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PAPER

Bit Error Rate and Power Allocation of Soft-Decision-and-Forward Cooperative Networks

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SUMMARY In this paper, the performance of the soft-decision-and-forward (SDF) protocol in the cooperative communication network with one source, one relay, and one destination, where each node has two transmit and receive antennas, is analyzed in terms of the bit error rate (BER) obtained from the pairwise error probability (PEP). Using the moment generating function and Q-function approximation, the PEP of SDF protocol is calculated and we confirm that the SDF with two antennas achieves the full diversity order. For the slow-varying Rayleigh fading channel, the optimal power allocation ratio can be determined so as to minimize the average PEP (or BER). Due to the difficulty of deriving the optimal value analytically, an alternative strategy of maximizing the product signal-to-noise ratio (SNR) of direct and relay links, which we call the suboptimal power allocation, is considered. Through a numerical analysis, we show that the performance gap between the suboptimal and optimal power allocation strategies is negligible in the high SNR region.

key words: *cooperative diversity, harmonic mean, power allocation, product signal-to-noise ratio (SNR), soft-decision-and-forward (SDF)*

1. Introduction

The performance of wireless communication systems can be improved if source and relays cooperate. In [1] and [2], Sendonaris, Erkip, and Aazhang proposed the cooperative diversity. In [3], cooperative communications and their relaying protocols such as amplify-and-forward (AF) and decode-and-forward (DF) protocols were overviewed. In [4], the authors proposed the process-and-forward (PF) protocol with two sources, one relay, and one destination, each equipped with a single antenna. They calculated the end-to-end signal-to-noise ratio (SNR) and symbol error rate (SER) for a single antenna at each node and showed that AF and PF protocols have the same SER performance.

Laneman and Wornell [5] applied the space-time coding [6] to the cooperative communication networks and derived the outage probability and diversity order. In [7], Jing and Hassibi analyzed the diversity order and power allocation of the distributed space-time coding based on AF protocol using the pairwise error probability (PEP). Maham and Hjörunes [8] analyzed the asymptotic performance of AF in a Nakagami-m fading channel. In [9], Lee and Kim proposed the decouple-and-forward protocol based on the squaring method [10] and evaluated the bit error rate (BER) by approximating the relay gain. But, in their system, there was no direct link from the source to the destination, and moreover, they ignored the noise correlation at the destination so that it cannot be considered as an optimal decoder. Yang, Song, No, and Shin [11] proposed the maximum-likelihood (ML) decoding for AF and soft-decision-and-forward (SDF) protocols with multiple antennas using orthogonal space-time codes [12], [13]. Although the SDF and AF protocols are identical for one antenna in each node, the difference between AF and SDF protocols is substantial in the case of multiple antennas. Relay nodes can separate the signals in SDF protocol while they simply amplify the received signals according to the power constraint in AF protocol. From the numerical results in [11], it has been shown that SDF protocol outperformed AF protocol under both the squaring method and ML decoding. However, performances of AF and SDF protocols were not analytically evaluated. Moreover, the SDF protocol has an advantage over DF protocol as follows: Though performance gap between DF and SDF is small, the decoding complexity of SDF in destination node is much smaller than that of DF.

Due to the limited resources in wireless communication systems, the power allocation is a very important matter. Hasna and Alouini [14] worked on the power allocation problem for AF and DF protocols for dual-hop relaying system. In [15] and [16], the power allocation strategy for AF protocol has been developed so as to maximize the average SNR and minimize the outage probability, which is equivalent to maximization of the product of the direct link and relay link SNRs in the high SNR region. In [17], the authors derived the SER for the dual-hop wireless communication with AF relaying and investigated into the optimal power allocation. In [18], the power allocation method for AF protocol with opportunistic relaying aiming at minimizing the outage probability was considered.

In this paper, we formulate the end-to-end SNR and derive the PEP of the SDF under the ML decoding proposed in [11]. From the derived PEP, the diversity order is derived by using the bounds on Q-function. And also, the BER of the SDF cooperative communication network, where each node has two antennas, is approximated by using Q-function approximation. For a slow-varying Rayleigh fading channel, the optimal and suboptimal power allocations are considered when the channel state information (CSI) is not fed back to the transmitter. The proposed power allocation strat-

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egy is to maximize the product SNR, called the suboptimal power allocation. In the high SNR region, it is shown that the performance using the suboptimal power allocation is approaching to the performance of the optimal power allocation in the slow-varying Rayleigh fading channel. This paper is organized as follows. In Sect. 2, the cooperative relaying protocol using SDF protocol is reviewed. And then, error performance of the SDF protocol with one relay node is evaluated in terms of PEP and BER in Sect. 3. The suboptimal power allocation is proposed and compared with the optimal power allocation scheme in Sect. 4. The numerical results are presented in Sect. 5. And finally, the concluding remarks are given in Sect. 6.

Throughout this paper, the following notations are used. $\mathbb{E}[\cdot]$ denotes the expectation of a random variable. $X \sim \mathcal{CN}(0, \sigma^2)$ means that X is a complex normal random variable with zero mean and variance $\sigma^2/2$ in both real and imaginary parts, respectively. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\dagger}$, and $\|\cdot\|$ denote the transpose of a matrix, the conjugate transpose of a matrix, and the Frobenius norm of a matrix or a vector, respectively. $\mathbf{0}_n$ and $\mathbf{I}_n$ are the zero matrix and the identity matrix of size n, and bold-face uppercase and lowercase letters denote matrices and vectors, respectively. For a complex number, $|\cdot|$, $\Re\{\cdot\}$, and $(\cdot)^*$ represent the modulus, the real part, and the complex conjugate, respectively. S, R, and D are used to denote source, relay, and destination nodes, respectively.

2. Soft-Decision-and-Forward Protocol

The SDF protocol [11] in the cooperative communication network with two antennas shown in Fig. 1 will be briefly reviewed. First, let us define the notations used in this section. The conventional Alamouti code $\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$ is denoted by $\mathbf{A}(a, b)$. For any 2×2 matrix $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, the 4×2 matrix $\mathbf{B}'$ and the vector $cv(\mathbf{B})$ are defined as $\mathbf{B}' = \begin{bmatrix} b_{11} & b_{21}^* & b_{12} & b_{22}^* \\ b_{21} & -b_{11}^* & b_{22} & -b_{12}^* \end{bmatrix}^{\mathrm{T}}$ and $cv(\mathbf{B}) = \begin{bmatrix} b_{11} & b_{21}^* & b_{12} & b_{22}^* \end{bmatrix}^{\mathrm{T}}$.

The name 'SDF' is originated from the notion that R obtains *soft-decision values*[†] from the received signals from S, encodes them into Alamouti codes, and forwards them

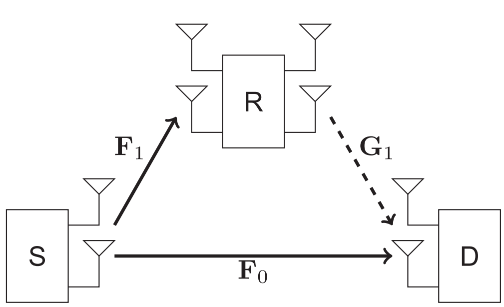

Fig. 1 Cooperative communication network composed of one source (S), one relay (R), and one destination (D) with two antennas at each node.

to D. The total transmit power P in the network is the sum of the transmit power P_1 at S and the transmit power P_2 at R. The channel gains of each link S $\to$ D, S $\to$ R, and R $\to$ D are Rayleigh-faded, i.e., $f_0^{ij} \sim \mathcal{CN}(0, \sigma_{\mathrm{SD}}^2)$, $f_1^{ij} \sim \mathcal{CN}(0, \sigma_{\mathrm{SR}}^2)$, and $g_1^{ij} \sim \mathcal{CN}(0, \sigma_{\mathrm{RD}}^2)$, where f_0^{ij}, f_1^{ij}, and g_1^{ij}, $1 \le i, j \le 2$, denote the path gains from the ith transmit antenna at S to the jth receive antenna at D, from the ith transmit antenna at S to the jth receive antenna at R, and from the ith transmit antenna at R to the jth receive antenna at D, respectively, and are the elements of the channel matrices $\mathbf{F}_0$, $\mathbf{F}_1$, and $\mathbf{G}_1$, respectively.

The transmission is composed of two phases. In the first phase, S transmits the signal using Alamouti code to R and D. Thus, the received signals at R and D are represented, respectively, as

$$\mathbf{Y}_{\mathrm{R}} = \sqrt{\frac{P_1}{2}}\mathbf{X}\mathbf{F}_1 + \mathbf{N}_{\mathrm{R}}$$
$$\mathbf{Y}_{\mathrm{D1}} = \sqrt{\frac{P_1}{2}}\mathbf{X}\mathbf{F}_0 + \mathbf{N}_{\mathrm{D1}}$$

where $\mathbf{X} = \mathbf{A}(x_1, x_2)$ is the transmit codeword at S in the first phase, $\mathbf{F}_0$ and $\mathbf{F}_1$ denote the channel matrices of S $\to$ D and S $\to$ R, respectively, and $\mathbf{N}_{\mathrm{R}}$ and $\mathbf{N}_{\mathrm{D1}}$ are the 2×2 AWGN matrices with zero-mean and unit-variance entries. During the intermediate decoding at R, R obtains the soft-decision values from the received signals using maximal ratio combining as

$$\tilde{\mathbf{x}} \triangleq \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 \end{bmatrix}^{\mathrm{T}} = \lambda \mathbf{F}_1'^{\dagger} cv(\mathbf{Y}_{\mathrm{R}})$$

where

$$cv(\mathbf{Y}_{\mathrm{R}}) = \begin{bmatrix} y_{11}^{(\mathrm{R})} & y_{21}^{(\mathrm{R})*} & y_{12}^{(\mathrm{R})} & y_{22}^{(\mathrm{R})*} \end{bmatrix}^{\mathrm{T}}$$
$$= \sqrt{\frac{P_1}{2}}\mathbf{F}_1'\mathbf{x} + cv(\mathbf{N}_{\mathrm{R}})$$
$$\lambda = \sqrt{\frac{2}{\|\mathbf{F}_1\|^2(P_1\|\mathbf{F}_1\|^2 + 2)}}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}}$ is the transmitted signal vector at S. And then, R transmits the following codeword into D

$$\mathbf{X}_{\mathrm{R}} = \mathbf{A}(\tilde{x}_1, \tilde{x}_2) = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 \\ -\tilde{x}_2^* & \tilde{x}_1^* \end{bmatrix}.$$

In the second phase, the received signal at D is expressed as

$$\mathbf{Y}_{\mathrm{D2}} = \sqrt{\frac{P_2}{2}}\mathbf{X}_{\mathrm{R}}\mathbf{G}_1 + \mathbf{N}_{\mathrm{D2}}$$

where $\mathbf{G}_1$ is the channel matrix of R $\to$ D and $\mathbf{N}_{\mathrm{D2}}$ denotes the 2×2 AWGN matrix with zero-mean and unit-variance entries. Converting the matrix form into the vector form gives the following alternative expression

[†]The soft-decision value is used as the opposite meaning of the hard-decision value.

$$cv(\mathbf{Y}_{\mathsf{D2}}) = \frac{\sqrt{P_1P_2}}{2}\lambda\|\mathbf{F}_1\|^2\mathbf{G}_1'\mathbf{x} + \sqrt{\frac{P_2}{2}}\lambda\mathbf{G}_1'\mathbf{F}_1'^{\dagger}cv(\mathbf{N}_{\mathsf{R}}) + cv(\mathbf{N}_{\mathsf{D2}}).$$

The received signal at D during two phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D1}}) \\ cv(\mathbf{Y}_{\mathsf{D2}}) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}}\begin{bmatrix} \mathbf{F}_0' \\ \sqrt{\frac{P_2}{2}}\lambda\|\mathbf{F}_1\|^2\mathbf{G}_1' \end{bmatrix}}_{\mathbf{H}}\mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}_{\mathsf{D1}}) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix}}_{\mathbf{n}} \quad (1)$$

where $cv(\mathbf{N}_{\mathsf{D}})$ means the equivalent noise at D in the vector form, which is given by

$$cv(\mathbf{N}_{\mathsf{D}}) = \sqrt{\frac{P_2}{2}}\lambda\mathbf{G}_1'\mathbf{F}_1'^{\dagger}cv(\mathbf{N}_{\mathsf{R}}) + cv(\mathbf{N}_{\mathsf{D2}}).$$

2.1 Maximum-Likelihood Decoding

The ML decoding rule for the SDF protocol can be written as

$$\begin{aligned}\hat{\mathbf{x}} &= \arg\min_{\mathbf{x}}[(\mathbf{y}-\mathbf{H}\mathbf{x})^{\dagger}\mathcal{K}_{\mathbf{n}}^{-1}(\mathbf{y}-\mathbf{H}\mathbf{x})] \\ &= \arg\min_{\mathbf{x}}\left[\mathbf{x}^{\dagger}\mathbf{H}^{\dagger}\mathcal{K}_{\mathbf{n}}^{-1}\mathbf{H}\mathbf{x} - 2\Re\{\mathbf{y}^{\dagger}\mathcal{K}_{\mathbf{n}}^{-1}\mathbf{H}\mathbf{x}\}\right]\end{aligned}$$

where $\mathcal{K}_{\mathbf{n}} = \mathcal{E}\left[\mathbf{n}\mathbf{n}^{\dagger}\right] = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \mathcal{K}_{cv(\mathbf{N}_{\mathsf{D}})} \end{bmatrix}$ with $\mathcal{K}_{cv(\mathbf{N}_{\mathsf{D}})} = \mathbf{I}_4 + P_2/(P_1\|\mathbf{F}_1\|^2+2)\cdot\mathbf{G}_1'\mathbf{G}_1'^{\dagger}$. The ML decoder for SDF protocol chooses $\hat{x}_i$ such that

$$\hat{x}_i = \arg\min_{x_i}\left[\gamma_{\mathrm{eq}|\mathbf{H}}\cdot|x_i|^2 - 2\Re\{\eta_i x_i\}\right]$$

where $\gamma_{\mathrm{eq}|\mathbf{H}}$ is the equivalent end-to-end SNR given in Sect. 3.1 and $\begin{bmatrix}\eta_1 & \eta_2\end{bmatrix} = \mathbf{y}^{\dagger}\mathcal{K}_{\mathbf{n}}^{-1}\mathbf{H}$. Note that the ML decoder for SDF protocol using equal energy signals such as phase-shift keying (PSK) is simplified as

$$\hat{x}_i = \arg\max_{x_i}\Re\{\eta_i x_i\}.$$

If the noise correlation between R and D is ignored, we have $\mathcal{K}_{\mathbf{n}} = \mathbf{I}_8$, which results in the same decoding method in [9]. Although the squaring method is much simpler than ML decoding, BER performance gap between two decoding methods becomes larger as SNR increases. From now on, all analyses are given for only ML decoding not the squaring method.

3. Error Rate Analysis

In this section, we compute the PEP conditioned on $\mathbf{H}$ using the end-to-end SNR, which is the sum of the direct link SNR and the relay link SNR. Taking the expectation to the conditional PEP gives the average PEP of the SDF protocol. Using the result in [19], we obtain the BER expression for the cooperative communication network with SDF protocol.

3.1 End-to-End SNR

Under the assumption of the ML decoding, the instantaneous end-to-end SNR, $\gamma_{\mathrm{eq}|\mathbf{H}}$, becomes the sum of two SNRs, i.e.,

$$\gamma_{\mathrm{eq}|\mathbf{H}} = \frac{P_1}{2}\|\mathbf{F}_0\|^2 + \frac{P_1P_2\|\mathbf{F}_1\|^2\|\mathbf{G}_1\|^2}{2(P_1\|\mathbf{F}_1\|^2 + P_2\|\mathbf{G}_1\|^2 + 2)}.$$

This result is similar to the end-to-end SNR of the conventional AF relaying with a single antenna. Letting $\gamma_0 = P_1\|\mathbf{F}_0\|^2/2$, $\gamma_1 = P_1\|\mathbf{F}_1\|^2/2$, and $\gamma_2 = P_2\|\mathbf{G}_1\|^2/2$, the instantaneous end-to-end SNR can be rewritten as

$$\gamma_{\mathrm{eq}|\mathbf{H}} = \gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1} \quad (2)$$

where $\gamma_0 \sim \mathcal{G}\left(4, \sigma_{\mathsf{SD}}^2P_1/2\right)$, $\gamma_1 \sim \mathcal{G}\left(4, \sigma_{\mathsf{SR}}^2P_1/2\right)$, and $\gamma_2 \sim \mathcal{G}\left(4, \sigma_{\mathsf{RD}}^2P_2/2\right)$, respectively. And also, (2) can be bounded upperly and lowerly as

$$\gamma_0 + \frac{\gamma_1\gamma_2}{c_k(\gamma_1+\gamma_2)} \le \gamma_{\mathrm{eq}|\mathbf{H}} \le \gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \quad (3)$$

where $c_k > 1 + (\gamma_1+\gamma_2)^{-1}$. This means that we can find an arbitrary constant c_k given γ_1 and γ_2.

3.2 Pairwise Error Probability and Diversity Order

In this subsection, we are going to derive the PEP of the SDF protocol through the MGF approach. Using (1), the conditional PEP can be written as

$$\Pr(\mathbf{x}\to\hat{\mathbf{x}}|\mathbf{H}) = Q\left(\sqrt{\frac{1}{2}\left\|\mathcal{K}_{\mathbf{n}}^{-\frac{1}{2}}\mathbf{H}(\hat{\mathbf{x}}-\mathbf{x})\right\|^2}\right) \quad (4)$$

where $Q(x) = \int_x^{\infty} e^{-u^2/2}/\sqrt{2\pi}du$. Since $Q(x)$ is monotonically decreasing for $x \ge 0$, substitution of (3) instead of (2) into (4) leads to the following inequalities as

$$\begin{aligned}\Pr(\mathbf{x}\to\hat{\mathbf{x}}|\mathbf{H}) &\le Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_1\gamma_2}{c_k(\gamma_1+\gamma_2)}\right\}\delta_{\mathbf{x}}^2}\right) \\ \Pr(\mathbf{x}\to\hat{\mathbf{x}}|\mathbf{H}) &\ge Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\right\}\delta_{\mathbf{x}}^2}\right)\end{aligned} \quad (5)$$

where $\delta_{\mathbf{x}}^2 = \|\hat{\mathbf{x}}-\mathbf{x}\|^2$ and $c_k > 1 + (\gamma_1+\gamma_2)^{-1}$.

Furthermore, the Q-function is bounded as

$$\sum_{n=1}^{N} a_n \exp(-b_{n-1}u) \le Q(\sqrt{u}) \le \sum_{n=1}^{N} a_n \exp(-b_n u) \quad (6)$$

for $a_n = (\theta_n - \theta_{n-1})/\pi$ and $b_n = 1/(2\sin^2\theta_n)$ for $n = 1, \cdots, N$ with $\theta_0 = 0$ and $\theta_N = \pi/2$. And also note that the upper bound in (6) can be used for approximation of Q-function

$$
\begin{aligned}
{}_2F_1\left(4,8;\frac{9}{2};-x\right) &= \int_0^1 t^3(1-t)^{-0.5}\,(1+xt)^{-8}\,dt \\
&= \frac{1}{98304(x+1)^7}\left(768x^3+5888x^2+20672x+49152-14980\frac{1}{x}-4060\frac{1}{x^2}-525\frac{1}{x^3}\right) \\
&\quad + \frac{\tanh^{-1}\left((1+1/x)^{-\frac{1}{2}}\right)}{98304(x+1)^7x^4\sqrt{1+1/x}}\left(58800x^3+17640x^2+4410x+525\right)
\end{aligned} \tag{10}
$$

for large N. Then, we can bound the PEP by averaging the conditional PEP in (5) over $\mathbf{H}$ using the Q-function inequality in (6). It gives the upper and lower bounds of the average PEP for the SDF protocol as

$$
\begin{aligned}
&\Pr(\mathbf{x}\to\hat{\mathbf{x}}) \\
&\leq \sum_{n=1}^{N}\mathbb{E}_{\mathbf{H}}\left[a_n\exp\left(-\frac{b_n\delta_{\mathbf{x}}^2}{2}\left\{\gamma_0+\frac{H(\gamma_1,\gamma_2)}{2c_k}\right\}\right)\right] \\
&= \sum_{n=1}^{N}a_n\mathcal{M}_{\gamma_0}\left(\frac{b_n\delta_{\mathbf{x}}^2}{2}\right)\mathcal{M}_{H(\gamma_1,\gamma_2)}\left(\frac{b_n\delta_{\mathbf{x}}^2}{4c_k}\right)
\end{aligned} \tag{7}
$$

and

$$
\begin{aligned}
&\Pr(\mathbf{x}\to\hat{\mathbf{x}}) \\
&\geq \sum_{n=1}^{N}\mathbb{E}_{\mathbf{H}}\left[a_n\exp\left(-\frac{b_{n-1}\delta_{\mathbf{x}}^2}{2}\left\{\gamma_0+\frac{H(\gamma_1,\gamma_2)}{2}\right\}\right)\right] \\
&= \sum_{n=1}^{N}a_n\mathcal{M}_{\gamma_0}\left(\frac{b_{n-1}\delta_{\mathbf{x}}^2}{2}\right)\mathcal{M}_{H(\gamma_1,\gamma_2)}\left(\frac{b_{n-1}\delta_{\mathbf{x}}^2}{4}\right).
\end{aligned} \tag{8}
$$

In order to check the diversity order of the SDF protocol, let us consider the special case of '*balanced*' channel [21], where $\sigma_{\mathsf{SD}}^2 = \sigma_{\mathsf{RD}}^2 = \sigma^2$ and $P_1 = P_2 = P/2$, i.e., $\Omega_{\min} = \Omega_{\max}$. Using (A· 3) and (A· 6), the PEP can be simplified as

$$
\begin{aligned}
\Pr(\mathbf{x}\to\hat{\mathbf{x}}) &\geq \sum_{n=1}^{N}a_n\frac{{}_2F_1\left(4,8;\frac{9}{2};-\rho_{n-1}\right)}{(1+4\rho_{n-1})^4} \\
\Pr(\mathbf{x}\to\hat{\mathbf{x}}) &\leq \sum_{n=1}^{N}a_n\frac{{}_2F_1\left(4,8;\frac{9}{2};-\frac{\rho_n}{c_k}\right)}{(1+4\rho_n)^4}
\end{aligned} \tag{9}
$$

where $\rho_n = \frac{b_n\delta_{\mathbf{x}}^2\sigma^2}{32}P$. The diversity order can be obtained by calculating the component term of the upper and lower bounds in (7) and (8). The Gauss' hypergeometric function in (9) can be exactly evaluated as (10).

In (10), the first part in the right hand side is dominant and gives the diversity order four. Thus, the $\mathsf{S}\to\mathsf{R}\to\mathsf{D}$ link contributes the diversity order four. Here, what we have shown is that the SDF protocol based on Alamouti code achieves the full diversity order for a specific power allocation, which is enough for us to conclude that the SDF protocol with Alamouti code using two receive antennas at R and D gives the diversity order eight if a proper power allocation is used.

3.3 Bit Error Rate

In this subsection, the BER of SDF protocol using Alamouti code with two antennas in each node is calculated. Let us assume that the minimum distance in the signal constellation is $2d$. And let $\mathbf{x}$ and $\hat{\mathbf{x}}$ be two message vectors, where all the corresponding components but one are equal, namely $x_l \neq \hat{x}_l$. Suppose $|\hat{x}_l - x_l| = 2md$. And it is assumed that $\sigma_{\mathsf{SD}}^2 = \sigma_{\mathsf{SR}}^2 = \sigma_{\mathsf{RD}}^2 = \sigma^2$.

Using (A· 1) and the one-point Padé approximation for (A· 4) in the appendix, the one-dimensional symbol error function for SDF cooperative network is given as

$$
\begin{aligned}
Q_P(md) &\triangleq \Pr(\mathbf{x}\to\hat{\mathbf{x}}) \\
&\approx \sum_{n=1}^{N}a_n\left(1+b_nm^2\sigma^2\alpha P\right)^{-4} \\
&\quad \times \mathcal{M}^{[N_p/N_q]}(b_nm^2\sigma^2)
\end{aligned} \tag{11}
$$

where $\Omega_{\max} = P/2\cdot\max\{\alpha,(1-\alpha)\}$ and $\Omega_{\min} = P/2\cdot\min\{\alpha,(1-\alpha)\}$, and α is a power allocation ratio defined by $\alpha = P_1/P$. In this paper, the following Q-function approximation in [20] is used

$$
Q(\sqrt{u}) \approx \sum_{n=1}^{N}a_n\exp(-b_nu)\,. \tag{12}
$$

Note that even a simple two-point approximation using $a_1 = 1/12$, $a_2 = 1/4$, $b_1 = 1/2$, and $b_2 = 2/3$ is tight in the hign SNR region. Figure 2 compares the BERs of SDF cooperative network with respect to power allocation, α, using both simulation and Padé approximation.

Next, it is assumed that the uniform power allocation is used between S and R, i.e., $\alpha = 1/2$. Then, using (9), the one-dimensional symbol error function of SDF protocol with the uniform power allocation is given as

$$
\begin{aligned}
Q_P(md) &\triangleq \Pr(\mathbf{x}\to\hat{\mathbf{x}}) \\
&\approx \sum_{n=1}^{N}a_n\left(1+\frac{b_n(md)^2\sigma^2}{2}P\right)^{-4} \\
&\quad \times {}_2F_1\left(4,8;\frac{9}{2};-\frac{b_n(md)^2\sigma^2}{8}P\right).
\end{aligned} \tag{13}
$$

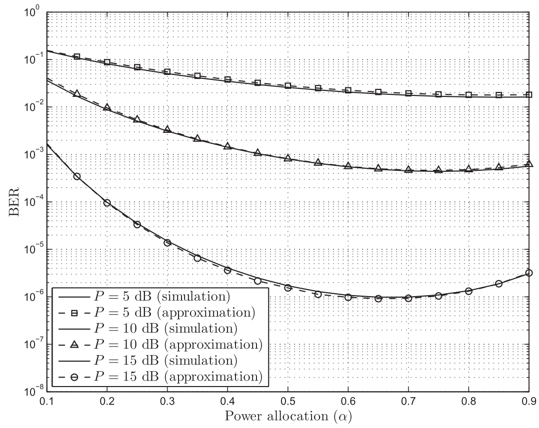

Fig. 2 BERs of SDF cooperative network with respect to power allocation using Padé approximation with order $(N_p, N_q) = (7, 8)$.

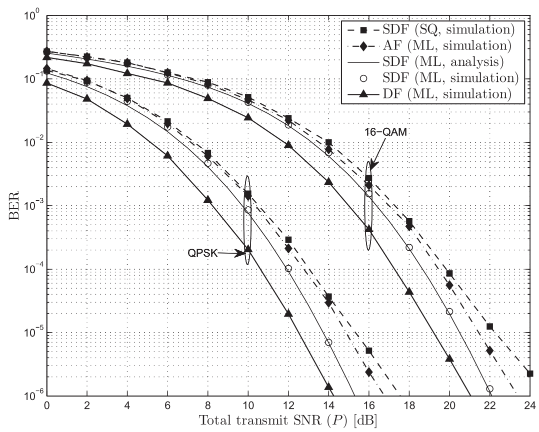

Fig. 3 Comparison of BERs for SDF, AF, and DF with no decoding error at R for QPSK and 16-QAM with uniform power allocation ($\alpha = 0.5$) in the symmetric Rayleigh fading channel.

Using the result in [19], BERs of quarternary PSK (QPSK) and 16-ary quadrature amplitude modulation (16-QAM) can be obtained from (13) as

$$\mathrm{BER}_{\mathrm{QPSK}} = Q_P(d)$$
$$\mathrm{BER}_{16\text{-QAM}} = \frac{3}{4}Q_P(d) + \frac{1}{2}Q_P(3d) - \frac{1}{4}Q_P(5d).$$

Note that d is chosen so as to make the average energy of the constellation one, e.g., $d = 1/\sqrt{2}$ and $d = 1/\sqrt{10}$ for QPSK and 16-QAM, respectively. Figure 3 shows the comparison between the simulation and the analytical results of the SDF protocol with uniform power allocation for two transmit and two receive antennas. Note that the total transmit SNR corresponds to the total transmit power because of the unit noise variance. We can observe only a small gap between simulation and analytical results. The BERs of OAF with ML decoding [11] and DF with perfect decoding at R under ML decoding are plotted. Note that DF protocol without decoding error at R under ML decoding is the same scheme as 4×2 STBC whose diversity order is eight. Thus, it can be shown that SDF protocol achieves the full diversity order through the comparison with DF with perfect decoding at R, which verifies the result in Sect. 3.2.

4. Power Allocation Strategy

In this section, the power allocation scheme for the SDF protocol is considered. The optimal power allocation ratio is selected so as to minimize the average BER (or SER), i.e.,

$$\alpha^{\circ} = \arg\min_{0 \le \alpha \le 1} \mathrm{BER}. \tag{14}$$

However, it is difficult to find the global minimum analytically. Thus, we use the alternative approach instead. The error rate in [15] and [16] is approximated by using the product SNR as follows:

$$\text{Error rate} \approx \frac{\text{constant}}{P^{\text{diversity order}}}\left(\mathbb{E}\left[\gamma_0\right]\mathbb{E}\left[\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}\right]\right)^{-1}. \tag{15}$$

In other words, the suboptimal power allocation is obtained by maximizing the product of two SNRs of the direct and relay links.

Thus, when the CSI is not available at S, it is worthwhile to consider the alternative, namely, the suboptimal power allocation in the slow-varying Rayleigh fading channel. Using (A· 2) in the appendix, the average product of two SNRs of the direct and relay links at D is calculated as

$$\begin{aligned}\Pi(\gamma_{\mathrm{eq}}) &\approx \mathbb{E}\left[\gamma_0\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}\right] = \mathbb{E}\left[\gamma_0\right]\mathbb{E}\left[\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}\right] \\ &= \frac{8}{9}\sigma_{\mathsf{SD}}^2 P_1 \frac{\Omega_{\min}^5}{\Omega_{\max}^4}\,{}_2F_1\left(9, 5; 10; 1 - \frac{\Omega_{\min}}{\Omega_{\max}}\right)\end{aligned}$$

where $\Omega_{\max} = \max(\Omega_1, \Omega_2)$ and $\Omega_{\min} = \min(\Omega_1, \Omega_2)$ for $\Omega_1 = \sigma_{\mathsf{SR}}^2\alpha P$ and $\Omega_2 = \sigma_{\mathsf{RD}}^2(1 - \alpha)P$. Thus, the average product SNR can be rearranged as (16).

For example, we consider the symmetric channel, i.e., $\sigma_{\mathsf{SD}}^2 = \sigma_{\mathsf{SR}}^2 = \sigma_{\mathsf{RD}}^2 = 1$. In this case, it is expected that the suboptimal power allocation ratio is determined for $\alpha > 0.5$. Letting $1-(1-\alpha)/\alpha \triangleq x$, (16) can be rewritten as a function of x

$$\Pi(\gamma_{\mathrm{eq}}) = \frac{4}{9}\frac{(1-x)^5}{(2-x)^2}P^2\,{}_2F_1\left(9, 5; 10; x\right).$$

Then, the suboptimal power allocation is obtained as

$$\begin{aligned}\alpha^{\star} &= \arg\max_{0 \le \alpha \le 1} \Pi(\gamma_{\mathrm{eq}}) \\ &= \left\{\frac{1}{2-x}\,\middle|\,\frac{d}{dx}\left[\frac{(1-x)^5}{(2-x)^2}\,{}_2F_1\left(9, 5; 10; x\right)\right] = 0\right\}.\end{aligned} \tag{17}$$

The resulting suboptimal power allocation ratio is $\alpha^{\star} = 0.6765$. We applied the similar approach to the asymmetric channels such as $\sigma_{\mathsf{RD}}^2/\sigma_{\mathsf{SR}}^2 = 4$ and $\sigma_{\mathsf{RD}}^2/\sigma_{\mathsf{SR}}^2 = 1/4$. The resulting suboptimal power allocation ratios are $\alpha^{\star} = 0.7688$ and $\alpha^{\star} = 0.5932$, respectively. As one can easily anticipate, we can observe that as the ratio $\sigma_{\mathsf{RD}}^2/\sigma_{\mathsf{SR}}^2$ becomes smaller,

$$\Pi(\gamma_{\rm eq}) = \begin{cases} \dfrac{4}{9}\dfrac{\sigma_{\rm SD}^2(\sigma_{\rm RD}^2)^5}{(\sigma_{\rm SR}^2)^4}\dfrac{(1-\alpha)^5}{\alpha^3}P^2{}_2F_1\left(9,5;10;1-\dfrac{\sigma_{\rm RD}^2(1-\alpha)}{\sigma_{\rm SR}^2\alpha}\right), & \sigma_{\rm SR}^2\alpha \geq \sigma_{\rm RD}^2(1-\alpha) \\ \dfrac{4}{9}\dfrac{\sigma_{\rm SD}^2(\sigma_{\rm SR}^2)^5}{(\sigma_{\rm RD}^2)^4}\dfrac{\alpha^6}{(1-\alpha)^4}P^2{}_2F_1\left(9,5;10;1-\dfrac{\sigma_{\rm SR}^2\alpha}{\sigma_{\rm RD}^2(1-\alpha)}\right), & \sigma_{\rm SR}^2\alpha \leq \sigma_{\rm RD}^2(1-\alpha) \end{cases} \tag{16}$$

the suboptimal power allocation ratio approaches to 0, and on the other hand, as the ratio $\sigma_{\rm RD}^2/\sigma_{\rm SR}^2$ becomes larger, the suboptimal power allocation ratio approaches to 1. This can be verified in Fig. 4 where shows the suboptimal power allocation with respect to the ratio of $\sigma_{\rm RD}^2$ and $\sigma_{\rm SR}^2$. For the comparison, the optimal power allocation ratios with respect to P and $(\sigma_{\rm SD}^2, \sigma_{\rm SR}^2, \sigma_{\rm RD}^2)$ are listed in Table 1 through the numerical experiments.

5. Numerical Results

A single relay cooperative communication network with two antennas at the transmitter and receiver is considered. The channel is assumed to be Rayleigh-faded and frequency-flat quasi-static, i.e., the channel state does not change in one phase but varies independently from phase to phase. We only consider QPSK and the total transmit power during two phases is set to P. For the sake of tractability, only three different channel conditions are considered: symmetric ($\sigma_{\rm SD}^2 = \sigma_{\rm SR}^2 = \sigma_{\rm RD}^2$) and asymmetric ($\sigma_{\rm SR}^2 = 4\sigma_{\rm RD}^2$ and $4\sigma_{\rm SR}^2 = \sigma_{\rm RD}^2$).

First, the channel statistics are given as $\sigma_{\rm SD}^2 = \sigma_{\rm SR}^2 = 1$ and various values of $\sigma_{\rm RD}^2$. Figures 5, 6, and 7 plot the BER performance of the SDF protocol in the different channel conditions. Figure 5 shows the BER of the SDF protocol in the symmetric channel, i.e., $\sigma_{\rm RD}^2 = 1$. From the numerical result, we can conclude that the performance degradation under the suboptimal power allocation instead of the optimal power allocation is negligible. And the optimal power allocation has about 0.5 dB performance gain over the uniform power allocation.

Figures 6 and 7 show the BERs for the asymmetric channel with $\sigma_{\rm RD}^2 = 4$ and $\sigma_{\rm RD}^2 = 0.25$, respectively. Figure 6 shows that the performances of the optimal and suboptimal power allocations are almost the same, which can be explained from the increase of the received SNR in the $\mathsf{R} \to \mathsf{D}$ link. On the other hand, if $\mathsf{R} \to \mathsf{D}$ link is not good, the direct transmission ($\alpha = 1$) is optimal in the low SNR region. As the total transmit power increases, the cooperative communication network with SDF protocol outperforms the direct transmission. One can see from Fig. 7 that the performance gap between the optimal and suboptimal BERs becomes negligible as the total transmit power increases. This

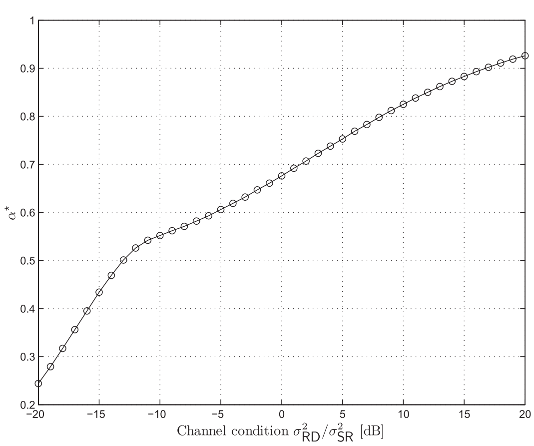

Fig. 4 Suboptimal power allocation with respect to channel condition.

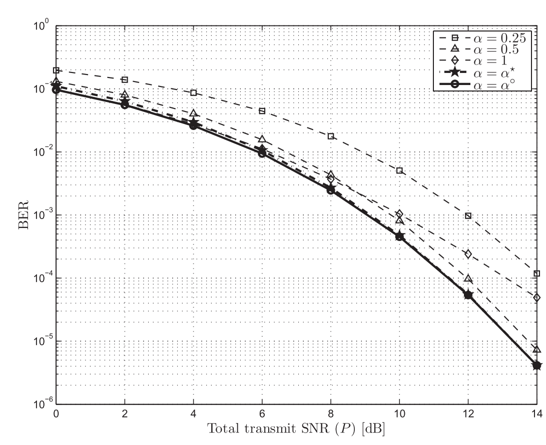

Fig. 5 Comparison of BERs of the SDF protocol with respect to different power allocation ratio $\alpha = 0.25,\ 0.5,\ 1,\ \alpha^\star$, and α° for QPSK in the symmetric Rayleigh fading channel ($\alpha^\star = 0.6765$ and α° in Table 1).

Table 1 The optimal power allocation ratios (α°) in different channel conditions.

$(\sigma_{\rm SD}^2, \sigma_{\rm SR}^2, \sigma_{\rm RD}^2)$ \ P [dB]	0	2	4	6	8	10	12	14	$\alpha^\star$
(1,1,1)	0.979	0.928	0.880	0.835	0.790	0.749	0.711	0.686	0.6765
(1,1,4)	0.829	0.820	0.809	0.801	0.788	0.782	0.778	0.774	0.7688
(1,1,0.25)	1	1	1	1	1	1	0.880	0.773	0.5932
(1,4,1)	0.841	0.794	0.736	0.691	0.652	0.615	0.584	0.556	0.5932
(1,4,4)	0.651	0.645	0.637	0.630	0.624	0.620	0.614	0.613	0.6765

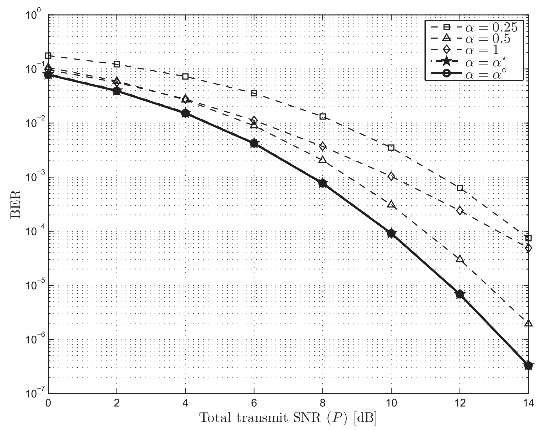

Fig. 6 Comparison of BERs of the SDF protocol with respect to different power allocation ratio $\alpha = 0.25,\ 0.5,\ 1,\ \alpha^\star$, and α° for QPSK in the asymmetric Rayleigh fading channel $((\sigma^2_{\mathsf{SD}}, \sigma^2_{\mathsf{SR}}, \sigma^2_{\mathsf{RD}}) = (1, 1, 4)$, $\alpha^\star = 0.7688$, and α° in Table 1).

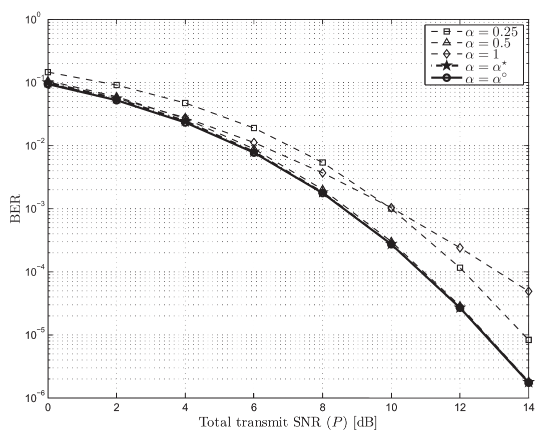

Fig. 7 Comparison of BERs of the SDF protocol with respect to different power allocation ratio $\alpha = 0.25,\ 0.5,\ 1,\ \alpha^\star$, and α° for QPSK in the asymmetric Rayleigh fading channel $((\sigma^2_{\mathsf{SD}}, \sigma^2_{\mathsf{SR}}, \sigma^2_{\mathsf{RD}}) = (1, 1, 0.25)$, $\alpha^\star = 0.5932$, and α° in Table 1).

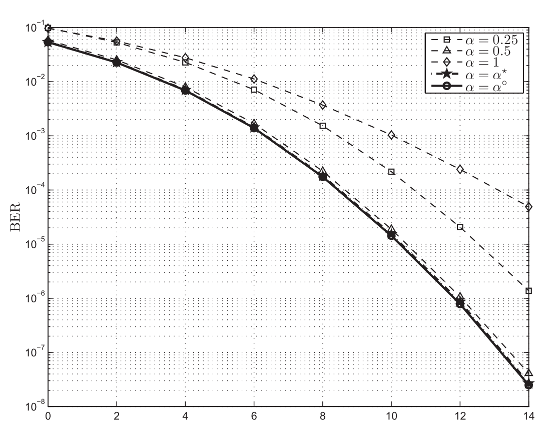

Fig. 8 Comparison of BERs of the SDF protocol with respect to different power allocation ratio $\alpha = 0.25,\ 0.5,\ 1,\ \alpha^\star$, and α° for QPSK in the asymmetric Rayleigh fading channel $((\sigma^2_{\mathsf{SD}}, \sigma^2_{\mathsf{SR}}, \sigma^2_{\mathsf{RD}}) = (1, 4, 1)$, $\alpha^\star = 0.5932$, and α° in Table 1).

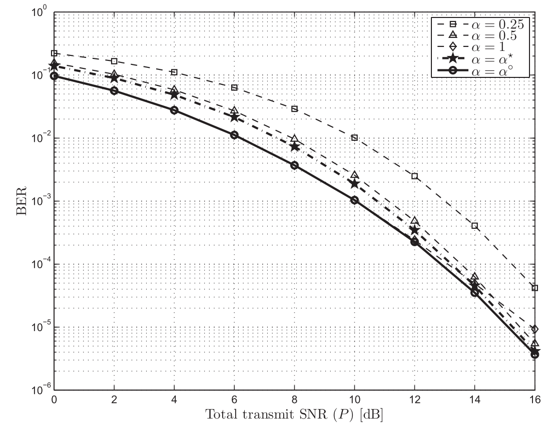

Fig. 9 Comparison of BERs of the SDF protocol with respect to different power allocation ratio $\alpha = 0.25,\ 0.5,\ 1,\ \alpha^\star_{\mathrm{a}}$, and $\alpha^\circ_{\mathrm{a}}$ for QPSK in the asymmetric Rayleigh fading channel $((\sigma^2_{\mathsf{SD}}, \sigma^2_{\mathsf{SR}}, \sigma^2_{\mathsf{RD}}) = (1, 4, 4)$, $\alpha^\star = 0.6765$, and α° in Table 1).

may imply that the maximization of the product SNR corresponds to the suboptimal power allocation.

Second, the channel statistics are given as $\sigma^2_{\mathsf{SD}} = 1$, $\sigma^2_{\mathsf{SR}} = 4$, and different σ^2_{RD}, i.e., better $\mathsf{S} \rightarrow \mathsf{R}$ link than $\mathsf{S} \rightarrow \mathsf{D}$ link is considered. Figures 8 and 9 show the BERs for different power allocations for the case of $\sigma^2_{\mathsf{RD}} = 1$ and $\sigma^2_{\mathsf{RD}} = 4$. It is shown that the performance gap between the optimal and the suboptimal power allocation is very small. Clearly, the cooperative communication network with $\sigma^2_{\mathsf{SR}} = 4$ outperforms that with $\sigma^2_{\mathsf{SR}} = 1$.

6. Concluding Remarks

In this paper, we reviewed the SDF protocol with a single relay based on the Alamouti code with two antennas at S, R, and D. We derived the approximate PEP of the SDF protocol using MGF of harmonic mean of independent gamma random variables and showed that the SDF protocol achieves the full diversity order if the proper power allocation is used. Using the PEP, we also calculated the approximate BERs of SDF protocol. The suboptimal power allocation strategy, obtained by maximizing the product SNR, for the slow-varying Rayleigh fading channel has been considered without feedback of the CSI. The numerical results show that the performance of SDF protocol with the suboptimal power allocation almost matches that with the optimal power allocation in the high SNR region. The performance analysis such as relay selection and power allocation strategy for the SDF cooperative communication network with multiple-relay remains as a future work.

Acknowledgement

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Appendix: Mathematical Backgrounds

In this appendix, the gamma distribution and its related properties are reviewed. We also review the Gauss' hypergeometric function (HGF), ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ and the expectation of the harmonic mean of gamma random variables. The sum X of K independent exponential random variables, each of which has a mean Ω, is a gamma-distributed random variable whose probability density function is expressed in terms of a shape parameter K and scale parameter Ω, denoted by $X \sim \mathcal{G}(K,\Omega)$,

$$f_X(x;K,\Omega) = x^{K-1}\frac{e^{-\frac{x}{\Omega}}}{\Omega^K\Gamma(K)},\quad K,\Omega > 0$$

where $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$. And its moment generating function (MGF) is given as

$$\mathcal{M}_X(s) = \mathbb{E}\left[e^{-sX}\right] = (1+\Omega s)^{-K}. \tag{A·1}$$

Suppose X_1 and X_2 are two independent gamma random variables, where $X_i \sim \mathcal{G}(K,\Omega_i)$. Then, the nth moment of the harmonic mean of X_1 and X_2 [22], $X \triangleq H(X_1,X_2) = \frac{2X_1X_2}{X_1+X_2}$, is given as

$$\begin{aligned}\mathbb{E}\left[X^n\right] =& {}_2F_1\left(2K+n, K+n; 2K+2n; 1-\frac{\Omega_{\min}}{\Omega_{\max}}\right)\\ &\times 2^n\frac{\Gamma(2K+n)}{\Gamma(K)^2}\mathcal{B}(K+n,K+n)\frac{\Omega_{\min}^{K+n}}{\Omega_{\max}^{K}}\end{aligned} \tag{A·2}$$

where $\Omega_{\max} = \max\{\Omega_1,\Omega_2\}$, $\Omega_{\min} = \min\{\Omega_1,\Omega_2\}$, $\mathcal{B}(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function, and the Gauss' HGF [23] denoted by ${}_2F_1(a,b;c;x)$ is defined as

$$\begin{aligned}&{}_2F_1(a,b;c;z)\\ &= \sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}\\ &= \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}dt\end{aligned} \tag{A·3}$$

$$\begin{aligned}\mathcal{M}_X(s) &= \mathbb{E}\left[e^{-sX}\right] = \mathbb{E}\left[\sum_{n=0}^{\infty}\frac{(-sX)^n}{n!}\right] = \sum_{n=0}^{\infty}\frac{(-1)^n\mathbb{E}\left[X^n\right]}{n!}s^n \\ &= \sum_{n=0}^{\infty}\underbrace{\frac{(-2)^n}{n!}\frac{\Gamma(2K+n)}{\Gamma(K)^2}\mathcal{B}(K+n,K+n)\frac{\Omega_{\min}^{K+n}}{\Omega_{\max}^{K}}{}_2F_1\left(2K+n,K+n;2K+2n;1-\frac{\Omega_{\min}}{\Omega_{\max}}\right)}_{r_n}s^n \end{aligned} \tag{A·4}$$

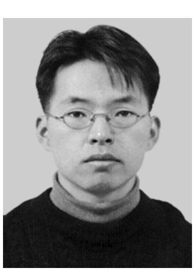
for $\Re\{c\} > \Re\{b\} > 0$ and $|\arg(1-z)| < \pi$, where $(x)_n = \Gamma(x+n)/\Gamma(x)$ is a Pochhammer symbol. From (A·2), the MGF of the harmonic mean X of two independent gamma random variables can be calculated using the Taylor series expansion of the exponential function $e^x = \sum_{n=0}^{\infty} x^n/(n!)$ as (A·4).

Although we have the exact representation of $\mathcal{M}_X(s)$, the actual evaluation involves the infinite series representation which does not gaurantee the convergence. In many applications, one may need to get an approximate closed-form expression for an infinite series representation. The one-point Padé approximation technique of order $[N_p/N_q]$ [24], [25] which approximates the infinite series representation of s as a rational function of s can be applied as follows:

$$\begin{aligned}\mathcal{M}^{[N_p/N_q]}(s) &= \sum_{n=0}^{N_p} p_n s^n \Big/ \sum_{n=0}^{N_q} q_n s^n \\ &= \sum_{n=0}^{N_p+N_q} r_n s^n + O\left(s^{N_p+N_q+1}\right)\end{aligned} \tag{A·5}$$

where $O(s^{n+1})$ means the terms of order higher than s^n. Note that if $\Omega_1 = \Omega_2 = \Omega$, using the result in [26], (A·4) reduces to

$$\mathcal{M}_X(s) = {}_2F_1\left(K, 2K; K+\frac{1}{2}; -\frac{\Omega}{4}s\right). \tag{A·6}$$



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