D}^{k_M}_0+L)| + |(\overline{D}^i_{\tau_2}+L) \cap \overline{D}^{k_M}_0|\right\}\\ &+\omega_4\left\{|D^i_{\tau_2} \cap (D^{k_M}_0+L)| + |(D^i_{\tau_2}+L) \cap D^{k_M}_0|\right\}\\ &-\omega_4\left\{|\overline{D}^i_{\tau_2} \cap (D^{k_M}_0+L)| + |(\overline{D}^i_{\tau_2}+L) \cap D^{k_M}_0|\right\}\\ &-\omega_4\left\{|D^i_{\tau_2} \cap (\overline{D}^{k_M}_0+L)| + |(D^i_{\tau_2}+L) \cap \overline{D}^{k_M}_0|\right\}.\end{aligned}$$

Let $R^B_{i,k_M}(L-\tau_2) = \sigma_5$ and $R^B_{i,k_M}(-L-\tau_2) = \sigma_6$. For $\tau_2 \ne 0 \mod L$, we have $|\sigma_5| \le \epsilon$ and $|\sigma_6| \le \epsilon$. Applying the relationship (1) to these correlation functions, we find

$$R_{i,k}(\tau) = \omega_4\left\{\sigma_5+\sigma_6\right\},$$

whose absolute value is smaller or equal to $2\epsilon$ for $\tau \equiv 1 \bmod 2$ and $0 < |\tau| < L$.

**Case 3)** $M \le i, k \le 2M-1$:

In this case, $A^i_0 = \overline{D}^{i_M}_0 + L$ and $A^k_0 = \overline{D}^{k_M}_0 + L$. Similarly to case 1), it can be proved that $|R_{i,k}(\tau)| \le 2\epsilon$ for $0 \le |\tau| < L$. From case 1)–case 3), the sequence set $\mathcal{Q}$ is an LCZ sequence set with parameters $(2N, 2M, L, 2\epsilon)$. □

Theorem 1 says that we can construct quaternary LCZ sequences sets with parameters $(2N, 2M, L, 2)$ given that an optimal or suboptimal $(N, M, L, 1)$ LCZ sequences set exists. If the employed binary LCZ sequence set with parameters $(N, M, L, 1)$ is optimal, it is proven straightforwardly that the quaternary LCZ sequence set with parameters $(2N, 2M, L, 2)$ defined in Theorem 1 is optimal with respect to Tang-Fan-Matsufuji bound [7] in the sense that its cardinality is the largest for the given length $2N$ and the correlation constraint $2\epsilon$.

Binary LCZ sequences sets applicable to Theorem 1 are listed in Table 1. Note that an optimal binary LCZ sequence set introduced by Jang, No, Chung, and Tang [8] yields the construction of an optimal quaternary LCZ sequence set. There is an example of newly constructed quaternary LCZ sequence sets, even though it is not an optimal one.

**Example 2:** Let $a(t)$ be a binary Legendre sequence of period 31 constructed from quadratic residue in $Z_{31}$ given by

$$a(t) = (1110110111100010101110000100100).$$

Let $\mathcal{L}$ be a binary LCZ sequence set with parameters $(N, M, L, \epsilon) = (1023, 16, 33, 1)$ constructed using the Legendre sequence and the trace function from $F_{2^{10}}$ to $F_{2^5}$ by Theorem 8 in [8]. Since $N = 1023 \equiv 3 \mod 4$, we can construct a quaternary LCZ sequence set $\mathcal{Q}$ with parameters $(2N, 2M, L, 2) = (2046, 32, 33, 2)$ using the binary LCZ sequence set.

## IV. CONCLUDING REMARKS

In this paper, we propose a simple construction of quaternary LCZ sequence sets with parameters $(2N, 2M, L, 2\epsilon)$ from binary LCZ sequence sets with parameters $(N, M, L, \epsilon)$. Any binary LCZ sequence sets with low correlation value $\epsilon = 1$ can be hired to derive new quaternary LCZ sequence sets with low correlation value $\epsilon = 2$. The constructed quaternary LCZ sequence set becomes optimal with respect to Tang-Fan-Matsufuji bound [7] if the binary LCZ sequence set employed is optimal. Therefore, the proposed construction provides optimal quaternary LCZ sequence sets from the optimal binary LCZ sequence sets given in [8].

## REFERENCES

[1] Y.-S. Kim, J.-W. Jang, J.-S. No, and H. Chung, "New design of low-correlation zone sequence sets," *IEEE Trans. Inf. Theory*, vol. 52, no. 10, pp. 4607–4616, 2006.

[2] R. De Gaudenzi, C. Elia, and R. Viola, "Bandlimited quasi-synchronous CDMA: A novel satellite access technique for mobile and personal communication systems," *IEEE J. Sel. Areas Commun.*, vol. 10, no. 2, pp. 328–343, 1992.

[3] B. Long, P. Zhang, and J. Hu, "A generalized QS-CDMA system and the design of new spreading codes," *IEEE Trans. Veh. Technol.*, vol. 47, no. 4, pp. 1268–1275, 1998.

[4] J. S. Cha, S. Kameda, M. Yokoyama, H. Nakase, K. Masu, and K. Tsubouchi, "New binary sequences with zero-correlation duration for approximately synchronised CDMA," *Electron. Lett.*, vol. 36, no. 11, pp. 991–993, May 2000.

[5] X. Tang and P. Fan, "A class of pseudonoise sequences over GF ($p$) with low correlation zone," *IEEE Trans. Inf. Theory*, vol. 47, no. 4, pp. 1644–1649, 2001.

[6] S.-H. Kim, J.-W. Jang, J.-S. No, and H. Chung, "New constructions of quaternary low correlation zone sequences," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1469–1477, 2005.

[7] X. Tang, P. Fan, and S. Matsufuji, "Lower bounds on correlation of spreading sequence set with low or zero correlation zone," *Electron. Lett.*, vol. 36, no. 6, pp. 551–552, 2000.

[8] J.-W. Jang, J.-S. No, H. Chung, and X. Tang, "New sets of optimal $p$-ary low-correlation zone sequences," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 815–821, 2007.

[9] X. Tang and P. Udaya, "New construction of low correlation zone sequences from Hadamard matrices," in *Proc. IEEE ISIT, Adelaide, Austrailia,* Sept. 4–9, 2005, pp. 482–486.

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