Soft-Decision-and-Forward Protocol for Cooperative Communication Networks with Multiple Antennas

Jae-Dong Yang, Kyoung-Young Song, Jong-Seon No, and Dong-Joon Shin

Abstract: In this paper, a cooperative relaying protocol called soft-decision-and-forward (SDF) with multiple antennas in each node is introduced. SDF protocol exploits the soft decision source symbol values from the received signal at the relay node. For orthogonal transmission (OT), orthogonal codes including Alamouti code are used and for non-orthogonal transmission (NT), distributed space-time codes are designed by using a quasi-orthogonal space-time block code. The optimal maximum likelihood (ML) decoders for the proposed protocol with low decoding complexity are proposed. For OT, the ML decoders are derived as symbolwise decoders while for NT, the ML decoders are derived as pairwise decoders. It can be seen through simulations that SDF protocol outperforms AF protocol for both OT and NT.

Index Terms: **Amplify-and-forward (AF), cooperative communications, distributed space-time code (DSTC), maximum-likelihood (ML) decoder, soft-decision-and-forward (SDF).**

I. INTRODUCTION

Multiple-antenna transmission has been considered as a core technology for wireless communication systems. Due to the constraint of the practical implementation, however, many wireless communication researchers have been attracted by cooperative communication networks. Through the cooperation of relay node with source node, the spectral efficiency and reliability of the wireless communication systems can be improved. Also, it can be used for the coverage extension by reducing the blanket areas. In general, cooperative communication networks consist of three types of nodes which are source nodes, relay nodes, and destination nodes [1]. In this paper, we consider a cooperative communication network with one source, one relay, and one destination node. For convenience, a source node is expressed as S, a relay node as R, and a destination node as D.

Manuscript received February 4, 2010; approved for publication by Meixia Tao, Division II Editor, October 13, 2010.

The part of this paper was presented in IEEE International Symposium on Information Theory [5].

This work was partly supported by the IT R&D program of MKE/IITA [2008-F-007-03, Intelligent Wireless Communication Systems in 3 Dimensional Environment] and the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MEST) (No. 2009-0081441).

J.-D. Yang is with Samsung Electronics Co., Ltd., Gyeonggi, Korea, email: jaedong.yang@samsung.com.

K.-Y. Song (corresponding author) is with the Advanced Communication Technology R&D Lab., LG Electronics Co., Ltd., Gyeonggi, Korea, email: kyoungyoung.song@lge.com.

J.-S. No are with the Department of EECS, INMC, Seoul National University, Seoul, Korea, email: jsno@snu.ac.kr.

D.-J. Shin is with the Department of ECE, Hanyang University, Seoul, Korea, email: djshin@hanyang.ac.kr.

There are many cooperative protocols which can be categorized according to the operation of R such as amplify-and-forward (AF) and decode-and-forward (DF) [1]–[4]. AF protocol allows R to amplify the received signals according to the power constraint and forward them to D. DF protocol permits R to decode and re-encode the received signals and then forward them to D. Since AF protocol amplifies the received signal, noise component is also amplified and forwarded. On the other hand, since DF protocol makes hard decision at R, the soft-information of the received signal is removed. In this paper, soft-decision-and-forward (SDF) protocol is proposed, which decodes the received signals into the soft-decision values at R and re-encodes them and then forwards them to D. The term 'soft-decision' value is used because soft decision values of source symbols are used at R.

We found that similar protocols to SDF protocol were presented in [6] and [7]. In [6], process-and-forward (PF) protocol, which is equipped with one antenna in each node, was proposed, which allows relay to perform space-time processing on the received signals in the manner of distributed space-time coding. It was shown that AF and PF protocols have the same performance for one antenna at R. However, for SDF protocol, multiple antennas are assumed at all nodes and thus, unlike PF protocol, it shows better performance than AF protocol. In [7], decouple-and-forward (DCF) protocol, which uses Alamouti coding with two-antenna transmission, for dual hop cooperative communications was proposed, which allows R to decouple the received signals. In spite of correlated noise at D, squaring method [8] was used for DCF protocol. However, for SDF protocol, it is assumed that D can hear S and the optimal maximum likelihood (ML) decoder is used. Furthermore, it is shown that the ML decoder can be simplified in spite of correlated noises at D.

In the first phase, S transmits, and in the second phase, R transmits and S transmits or not. If S does not transmit in the second phase, it is called orthogonal transmission (OT). Otherwise, it is called non-orthogonal transmission (NT). In this paper, four schemes are considered, that is, OT-AF, OT-SDF, NT-AF, and NT-SDF, and their ML decoders are derived as simplified forms. For the ease of processing at R, the orthogonal space-time block codes are used, which is appropriate to decouple the signals to obtain the soft-decision source symbol values.

In this paper, the following notations are used: A capital boldface letter denotes a matrix, a small boldface letter denotes a vector, $\Re(\cdot)$ denotes the real part of a complex number, $(\cdot)^*$ denotes the complex conjugate, $(\cdot)^T$ denotes the transpose of a matrix, $(\cdot)^H$ denotes the complex conjugate and transpose of a matrix, $|\cdot|$ denotes the norm of a complex number, $\|\cdot\|$ denotes the Frobenius norm of a matrix, $\mathbf{I}_n$ denotes the $n \times n$ identity matrix, $\mathbf{0}$ denotes the all zero matrix. $\mathrm{diag}(\cdot)$ means the diago-

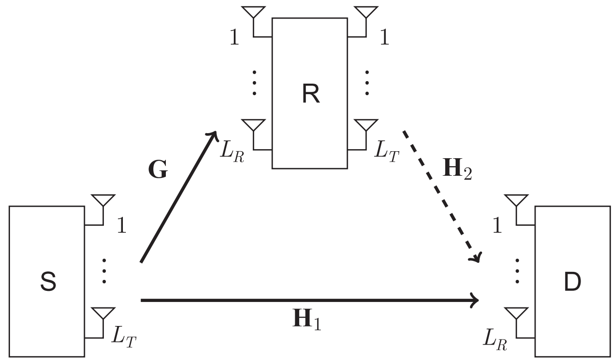


Fig. 1. Cooperative communication network with multiple antennas.

nal matrix.

This paper is organized as follows. In Section II, the cooperative communication network is explained. In Section III, four schemes such as OT-AF, OT-SDF, NT-AF, and NT-SDF are investigated. In Section IV, the optimal ML decoder for each scheme is derived. The extension to more than two-antenna transmission is addressed in Section V. In Section VI, numerical results are shown and the conclusion is given.

II. SYSTEM MODEL

We consider a cooperative communication network which consists of three nodes S, D, and R. We assume that all nodes have multiple antennas, and furthermore, the number of transmit and receive antennas is the same, i.e., $L_T = L_R = L$. Fig. 1 shows such cooperative communication network. Let $\mathbf{G}$ be the fading channel matrix between S and R, $\mathbf{H}_1$ be the fading channel matrix between S and D, and $\mathbf{H}_2$ be the fading channel matrix between R and D. All the fading channel matrices are $L \times L$ matrices whose entries are independent complex Gaussian random variables with zero mean and unit variance. Note that g^{ij}, h_1^{ij}, and h_2^{ij}, for $i, j = 1, 2$ are the fading channel coefficients from the ith transmit antenna to the jth receive antenna. We assume that all the channels are quasi-static Rayleigh fading. Furthermore, perfect channel state information is assumed to be known only at the receiver, that is, R knows g^{ij} and D knows g^{ij}, h_1^{ij}, and h_2^{ij}.

We also assume half-duplex communications, i.e., all nodes can either transmit or receive. In the first phase, S transmits signals bearing the information. Since S broadcasts its signals, it is sometimes called broadcast phase. In the second phase, R transmits signals to D to help the communication between S and D, i.e., cooperation. Thus, the second phase is sometimes called cooperation phase. In the second phase, S can either transmit or not. In this paper, we consider both NT and OT, and for NT, source antenna switching (SAS) [4] is considered such that S utilizes two RF chains, where each RF chain has two transmit antennas. Thus, there are four transmit antennas at S. In the first phase, one transmit antenna at each RF chain is used and then in the second phase the other two antennas are used. By using SAS for DF protocol, the additional diversity gain can be obtained [4].

Relaying protocols are classified by the operation at R. There are two well-known protocols, AF and DF protocols. For AF protocol, R amplifies the received signals and forwards them to D. Therefore, noise component is also amplified and forwarded. For DF protocol, R decodes the received signals, re-encodes, and transmits them to D. Since R performs hard decision, the soft information of the received signal cannot be fully utilized. Therefore, we propose the SDF protocol which exploits the soft-decision source symbol values. In the next section, combining OT and NT with AF and SDF, we investigate four schemes, that is, OT-AF, OT-SDF, NT-AF, and NT-SDF.

III. AF AND SDF PROTOCOLS

In this section, we consider the cooperative communication network with two antennas, i.e., $L = 2$. For AF protocol, since two transmit and receive antennas at each node are considered, we assume that the received signal at the first (second) receive antenna of R is amplified and forwarded through the first (second) transmit antenna of R. For SDF protocol, R decodes the received signals into the soft decision source symbol values. R re-encodes and transmits them to D. Since the transmission of the soft values can improve the performance compared with AF protocol, it is useful for the cooperative relay network. In this section, we assume that S transmits Alamouti code [9] and four schemes are described, which are OT-AF, OT-SDF, NT-AF, and NT-SDF. To explain each scheme in Sections III and IV, the following definitions will be used.

Definition. *Alamouti operation and complex vectorization:*

- *Alamouti operation for two symbols a and b is represented as* $\mathbf{A}(a,b) = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$.
- *For 2×2 matrices* $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ *and* $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, *let* $\mathbf{B}' = \begin{bmatrix} \mathbf{M} \cdot \mathbf{A}(b_{11}, b_{21}) \\ \mathbf{M} \cdot \mathbf{A}(b_{12}, b_{22}) \end{bmatrix}$.
- *Let $cv(\cdot)$ denote complex vectorization operation for 2×2 matrix, i.e.,* $cv\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [a\ c^*\ b\ d^*]^T$. ■

A. AF Protocol with OT

In the first phase, S transmits an Almouti code which consists of two independent symbols x_1 and x_2, i.e., $\mathbf{X} = \mathbf{A}(x_1, x_2)$. The received signals at R and D in the first phase can be expressed as

$$\mathbf{Y}_{\mathsf{R}} = \sqrt{P_1}\mathbf{X}\mathbf{G} + \mathbf{N}_{\mathsf{R}}$$
$$\mathbf{Y}_{\mathsf{D1}} = \sqrt{P_1}\mathbf{X}\mathbf{H}_1 + \mathbf{N}_{\mathsf{D1}} \quad (1)$$

where $\mathbf{Y}_{\mathsf{R}}$ is the received signal matrix at R, $\mathbf{Y}_{\mathsf{D1}}$ is the received signal at D, $\mathbf{N}_{\mathsf{R}}$ and $\mathbf{N}_{\mathsf{D1}}$ are additive white Gaussian noise (AWGN) matrices with entries of zero mean and unit variance complex Gaussian random variables. Note that P_1 denotes the average power per antenna at S.

In the second phase, R transmits signals by regenerating the received signals in the first phase in the two manners: AF and SDF. In this subsection, we consider AF protocol, that is, R amplifies the received signal. The transmitted signal at R is given as

$$\mathbf{X}_{\mathsf{R}} = \mathbf{Y}_{\mathsf{R}}\boldsymbol{\beta}$$

where $\mathbf{X}_{\mathsf{R}}$ is the transmitted signal and $\boldsymbol{\beta} = \mathrm{diag}(\beta_1, \beta_2)$ is the amplification matrix at R. That is, the received signal at the first (second) antenna at R is amplified by β_1 (β_2) and forwarded to D. It can be easily shown that

$$\beta_i = \sqrt{\frac{P_2}{P_1(|g^{1i}|^2 + |g^{2i}|^2) + 1}}$$

ensures the average power per antenna at R to be P_2 for $i = 1, 2$.

In OT-AF, S is silent in the second phase and thus, only R transmits in the second phase. The received signal at D in the second phase is given as

$$\begin{aligned}\mathbf{Y}_{\mathsf{D}2} &= \mathbf{Y}_R\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2}\\ &= \sqrt{P_1}\mathbf{X}\mathbf{F} + \mathbf{N}_{\mathsf{D}}\end{aligned} \tag{2}$$

where $\mathbf{F} = \mathbf{G}\boldsymbol{\beta}\mathbf{H}_2$, $\mathbf{N}_{\mathsf{D}} = \mathbf{N}_{\mathsf{R}}\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2}$, and $\mathbf{N}_{\mathsf{D}2}$ is AWGN matrix with entries of zero mean and unit variance complex Gaussian random variables.

Equivalent vector model is useful to derive the ML decoder. By using (1), (2), and the Almouti code structure, the equivalent model for OT-AF can be expressed as

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D}1}) \\ cv(\mathbf{Y}_{\mathsf{D}2}) \end{bmatrix}}_{\mathbf{y}_e} = \underbrace{\begin{bmatrix} \sqrt{P_1}\mathbf{H}_1' \\ \sqrt{P_1}\mathbf{F}' \end{bmatrix}}_{\mathbf{H}_e} \mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}_{\mathsf{D}1}) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix}}_{\mathbf{n}_e} \tag{3}$$

where $\mathbf{x} = [x_1\ x_2]^T$. Equation (3) has similar form to the conventional multiple-input multiple-output (MIMO) systems.

B. SDF Protocol with OT

In the first phase, we have the same situation as in Subsection III-A. Thus, the received signals at R and D in the first phase are given in (1).

However, the operation of R is different from OT-AF. For SDF protocol, R decodes the received signals into the soft decision values, and re-encodes them, and transmits the re-encoded signals. Since S transmits Alamouti code, we have soft decision values for x_1 and x_2 at R as

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \gamma(\mathbf{G}')^H cv(\mathbf{Y}_{\mathsf{R}}) \tag{4}$$

where γ is the power gain at the relay. It can be easily shown that we have $\gamma = \sqrt{P_2/(\|\mathbf{G}\|^2(1 + P_1\|\mathbf{G}\|^2))}$ to make the average power per antenna at R to be P_2. Using the soft decision values in (4), R re-encodes them by using Alamouti code and thus, we have $\mathbf{X}_{\mathsf{R}} = \mathbf{A}(\hat{x}_1, \hat{x}_2)$.

The received signal at D in the second phase is given as

$$\mathbf{Y}_{\mathsf{D}2} = \mathbf{X}_{\mathsf{R}}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2}.$$

Using (4) and after some manipulations, we have the following equivalent model of OT-SDF.

$$\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D}1}) \\ cv(\mathbf{Y}_{\mathsf{D}2}) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1}\mathbf{H}_1' \\ a\mathbf{H}_2' \end{bmatrix} \mathbf{x} + \begin{bmatrix} cv(\mathbf{N}_{\mathsf{D}1}) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix} \tag{5}$$

where $a = \gamma\sqrt{P_1}\|\mathbf{G}\|^2$, $cv(\mathbf{N}_{\mathsf{D}}) = \gamma\mathbf{H}_2'(\mathbf{G}')^H cv(\mathbf{N}_{\mathsf{R}}) + cv(\mathbf{N}_{\mathsf{D}2})$, and $\mathbf{x} = [x_1\ x_2]^T$. Equation (5) also has similar form to conventional MIMO systems.

C. AF Protocol with NT

In this subsection, NT is considered and thus, S transmits signals in the second phase. Since S and R transmit signals in the second phase simultaneously, we design a distributed space-time code (DSTC) for cooperation between S and R. Since there are two antennas at each node, we consider a 4×4 quasi-orthogonal space-time block code (QO-STBC) shown in [10], which is spread over S and R. For the use of DSTC in the second phase, we assume that S transmits two Alamouti codes, i.e., $\mathbf{X}_1 = \mathbf{A}(x_1, x_2)$ and $\mathbf{X}_2 = \mathbf{A}(x_3, x_4)$, in the first phase. Thus, the received signals at R and D in the first phase are given as

$$\begin{aligned}\mathbf{Y}_{\mathsf{R}1} &= \sqrt{P_1}\mathbf{X}_1\mathbf{G} + \mathbf{N}_{\mathsf{R}1}\\ \mathbf{Y}_{\mathsf{R}2} &= \sqrt{P_1}\mathbf{X}_2\mathbf{G} + \mathbf{N}_{\mathsf{R}2}\\ \mathbf{Y}_{\mathsf{D}1,1} &= \sqrt{P_1}\mathbf{X}_1\mathbf{H}_1 + \mathbf{N}_{\mathsf{D}1,1}\\ \mathbf{Y}_{\mathsf{D}1,2} &= \sqrt{P_1}\mathbf{X}_2\mathbf{H}_1 + \mathbf{N}_{\mathsf{D}1,2}\end{aligned} \tag{6}$$

where $\mathbf{Y}_{\mathsf{R}1}$ and $\mathbf{Y}_{\mathsf{R}2}$ are 2×2 received signal matrices at R, $\mathbf{Y}_{\mathsf{D}1,1}$ and $\mathbf{Y}_{\mathsf{D}1,2}$ are received signal matrices at D, and $\mathbf{N}_{\mathsf{R}1}$, $\mathbf{N}_{\mathsf{R}2}$, $\mathbf{N}_{\mathsf{D}1,1}$, and $\mathbf{N}_{\mathsf{D}1,2}$ are 2×2 AWGN matrices.

Similar to Subsection III-A, R amplifies the received signals and forwards them to D. Thus, R transmits $\mathbf{R}_\theta\mathbf{Y}_{\mathsf{R}2}\boldsymbol{\beta}$ at first and then transmits $\mathbf{Y}_{\mathsf{R}1}\boldsymbol{\beta}$. Note that $\mathbf{R}_\theta = \mathbf{A}(e^{j\theta}, 0)$ and θ is the constellation rotation angle for full diversity in QO-STBC [11]. In this paper, θ is assumed to be $\pi/4$. S transmits $\mathbf{X}_1$ at first and then $\mathbf{R}_\theta\mathbf{X}_2$. The received signal model in the second phase can be expressed as

$$\begin{aligned}\mathbf{Y}_{\mathsf{D}2,1} &= \sqrt{P_3}\mathbf{X}_1\mathbf{H}_1 + \mathbf{R}_\theta\mathbf{Y}_{\mathsf{R}2}\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2,1}\\ &= \sqrt{P_3}\mathbf{X}_1\mathbf{H}_1 + \sqrt{P_1}\mathbf{R}_\theta\mathbf{X}_2\mathbf{F} + \mathbf{N}_{e1}\\ \mathbf{Y}_{\mathsf{D}2,2} &= \sqrt{P_3}\mathbf{R}_\theta\mathbf{X}_2\mathbf{H}_1 + \mathbf{Y}_{\mathsf{R}1}\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2,2}\\ &= \sqrt{P_3}\mathbf{R}_\theta\mathbf{X}_2\mathbf{H}_1 + \sqrt{P_1}\mathbf{X}_1\mathbf{F} + \mathbf{N}_{e2}\end{aligned} \tag{7}$$

where P_2 and P_3 are the average power per antenna at R and S, respectively, $\mathbf{N}_{\mathsf{D}2,1}$ and $\mathbf{N}_{\mathsf{D}2,2}$ are 2×2 AWGN matrices, $\mathbf{F} = \mathbf{G}\boldsymbol{\beta}\mathbf{H}_2$, $\mathbf{N}_{e1} = \mathbf{R}_\theta\mathbf{N}_{\mathsf{R}2}\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2,1}$, and $\mathbf{N}_{e2} = \mathbf{N}_{\mathsf{R}1}\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2,2}$. From (7), we can see that a QO-STBC is transmitted to D in the second phase.

For NT-AF, the equivalent vector model can be expressed as

$$\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D}1,1}) \\ cv(\mathbf{Y}_{\mathsf{D}1,2}) \\ cv(\mathbf{Y}_{\mathsf{D}2,1}) \\ cv(\mathbf{Y}_{\mathsf{D}2,2}) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1}\mathbf{H}_1' & \mathbf{0} \\ \mathbf{0} & \sqrt{P_1}\mathbf{H}_1' \\ \sqrt{P_3}\mathbf{H}_1' & \sqrt{P_1}e^{j\theta}\mathbf{F}' \\ \sqrt{P_1}\mathbf{F}' & \sqrt{P_3}e^{j\theta}\mathbf{H}_1' \end{bmatrix} \mathbf{x} + \begin{bmatrix} cv(\mathbf{N}_{\mathsf{D}1,1}) \\ cv(\mathbf{N}_{\mathsf{D}1,2}) \\ cv(\mathbf{N}_{e1}) \\ cv(\mathbf{N}_{e2}) \end{bmatrix} \tag{8}$$

where $\mathbf{x} = [x_1\ x_2\ x_3\ x_4]^T$.

D. SDF Protocol with NT

In the similar way as Subsection III-C, for NT-SDF, we assume that S transmits $\mathbf{X}_1$ and $\mathbf{X}_2$ in the first phase, and a QO-STBC is used in the second phase. Since there is no difference

$$\mathbf{P}^H\mathbf{K}_a\mathbf{P} = \{p(|a|^2+|b|^2)+r(|c|^2+|d|^2)+2\Re[(a^*c+b^*d)q]\}\mathbf{I}_2 \quad (11)$$

$$\mathbf{P}^H\mathbf{K}_s\mathbf{P} = \{x(|a|^2+|b|^2)+y(|c|^2+|d|^2)+2\Re[(a^*c+b^*d)u+(a^*d^*-b^*c^*)v]\}\mathbf{I}_2 \quad (12)$$

$$\mathbf{P}^H\mathbf{K}_a\mathbf{Q}+\mathbf{Q}^H\mathbf{K}_a\mathbf{P} = \{2p\Re[a^*e+bf^*]+2r\Re[c^*g+dh^*]+2\Re[(a^*g+b^*h+ce^*+df^*)q]\}\mathbf{I}_2 \quad (13)$$

$$\begin{aligned}\mathbf{P}^H\mathbf{K}_s\mathbf{Q}+\mathbf{Q}^H\mathbf{K}_s\mathbf{P} = \{&2x\Re[a^*e+bf^*]+2y\Re[c^*g+dh^*]+2\Re[(a^*g+b^*h+ce^*+df^*)u\\ &+(a^*h^*-b^*g^*+d^*e^*-c^*f^*)v]\}\mathbf{I}_2\end{aligned} \quad (14)$$

between NT-AF and NT-SDF in the first phase, the received signals at R and D are given in (6).

However, the operation at R is different from Subsection III-C. In this subsection, we consider SDF protocol and thus, R performs the similar operation in Subsection III-B. R decodes the received signals into the soft decision values as follows:

$$\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} = \gamma(\mathbf{G}')^H cv(\mathbf{Y}_{\mathsf{R}1}),\quad \begin{bmatrix}\hat{x}_3\\ \hat{x}_4\end{bmatrix} = \gamma(\mathbf{G}')^H cv(\mathbf{Y}_{\mathsf{R}2})$$

where γ is the same as in (4). Similar to Subsection III-C, to construct a QO-STBC in the second phase, R transmits $\mathbf{R}_\theta\mathbf{A}(\hat{x}_3,\hat{x}_4)$ at first and then transmits $\mathbf{A}(\hat{x}_1,\hat{x}_2)$. S transmits $\mathbf{X}_1$ at first and then $\mathbf{R}_\theta\mathbf{X}_2$ in the second phase. This operation guarantees that the power of the transmitted signal per antenna of R and S in every time slot of the second phase is the P_2 and P_3, respectively.

The equivalent vector model can be expressed as

$$\begin{bmatrix}cv(\mathbf{Y}_{\mathsf{D}1,1})\\ cv(\mathbf{Y}_{\mathsf{D}1,2})\\ cv(\mathbf{Y}_{\mathsf{D}2,1})\\ cv(\mathbf{Y}_{\mathsf{D}2,2})\end{bmatrix} = \begin{bmatrix}\sqrt{P_1}\mathbf{H}_1' & \mathbf{0}\\ \mathbf{0} & \sqrt{P_1}\mathbf{H}_1'\\ \sqrt{P_3}\mathbf{H}_1' & ae^{j\theta}\mathbf{H}_2'\\ a\mathbf{H}_2' & \sqrt{P_3}e^{j\theta}\mathbf{H}_1'\end{bmatrix}\mathbf{x} + \begin{bmatrix}cv(\mathbf{N}_{\mathsf{D}1,1})\\ cv(\mathbf{N}_{\mathsf{D}1,2})\\ cv(\mathbf{N}_{e1})\\ cv(\mathbf{N}_{e2})\end{bmatrix} \quad (9)$$

where a is the same as in (5), $\mathbf{x} = [x_1\ x_2\ x_3\ x_4]^T$, and

$$\begin{aligned}cv(\mathbf{N}_{e1}) &= \gamma e^{j\theta}\mathbf{H}_2'(\mathbf{G}')^H cv(\mathbf{N}_{\mathsf{R}2}) + cv(\mathbf{N}_{\mathsf{D}2,1})\\ cv(\mathbf{N}_{e2}) &= \gamma\mathbf{H}_2'(\mathbf{G}')^H cv(\mathbf{N}_{\mathsf{R}1}) + cv(\mathbf{N}_{\mathsf{D}2,2}).\end{aligned}$$

IV. OPTIMAL ML DECODERS

In the previous section, four schemes have been explained. Now, we want to derive the optimal ML decoder for each scheme. To this end, we have derived the equivalent vector model for each scheme. The four equivalent models (3), (5), (8), and (9) can be commonly expressed as

$$\mathbf{y}_e = \mathbf{H}_e\mathbf{x} + \mathbf{n}_e. \quad (10)$$

Since the noise at R is transmitted to both receive antennas at D, entries of equivalent noise vector $\mathbf{n}_e$ are correlated. To derive the ML decoder for each scheme, we introduce some properties in the next subsection and we derive the ML decoders in the following subsection.

A. Preliminaries for Deriving ML Decoders

In this paper, we use Alamouti code as a basic building block and thus, we defined Alamouti operation in the previous section. The following properties are useful to derive the ML decoder for each scheme.

Properties of Alamouti operation:
1. $\mathbf{A}^H(a,b) = \mathbf{A}(a^*,-b)$
2. $\mathbf{A}(a,b)+\mathbf{A}(c,d) = \mathbf{A}(a+c,b+d)$
3. $\mathbf{A}(a,b)\cdot\mathbf{A}(c,d) = \mathbf{A}(ac-bd^*, ad+bc^*)$
4. $\mathbf{A}^H(a,b)\cdot\mathbf{A}(a,b) = \mathbf{A}(a,b)\cdot\mathbf{A}^H(a,b) = \mathbf{A}(|a|^2+|b|^2,0) = (|a|^2+|b|^2)\mathbf{I}_2$ ■

Let $\mathbf{P} = \begin{bmatrix}\mathbf{M}\cdot\mathbf{A}(a,b)\\ \mathbf{M}\cdot\mathbf{A}(c,d)\end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix}\mathbf{M}\cdot\mathbf{A}(e,f)\\ \mathbf{M}\cdot\mathbf{A}(g,h)\end{bmatrix}$, $\mathbf{K}_a = \begin{bmatrix}p\mathbf{I}_2 & \mathbf{A}(q,0)\\ \mathbf{A}(q^*,0) & r\mathbf{I}_2\end{bmatrix}$, and $\mathbf{K}_s = \begin{bmatrix}x\mathbf{I}_2 & \mathbf{A}(u,v)\\ \mathbf{A}(u^*,-v) & y\mathbf{I}_2\end{bmatrix}$. Then, we can obtain the results in (11)–(14) (at the top of this page), which can be easily shown by using the properties of Alamouti operation.

Consider the communication system model in (10). Let $\mathbf{K}_{\mathbf{n}_e}$ be the covariance matrix of $\mathbf{n}_e$. It is known that the ML decoder of (10) can be derived as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}}\left[(\mathbf{y}_e-\mathbf{H}_e\mathbf{x})^H\mathbf{K}_{\mathbf{n}_e}^{-1}(\mathbf{y}_e-\mathbf{H}_e\mathbf{x})\right]. \quad (15)$$

Thus, we need to derive the inverse of the equivalent noise covariance matrix and calculate the term inside the bracket in (15).

For AF and SDF protocols, we should derive the covariance matrix of $cv(\mathbf{N}_\mathsf{D})$ which corresponds to the equivalent noise at D in the second phase under OT. Considering the AF protocol, the covariance matrix of $cv(\mathbf{N}_\mathsf{D})$ in (3) is given as

$$\begin{aligned}\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})} &= E\left[cv(\mathbf{N}_\mathsf{D})(cv(\mathbf{N}_\mathsf{D}))^H\right]\\ &= \begin{bmatrix}p\mathbf{I}_2 & \mathbf{A}(q,0)\\ \mathbf{A}(q^*,0) & r\mathbf{I}_2\end{bmatrix}\end{aligned} \quad (16)$$

where

$$\begin{aligned}p &= \beta_1^2|h_2^{11}|^2+\beta_2^2|h_2^{21}|^2+1\\ q &= \beta_1^2 h_2^{11}(h_2^{12})^* + \beta_2^2 h_2^{21}(h_2^{22})^*\\ r &= \beta_1^2|h_2^{12}|^2+\beta_2^2|h_2^{22}|^2+1.\end{aligned}$$

If we consider SDF protocol, the covariance matrix of $cv(\mathbf{N}_\mathsf{D})$ in (5) is given as

$$\begin{aligned}\mathbf{K}_{cv(\mathbf{N})} &= E\left[cv(\mathbf{N}_\mathsf{D})(cv(\mathbf{N}_\mathsf{D}))^H\right]\\ &= \frac{P_2}{1+P_1\|\mathbf{G}\|^2}\mathbf{H}_2'(\mathbf{H}_2')^H + \mathbf{I}_4\end{aligned}$$

$$= \begin{bmatrix} x\mathbf{I}_2 & \mathbf{A}(u,v) \\ \mathbf{A}(u^*,-v) & y\mathbf{I}_2 \end{bmatrix} \quad (17)$$

where

$$x = \frac{P_2(|h_2^{11}|^2 + |h_2^{21}|^2)}{1+P_1\|\mathbf{G}\|^2} + 1,\ y = \frac{P_2(|h_2^{12}|^2 + |h_2^{22}|^2)}{1+P_1\|\mathbf{G}\|^2} + 1$$
$$u = \frac{P_2(h_2^{11}h_2^{12*} + h_2^{21}h_2^{22*})}{1+P_1\|\mathbf{G}\|^2},\ v = \frac{P_2(h_2^{11}h_2^{22} - h_2^{21}h_2^{12})}{1+P_1\|\mathbf{G}\|^2}.$$

B. ML Decoders

The ML decoder for correlated noises is given as in (15). Thus, in this subsection, we calculate the term inside the bracket in (15) for OT-AF, OT-SDF, NT-AF, and NT-SDF. For each scheme, we have to derive the inverse covariance matrix of equivalent noise $\mathbf{K}_{\mathbf{n}_e}^{-1}$ and equivalent channel $\mathbf{H}_e$. Then we will show that although there are correlated noises at D, the ML decoder for OT (NT) can be simplified to a symbolwise (pairwise) decoder.

For OT-AF, the covariance matrix of equivalent noise $\mathbf{n}_e$ is given as

$$\mathbf{K}_{\mathbf{n}_e} = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})} \end{bmatrix} \quad (18)$$

where $\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}$ is in (16). The inverse matrix of (18) can be obtained as

$$\mathbf{K}_{\mathbf{n}_e}^{-1} = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1} \end{bmatrix}$$
$$\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1} = \frac{1}{pr-|q|^2}\begin{bmatrix} \mathbf{A}(r,0) & \mathbf{A}(-q,0) \\ \mathbf{A}(-q^*,0) & \mathbf{A}(p,0) \end{bmatrix}. \quad (19)$$

It can be easily shown that this inverse matrix always exists.

Thus, using $\mathbf{H}_e$ in (3), (11), and $\mathbf{K}_{\mathbf{n}_e}^{-1}$ in (19), we have

$$\mathbf{H}_e^H\mathbf{K}_{\mathbf{n}_e}^{-1}\mathbf{H}_e = (P_1\|\mathbf{H}_1\|^2 + t_a)\mathbf{I}_2 \quad (20)$$

where

$$t_a = \frac{P_1 s_a}{pr-|q|^2}$$
$$s_a = (|f^{12}|^2 + |f^{22}|^2)p + (|f^{11}|^2 + |f^{21}|^2)r - 2\Re[((f^{11})^*f^{12} + (f^{21})^*f^{22})q].$$

In the above equation, f^{ij}'s are the elements of equivalent channel matrix $\mathbf{F}$ in (8). Let $\mathbf{y}_e^H\mathbf{K}_{\mathbf{n}_e}^{-1}\mathbf{H}_e = [\eta_1\ \eta_2]$. Since (20) is a diagonal matrix, then the ML decoder is simplified to symbolwise decoder. After some manipulation, the ML decoder for OT-AF can be given as

$$\tilde{x}_i = \arg\min_{x_i}\left[(P_1\|\mathbf{H}_1\|^2 + t_a)|x_i|^2 - 2\Re[\eta_i x_i]\right]$$

for $i = 1, 2$.

For OT-SDF, the covariance matrix is given as

$$\mathbf{K}_{\mathbf{n}_e} = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})} \end{bmatrix} \quad (21)$$

where $\mathbf{K}_{cv(\mathbf{N})}$ is in (17). The inverse matrix of (21) can be obtained as

$$\mathbf{K}_{\mathbf{n}_e}^{-1} = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1} \end{bmatrix}$$
$$\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1} = \frac{1}{xy-|u|^2-|v|^2}\begin{bmatrix} \mathbf{A}(y,0) & \mathbf{A}(-u,-v) \\ \mathbf{A}(-u^*,v) & \mathbf{A}(x,0) \end{bmatrix}. \quad (22)$$

It can be easily shown that the above inverse matrix always exists.

Thus, using $\mathbf{H}_e$ in (5), (12), and $\mathbf{K}_{\mathbf{n}_e}^{-1}$ in (22), we have

$$\mathbf{H}_e^H\mathbf{K}_{\mathbf{n}_e}^{-1}\mathbf{H}_e = (P_1\|\mathbf{H}_1\|^2 + t_s)\mathbf{I}_2 \quad (23)$$

where

$$t_s = \frac{a^2 s_s}{xy-|u|^2-|v|^2}$$
$$s_s = (|h_2^{12}|^2 + |h_2^{22}|^2)x + (|h_2^{11}|^2 + |h_2^{21}|^2)y - 2\Re[((h_2^{11})^*h_2^{12} + (h_2^{21})^*h_2^{22})u + ((h_2^{11})^*(h_2^{22})^* - (h_2^{12})^*(h_2^{21})^*)v].$$

Similar to OT-AF, from (23), we can see that the ML decoder can be derived as symbolwise decoder. We define $[\eta_1\ \eta_2] = \mathbf{y}_e^H\mathbf{K}_{\mathbf{n}_e}^{-1}\mathbf{H}_e$ and then the ML decoder for OT-SDF is derived as

$$\tilde{x}_i = \arg\min_{x_i}\left[(P_1\|\mathbf{H}_1\|^2 + t_s)|x_i|^2 - 2\Re[\eta_i x_i]\right]$$

for $i = 1, 2$.

The same procedure can be applied to NT-AF and NT-SDF. For NT-AF, covariance matrix of equivalent noise is given as

$$\mathbf{K}_{\mathbf{n}_e} = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})} \end{bmatrix} \quad (24)$$

where $\mathbf{K}_{cv(\mathbf{N})}$ is in (16). The inverse covariance matrix is given as

$$\mathbf{K}_{\mathbf{n}_e}^{-1} = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1} \end{bmatrix} \quad (25)$$

where $\mathbf{K}_{cv(\mathbf{N})}^{-1}$ is the same as in (19). Using (8), (13), and (25), we have

$$\mathbf{H}_e^H\mathbf{K}_{\mathbf{n}_e}^{-1}\mathbf{H}_e = \begin{bmatrix} \alpha_a\mathbf{I}_2 & \beta_a\mathbf{I}_2 \\ \beta_a^*\mathbf{I}_2 & \alpha_a\mathbf{I}_2 \end{bmatrix} \quad (26)$$

where

$$\alpha_a = P_1\|\mathbf{H}_1\|^2 + \frac{P_3 s_{a1}}{pr-|q|^2} + \frac{P_1 s_{a2}}{pr-|q|^2}$$
$$\beta_a = \sqrt{P_1P_3}e^{j\theta}z_a$$

and s_{a1}, s_{a2}, and z_a are represented at the top in the next page.

$$
\begin{aligned}
s_{a1} =&(|h_1^{12}|^2 + |h_1^{22}|^2)p + (|h_1^{11}|^2 + |h_1^{21}|^2)r - 2\Re[((h_1^{11})^* h_1^{12} + (h_1^{21})^* h_1^{22})q]\\
s_{a2} =&(|f^{12}|^2 + |f^{22}|^2)p + (|f^{11}|^2 + |f^{21}|^2)r - 2\Re[((f^{11})^* f^{12} + (f^{21})^* f^{22})q]\\
z_a =&\frac{1}{(pr-|q|^2)}\Big[2r\Re[(h_1^{11})^* f^{11} + h_1^{21}(f^{21})^*] + 2p\Re[(h_1^{12})^* f^{12} + h_1^{22}(f^{22})^*] - 2\Re[((h_1^{11})^* f^{12}\\
&+ (h_1^{21})^* f^{22} + h_1^{12}(f^{11})^* + h_1^{22}(f_{21})^*)q]\Big]
\end{aligned}
$$

$$
\begin{aligned}
s_{s1} =&(|h_1^{12}|^2 + |h_1^{22}|^2)x + (|h_1^{11}|^2 + |h_1^{21}|^2)y - 2\Re[((h_1^{11})^* h_1^{12} + (h_1^{21})^* h_1^{22})u + ((h_1^{11})^*(h_1^{22})^* - (h_1^{12})^*(h_1^{21})^*)v]\\
s_{s2} =&(|h_2^{12}|^2 + |h_2^{22}|^2)x + (|h_2^{11}|^2 + |h_2^{21}|^2)y - 2\Re[((h_2^{11})^* h_2^{12} + (h_2^{21})^* h_2^{22})u\\
&+ ((h_2^{11})^*(h_2^{22})^* - (h_2^{12})^*(h_2^{21})^*)v]\\
z_s =&\frac{1}{(xy-|u|^2-|v|^2)}\Big[2y\Re[(h_1^{11})^* h_2^{11} + h_1^{21}(h_2^{21})^*] + 2x\Re[(h_1^{12})^* h_2^{12} + h_1^{22}(h_2^{22})^*]\\
&- 2\Re[((h_1^{11})^* h_2^{12} + (h_1^{21})^* h_2^{22} + h_1^{12}(h_2^{11})^* + h_1^{22}(h_2^{21})^*)u + ((h_1^{11})^*(h_2^{22})^*\\
&- (h_1^{21})^*(h_2^{12})^* + (h_1^{22})^*(h_2^{11})^* - (h_1^{12})^*(h_2^{21})^*)v]\Big]
\end{aligned}
$$

From (26), we can see that we should jointly decode two symbols (x_1, x_3) and (x_2, x_4), independently. Let $\mathbf{y}_e^H \mathbf{K}_{\mathbf{n}_e}^{-1} \mathbf{H}_e = [\eta_1\ \eta_2\ \eta_3\ \eta_4]$. Then, the ML decoder for NT-AF can be derived into pairwise decoder as

$$
\begin{aligned}
(\tilde{x}_i, \tilde{x}_{i+2}) =& \arg\min_{x_i, x_{i+2}} \Big[\alpha_a(|x_i|^2 + |x_{i+2}|^2)\\
&+ 2\Re[\beta_a^* x_i x_{i+2}^* - \eta_i x_i - \eta_{i+2} x_{i+2}]\Big]
\end{aligned}
$$

for $i = 1, 2$.

For NT-SDF, covariance matrix of equivalent noise is the same as in (24) except for $\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}$. In this case, $\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}$ is given in (17). The inverse covariance matrix is also the same as (25) except for $\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1}$. For NT-SDF, we use $\mathbf{K}_{cv(\mathbf{N}_\mathsf{D})}^{-1}$ in (22).

In the same manner as NT-AF, we have

$$
\mathbf{H}_e^H \mathbf{K}_{\mathbf{n}_e}^{-1} \mathbf{H}_e = \begin{bmatrix} \alpha_s \mathbf{I}_2 & \beta_s \mathbf{I}_2 \\ \beta_s^* \mathbf{I}_2 & \alpha_s \mathbf{I}_2 \end{bmatrix}
$$

where

$$
\begin{aligned}
\alpha_s =& P_1\|\mathbf{H}_1\|^2 + \frac{P_3 s_{s1}}{xy-|u|^2-|v|^2} + \frac{a^2 s_{s2}}{xy-|u|^2-|v|^2}\\
\beta_s =& a\sqrt{P_3} e^{j\theta} z_s.
\end{aligned}
$$

Note that s_{s1}, s_{s2}, and z_s are represented at the second top in this page.

Let $\mathbf{y}_e^H \mathbf{K}_{\mathbf{n}_e}^{-1} \mathbf{H}_e = [\eta_1\ \eta_2\ \eta_3\ \eta_4]$. Then, the ML decoder for NT-SDF is derived as pairwise decoder as

$$
\begin{aligned}
(\tilde{x}_i, \tilde{x}_{i+2}) =& \arg\min_{x_i, x_{i+2}} \Big[\alpha_s(|x_i|^2 + |x_{i+2}|^2)\\
&+ 2\Re[\beta_s^* x_i x_{i+2}^* - \eta_i x_i - \eta_{i+2} x_{i+2}]\Big]
\end{aligned}
$$

for $i = 1, 2$.

V. EXTENSION TO THE CASE OF MORE THAN TWO ANTENNAS

It is possible to extend AF and SDF protocols to the multiple-antenna case by using the orthogonal space-time coding (STC) [12] which is determined by the number of antennas. The construction method is similar to the case of two antennas. In this section, three-antenna case, i.e., $L = 3$, is only considered. In this case, an orthogonal code with the transmission rate $R = 3/4$ can be used, which is given by

$$
\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ x_3^* & 0 & -x_1^* \\ 0 & -x_3^* & x_2^* \end{bmatrix}. \tag{27}
$$

A. AF Protocol with OT

We first consider the AF protocol with OT. In the first phase, the received signals at R and D can be written as

$$
\begin{aligned}
\mathbf{Y}_\mathsf{R} &= \sqrt{P_1}\mathbf{X}\mathbf{G} + \mathbf{N}_\mathsf{R}\\
\mathbf{Y}_{\mathsf{D}1} &= \sqrt{P_1}\mathbf{X}\mathbf{H}_1 + \mathbf{N}_{\mathsf{D}1}
\end{aligned} \tag{28}
$$

where $\mathbf{G}$ and $\mathbf{H}_1$ are 3×3 channel matrices of $\mathsf{S} \to \mathsf{R}$ and $\mathsf{S} \to \mathsf{D}$, respectively.

Then, the regenerated signals can be obtained by multiplying the amplification matrix $\boldsymbol{\beta} = \mathrm{diag}(\beta_1, \beta_2, \beta_3)$ where

$$
\beta_i = \sqrt{\frac{P_2}{P_1(|g^{1i}|^2 + |g^{2i}|^2 + |g^{3i}|^2) + 1}}
$$

for $i = 1, 2, 3$. The transmitted signal at R is given as

$$
\mathbf{X}_\mathsf{R} = \mathbf{Y}_\mathsf{R}\boldsymbol{\beta}.
$$

This operation guarantees that the power of transmitted signal per antenna is P_2. Since S in OT-AF is silent in the second phase, the received signal at D in the second phase is given as

$$
\mathbf{Y}_{\mathsf{D}2} = \mathbf{Y}_\mathsf{R}\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2}
$$

$$= \sqrt{P_1}\mathbf{X}\mathbf{F} + \mathbf{N}_{\mathsf{D}} \tag{29}$$

where $\mathbf{F} = \mathbf{G}\boldsymbol{\beta}\mathbf{H}_2$ and $\mathbf{N}_{\mathsf{D}} = \mathbf{N}_{\mathsf{R}}\boldsymbol{\beta}\mathbf{H}_2 + \mathbf{N}_{\mathsf{D}2}$. Transforming the matrix model into the vector form, the following equivalent model for OT-AF with three antennas can be obtained as

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D}1}) \\ cv(\mathbf{Y}_{\mathsf{D}2}) \end{bmatrix}}_{\mathbf{y}_e} = \underbrace{\begin{bmatrix} \sqrt{P_1}\mathbf{H}_1' \\ \sqrt{P_1}\mathbf{F}' \end{bmatrix}}_{\mathbf{H}_e} \mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}_1) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix}}_{\mathbf{n}_e} \tag{30}$$

where $\mathbf{x} = [x_1\ x_2\ x_3]^T$ and each vector is represented at the top of the next page. Note that the complex vectorization in this section is different from that in Section III because different codes are used: Alamouti code vs. orthogonal code with rate $R = 3/4$.

For the optimal decoder, the covariance matrix of the equivalent noise $\mathbf{n}_e$ should be calculated. The covariance matrix of $cv(\mathbf{N}_{\mathsf{D}})$ is given as

$$\begin{aligned} \mathbf{K}_{cv(\mathbf{N}_{\mathsf{D}})} &= E\left[cv(\mathbf{N}_{\mathsf{D}})cv(\mathbf{N}_{\mathsf{D}})^H\right] \\ &= \begin{bmatrix} \mathbf{D}(k_1) & \mathbf{D}(k_4^*) & \mathbf{D}(k_6^*) \\ \mathbf{D}(k_4) & \mathbf{D}(k_2) & \mathbf{D}(k_5^*) \\ \mathbf{D}(k_6) & \mathbf{D}(k_5) & \mathbf{D}(k_3) \end{bmatrix} \end{aligned} \tag{31}$$

where $\mathbf{D}(x) = \mathrm{diag}(x, x^*, x^*, x^*)$ and the terms are represented as

$$\begin{aligned} k_1 &= \beta_1^2|h_2^{11}|^2 + \beta_2^2|h_2^{21}|^2 + \beta_3^2|h_2^{31}|^2 + 1 \\ k_2 &= \beta_1^2|h_2^{12}|^2 + \beta_2^2|h_2^{22}|^2 + \beta_3^2|h_2^{32}|^2 + 1 \\ k_3 &= \beta_1^2|h_2^{13}|^2 + \beta_2^2|h_2^{23}|^2 + \beta_3^2|h_2^{33}|^2 + 1 \\ k_4 &= \beta_1^2 h_2^{12} h_2^{11*} + \beta_2^2 h_2^{22} h_2^{21*} + \beta_3^2 h_2^{32} h_2^{31*} \\ k_5 &= \beta_1^2 h_2^{13} h_2^{12*} + \beta_2^2 h_2^{23} h_2^{22*} + \beta_3^2 h_2^{33} h_2^{32*} \\ k_6 &= \beta_1^2 h_2^{13} h_2^{11*} + \beta_2^2 h_2^{23} h_2^{21*} + \beta_3^2 h_2^{33} h_2^{31*}. \end{aligned}$$

B. SDF Protocol with OT

The received signal at D is almost the same as that in Subsection III-B except the size of matrices. Thus, the equivalent received signals can be written as

$$\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D}1}) \\ cv(\mathbf{Y}_{\mathsf{D}2}) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1}\mathbf{H}_1' \\ a\mathbf{H}_2' \end{bmatrix} \mathbf{x} + \begin{bmatrix} cv(\mathbf{N}_{\mathsf{D}1}) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix} \tag{32}$$

where $a = \gamma\sqrt{P_1}\|\mathbf{G}\|^2$, $cv(\mathbf{N}_{\mathsf{D}}) = \gamma\mathbf{H}_2'(\mathbf{G}')^H cv(\mathbf{N}_{\mathsf{R}}) + cv(\mathbf{N}_{\mathsf{D}2})$, and $\mathbf{x} = [x_1\ x_2\ x_3]^T$.

The covariance matrix of $cv(\mathbf{N}_{\mathsf{D}})$ is given as

$$\mathbf{K}_{cv(\mathbf{N}_{\mathsf{D}})} = \frac{P_2}{1 + P_1\|\mathbf{G}\|^2}\mathbf{H}_2'(\mathbf{H}_2')^H + \mathbf{I}_{12}. \tag{33}$$

Note that the only difference between (17) and (33) is the size of the matrices.

VI. NUMERICAL RESULTS AND CONCLUSION

We assume that all the channels are quasi-static Rayleigh fading channels and QPSK and 16 quadrature amplitude modulation (16QAM) are used. The average power of the transmitted symbol is assumed to be 1.

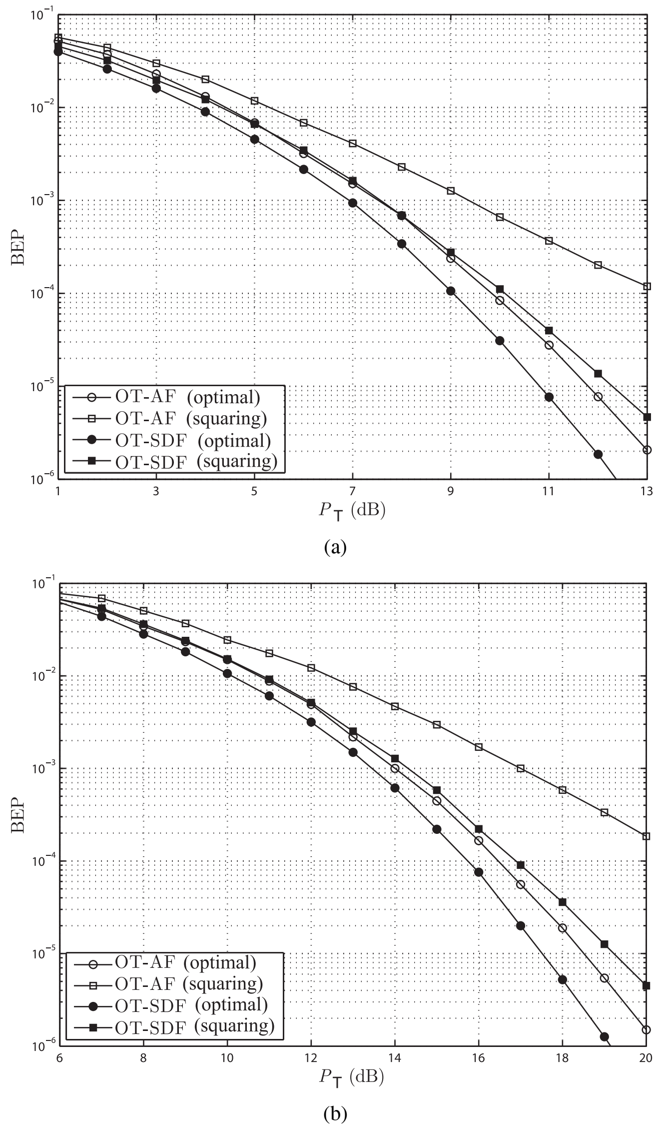


Fig. 2. Performance comparison of various orthogonal protocols for $L = 2$: (a) QPSK and (b) 16QAM.

For OT, we assume equal power distribution, i.e., $P_1 = P_2$. Fig. 2 shows the performance of OT-AF and OT-SDF with QPSK and 16QAM, respectively. The horizontal axis P_T represents the total power per antenna, i.e., $P_T = P_1 + P_2$. "Optimal" means that D uses the optimal ML decoder derived in the previous section and "squaring" means that D uses squaring method as shown in [7]. It is shown that SDF protocol outperforms AF protocol. Also, the optimal decoder shows better performance than squaring decoder, although they have the same decoding complexity at D.

From the results in Fig. 2, we can observe that the AF and SDF protocols under the squaring method lose diversity order. This is because the squaring method does not guarantee the optimal performance when the correlated noise exists in the cooperative communication system [13]. In both AF and SDF protocols, the signals transmitted at R are not the same as the signals transmitted at S, i.e., $\mathbf{x}_{\mathsf{R}} \neq \mathbf{x}_{\mathsf{S}}$. This causes the performance degradation if the squaring method is used at D. In con-

$$cv(\mathbf{Y}_{\mathsf{D}1}) = \left[\begin{array}{cccccccccccc} y_{\mathsf{D}1}^{11} & y_{\mathsf{D}1}^{21*} & y_{\mathsf{D}1}^{31*} & y_{\mathsf{D}1}^{41*} & y_{\mathsf{D}1}^{12} & y_{\mathsf{D}1}^{22*} & y_{\mathsf{D}1}^{32*} & y_{\mathsf{D}1}^{42*} & y_{\mathsf{D}1}^{13} & y_{\mathsf{D}1}^{23*} & y_{\mathsf{D}1}^{33*} & y_{\mathsf{D}1}^{43*} \end{array}\right]$$

$$cv(\mathbf{Y}_{\mathsf{D}2}) = \left[\begin{array}{cccccccccccc} y_{\mathsf{D}2}^{11} & y_{\mathsf{D}2}^{21*} & y_{\mathsf{D}2}^{31*} & y_{\mathsf{D}2}^{41*} & y_{\mathsf{D}2}^{12} & y_{\mathsf{D}2}^{22*} & y_{\mathsf{D}2}^{32*} & y_{\mathsf{D}2}^{42*} & y_{\mathsf{D}2}^{13} & y_{\mathsf{D}2}^{23*} & y_{\mathsf{D}2}^{33*} & y_{\mathsf{D}2}^{43*} \end{array}\right]$$

$$cv(\mathbf{N}_{\mathsf{D}1}) = \left[\begin{array}{cccccccccccc} n_{\mathsf{D}1}^{11} & n_{\mathsf{D}1}^{21*} & n_{\mathsf{D}1}^{31*} & n_{\mathsf{D}1}^{41*} & n_{\mathsf{D}1}^{12} & n_{\mathsf{D}1}^{22*} & n_{\mathsf{D}1}^{32*} & n_{\mathsf{D}1}^{42*} & n_{\mathsf{D}1}^{13} & n_{\mathsf{D}1}^{23*} & n_{\mathsf{D}1}^{33*} & n_{\mathsf{D}1}^{43*} \end{array}\right]$$

$$cv(\mathbf{N}_{\mathsf{D}}) = \left[\begin{array}{cccccccccccc} n_{\mathsf{D}}^{11} & n_{\mathsf{D}}^{21*} & n_{\mathsf{D}}^{31*} & n_{\mathsf{D}}^{41*} & n_{\mathsf{D}}^{12} & n_{\mathsf{D}}^{22*} & n_{\mathsf{D}}^{32*} & n_{\mathsf{D}}^{42*} & n_{\mathsf{D}}^{13} & n_{\mathsf{D}}^{23*} & n_{\mathsf{D}}^{33*} & n_{\mathsf{D}}^{43*} \end{array}\right]$$

$$\mathbf{H}_1' = \left[\begin{array}{cccccccccccc} h_1^{11} & h_1^{21*} & -h_1^{31*} & 0 & h_1^{12} & h_1^{22*} & -h_1^{32*} & 0 & h_1^{13} & h_1^{23*} & -h_1^{33*} & 0 \\ h_1^{21} & -h_1^{11*} & 0 & h_1^{31*} & h_1^{22} & -h_1^{12*} & 0 & h_1^{32*} & h_1^{23} & -h_1^{13*} & 0 & h_1^{33*} \\ h_1^{31} & 0 & h_1^{11*} & -h_1^{21*} & h_1^{32} & 0 & h_1^{12*} & -h_1^{22*} & h_1^{33} & 0 & h_1^{13*} & -h_1^{23*} \end{array}\right]^T$$

$$\mathbf{F}' = \left[\begin{array}{cccccccccccc} f^{11} & f^{21*} & -f^{31*} & 0 & f^{12} & f^{22*} & -f^{32*} & 0 & f^{13} & f^{23*} & -f^{33*} & 0 \\ f^{21} & -f^{11*} & 0 & f^{31*} & f^{22} & -f^{12*} & 0 & f^{32*} & f^{23} & -f^{13*} & 0 & f^{33*} \\ f^{31} & 0 & f^{11*} & -f^{21*} & f^{32} & 0 & f^{12*} & -f^{22*} & f^{33} & 0 & f^{13*} & -f^{23*} \end{array}\right]^T$$

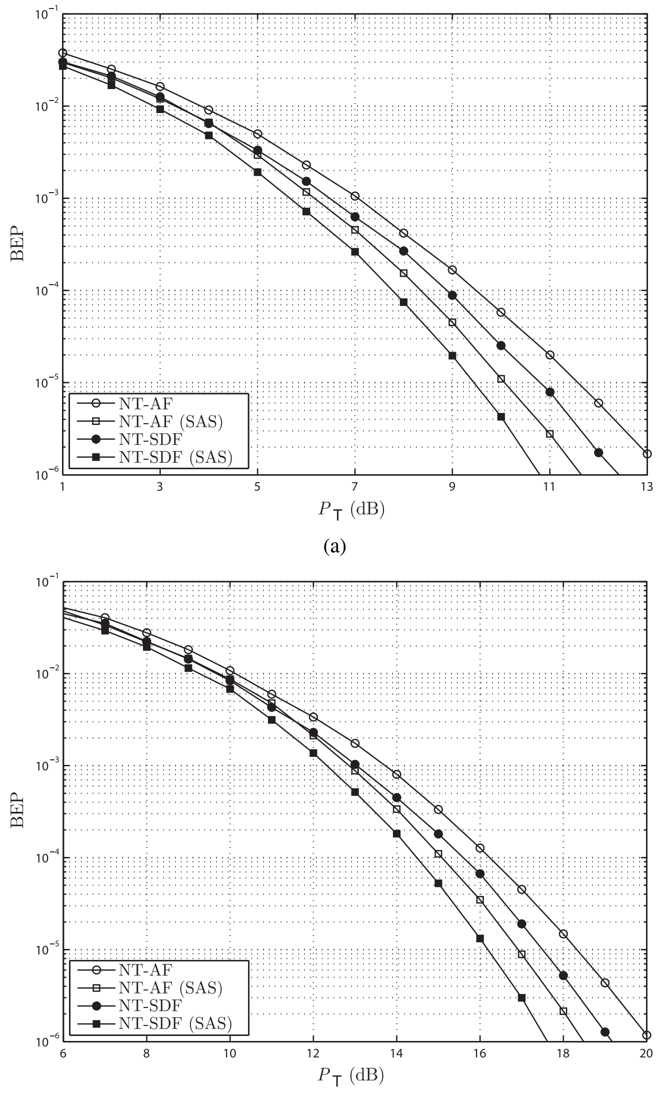


Fig. 3. Performance comparison of various non-orthogonal protocols for $L = 2$: (a) QPSK and (b) 16QAM.

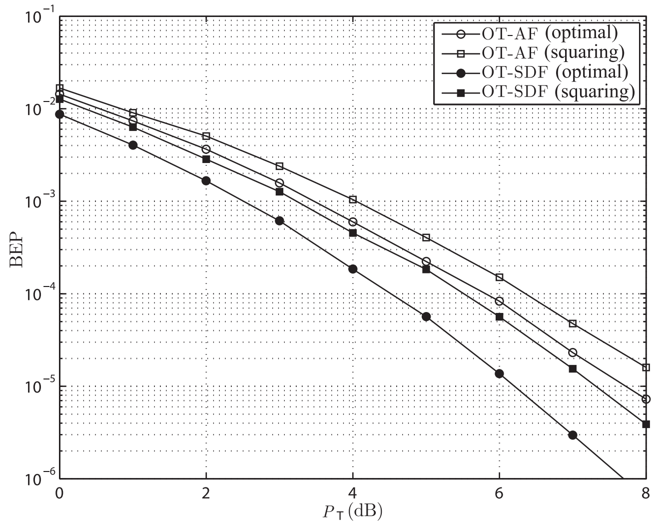


Fig. 4. Performance comparison of various orthogonal protocols with QPSK for $L = 3$.

trast to the squaring method, the optimal decoder considers the noise correlation, which compensates the performance degradation by the correlated noise at R. And the additional complexity of SDF protocol compared to AF protocol is due to a maximal combiner at R. However SDF protocol improves the diversity with the same decoding complexity at D. Finally, we can obtain more diversity order with ML decoder compared to the squaring method. In [14], they showed that OT-SDF achieves the maximum diversity order if power between S and R is appropriately allocated.

For NT, the power of the first phase is assumed to be equal to the total power of the second phase. And S and R use the same power in the second phase. Thus, we assume $P_T = P_1+P_2+P_3$, $P_1 = P_2 + P_3$, and $P_2 = P_3$. And we apply SAS technique to achieve better performance. Although, in [4], DF protocol is considered, SAS can be applied to AF and SDF protocols. Fig. 3 shows the performance of NT-AF and NT-SDF with QPSK and 16QAM, respectively. It can be seen that SDF protocol also shows better performance than AF protocol. Further, it can be seen that SAS technique enhances the diversity order of the cooperative communication networks.

In Fig. 4, BEPs of OT-AF and OT-SDF are shown for $L = 3$. The tendency of BEP is similar to that of the two-antenna case, i.e., OT-SDF outperforms OT-AF for the squaring and optimal decoder, respectively. Since more diversity can be used compared to the two-antenna case, BEP performance is significantly improved as SNR increases.

In this paper, we introduce SDF protocol which decodes the received signals into soft decision source symbol values at R sacrificing a little additional decoding complexity at R. Furthermore, we design DSTCs (QO-STBC) for the cooperation between S and R. We show that the ML decoders for AF and SDF protocols with both OT and NT can be simplified although the noises are correlated at D. From numerical analysis, the performances of SDF and AF can be significantly enhanced using the proposed ML decoder in comparison with those under the squaring decoder. It is also shown that SDF protocol outperforms AF protocol for both squaring decoder and the proposed optimal decoder.

REFERENCES

[1] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. IT-23, no. 5, pp. 572–584, Sept. 1979.

[2] A. Nosratinia and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74–80, Oct. 2004.

[3] C. Hucher, G. R. Rekaya, and J. Belfiore, "AF and DF protocols based on Alamouti ST code," in *Proc. IEEE ISIT*, June 2007, pp. 1526–1530.

[4] X. Jin, J.-D. Yang, J.-S. No, and D.-J. Shin, "Distributed space-time coded non-orthogonal DF protocol with source antenna switching," *J. Commun. Netw.*, vol. 12, no. 5, pp. 492–498, Oct. 2010.

[5] J.-D. Yang, K.-Y. Song, J.-S. No, and D.-J. Shin, "Soft-decision-and-forward protocol for cooperative communication networks based on Alamouti code," in *Proc. IEEE ISIT*, Seoul, Korea, June 28-July 3, 2009, pp. 1016–1019.

[6] J. Vazifehdan and H. Shafiee, "Cooperative diversity in space-time coded wireless networks," in *Proc. ICCS*, Sept. 2004, pp. 215–219.

[7] I.-H. Lee and D.-W. Kim, "Decouple-and-forward relaying for dual-hop Alamouti transmissions," *IEEE Commun. Lett.*, vol. 12, no. 2, pp. 97–99, Feb. 2008.

[8] X. Li, T. Luo, G. Yue, and C. Yin, "A squaring method to simplify the decoding of orthogonal space-time block codes," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1700–1703, Oct. 2001.

[9] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Aug. 1998.

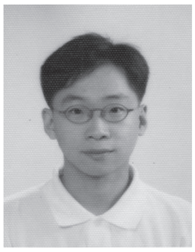
[10] O. Tirkkonen, A. Boariu, and A. Hottinen, "Minimal nonorthogonality rate 1 space-time block code for 3+ Tx antennas," in *Proc. IEEE ISSSTA*, Sept. 2000, pp. 429–432.

[11] W. Su and X. Xia, "Signal constellations for quasi-orthogonal space-time block codes with full diversity," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2331–2347, Oct. 2004.

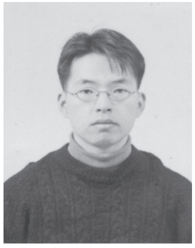
[12] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[13] T. Wang, A. Cano, G. B. Giannakis, and J. N. Laneman, "High-performance cooperative demodulation with decode-and-forward relays," *IEEE Trans. Commun.*, vol. 55, no. 7, pp. 1427–1438, July 2006.

[14] K.-Y. Song, J.-S. No, and H. Chung, "Bit error rate and power allocation of soft-decision-and-forward cooperative networks," *IEICE Trans. Wireless Commun.*, vol. E94-B, no. 1, Jan. 2011.



Jae-Dong Yang received the B.S. degree in Electronic Engineering, Seoul National University in 2002 and received Ph.D. degree in the Department of Electrical Engineering and Computer Science, Seoul National University in 2009. He is now a Senior Engineer at Samsung Electronics. His research interests include space-time codes, MIMO, coding theory, information theory, and wireless communication systems.



Kyoung-Young Song received the B.S. degree in Electrical Engineering (major) and Mathematics (minor) from Korea University, Seoul, Korea, in 2004. He received the Ph.D. degree in Electrical Engineering from Seoul National University in August 2010. From August 2010, he is now a Senior Research Engineer at LG Electronics. His research interests include algebraic coding theory, cryptography, and wireless communication systems such as MIMO communication and space-time coding.

Jong-Seon No received the B.S. and M.S.E.E. degrees in Electronics Engineering from Seoul National University, Seoul, Korea, in 1981 and 1984, respectively, and the Ph.D. degree in Electrical Engineering from the University of Southern California, Los Angeles, in 1988. He was a Senior MTS with Hughes Network Systems, Germantown, MD, from February 1988 to July 1990. He was an Associate Professor with the Department of Electronic Engineering, Konkuk University, Seoul, from September 1990 to July 1999. He joined the Faculty of the Department of Electrical Engineering and Computer Science, Seoul National University, in August 1999, where he is currently a Professor. His research interests include error-correcting codes, sequences, cryptography, space-time codes, LDPC codes, network coding, and wireless communication systems.

Dong-Joon Shin received the B.S. degree in Electronics Engineering from Seoul National University, Seoul, Korea, the M.S. degree in Electrical Engineering from Northwestern University, Evanston, IL, and the Ph.D. degree in Electrical Engineering from the University of Southern California, Los Angeles. From 1999 to 2000, he was a Member of Technical Staff with the Wireless Network Division and Satellite Network Division, Hughes Network Systems, Germantown, MD. Since September 2000, he has been an Associate Professor with the Department of Electronics and Computer Engineering, Hanyang University, Seoul. His current research interests include digital communication systems, error-correcting codes, sequences, and discrete mathematics.