PAPER

New Construction of Quaternary Sequences with Good Correlation Using Binary Sequences with Good Correlation

Taehyung LIM[†a)], ***Nonmember***, **Jong-Seon NO**[†b)], ***Member***, ***and*** **Habong CHUNG**[††c)], ***Nonmember***

SUMMARY In this paper, a new construction method of quaternary sequences of even period $2N$ having the ideal autocorrelation and balance properties is proposed. These quaternary sequences are constructed by applying the inverse Gray mapping to binary sequences of odd period N with the ideal autocorrelation. Autocorrelation distribution of the proposed quaternary sequences is derived. These sequences can be used to construct quaternary sequence families of even period $2N$. Family size and the maximum absolute value of correlation spectrum of the proposed quaternary sequence families are also derived.

key words: *quaternary sequences, inverse Gray mapping, autocorrelation, cross-correlation*

1. Introduction

Pseudorandom sequences with good correlation property play an important role in designing digital communication systems. This is because the good correlation property guarantees less interferences in the wireless communication systems. Especially, binary and quaternary sequences have been paid more attention to because the binary and quadrature modulations are widely used in the wireless communication systems.

There have been numerous researches on the binary sequences with the ideal autocorrelation property, which include m-sequences [1], GMW sequences [2], and Legendre sequences [3]. Also, binary sequence families with a good cross-correlation property have been extensively studied [1], [4]. Gold sequence families [1] are optimal for odd n with respect to the Sidelnikov bound. A small set of Kasami sequences [1] and a set of No sequences [4] are optimal with respect to the Welch bound.

M-ary phase-shift keying (PSK) modulation schemes are frequently used for the high-speed data transmission. Especially, quadrature phase shift keying (QPSK) modulation is adopted as a standard for the third generation wireless communication systems. Binary codes can be used for this purpose by separating each quaternary symbol into two binary symbols. However, this shows an inferior spreading performance when compared to the case that quaternary codes are used for the same purpose. Therefore, quaternary sequences are recommended for QPSK modulation rather than binary ones. As a result, it becomes more important to find quaternary codes with good correlation property.

Various results have been reported on the quaternary sequences with good autocorrelation property [5]–[7]. Schotten's complementary-based sequences [7] have a good autocorrelation property for odd period. Luke, Schotten, and Hadinejad-Mahram constructed quaternary sequences with good autocorrelation property for even period [7]. However, balance property of these sequences is not good because they are almost binary sequences. Jang, Kim, Kim, and No constructed the quaternary sequences with the ideal autocorrelation and balance properties [5], [6]. Although these sequences have good autocorrelation property, they have a weak point in their symbol distribution, that is, the sequences take the symbols 0 and 2 at the even indices and the symbols 1 and 3 at the odd indices. This characteristic reduces the randomness of the sequences. Chung, Han, and Yang recently proposed a new method to construct quaternary sequences from a binary sequence [8]. These sequences also have good autocorrelation property, but they use a single binary sequence in order to construct quaternary sequences. This characteristic decreases the flexibility of constructing quaternary sequences.

Some quaternary sequence families with good correlation are listed in Table 1. The quaternary sequence families $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{S}$, $\mathcal{U}$, and $\mathcal{E}$ [9] are constructed on Galois rings rather than finite fields. Therefore, construction processes of them are rather complex. The quaternary sequence family $\mathcal{L}$ [10] is generated by using the quaternary Sidel'nikov sequences. Although this sequence family has a large family size and an easy construction method, its maximal correlation is three times of the optimal maximum correlation.

In this paper, a new construction method of quaternary sequences of even period $2N$ with the ideal autocorrelation and balance properties is proposed by applying the inverse Gray mapping to binary sequences of odd period N with

Table 1 Comparison of the quaternary sequence families.

Family	Period N	Family size	Maximal correlation
$\mathcal{A}$,$\mathcal{S}$ [9]	2^n-1	2^n+1	$2^{\frac{n}{2}}+1 \simeq \sqrt{N}$
$\mathcal{B}$,$\mathcal{C}$ [9]	$2(2^n-1)$	2^{n-1}	$2^{\frac{n+1}{2}}+2 \simeq \sqrt{N}$
$\mathcal{D}$,$\mathcal{U}$,$\mathcal{E}$ [9]	$2(2^n-1)$	2^n	$2^{\frac{n+1}{2}}+2 \simeq \sqrt{N}$
$\mathcal{L}$ [10]	p^n-1	$\frac{9(p^n-3)}{2}+6$	$3p^{\frac{n}{2}}+5 \simeq 3\sqrt{N}$

Manuscript received August 20, 2010.
Manuscript revised February 24, 2011.

[†]The authors are with the Department of Electrical Engineering and Computer Science, INMC, Seoul National University, Seoul 151-742, Korea.
[††]The author is with the Department of Electronic and Electrical Engineering, Hongik University, Seoul 121-791, Korea.
a) E-mail: jayelish@hotmail.com
b) E-mail: jsno@snu.ac.kr
c) E-mail: habchung@hongik.ac.kr

DOI: 10.1587/transfun.E94.A.1701

Copyright © 2011 The Institute of Electronics, Information and Communication Engineers

the ideal autocorrelation. Autocorrelation distribution of the proposed quaternary sequence is also derived. Moreover, a new construction method of quaternary sequence families of even period $2N$ is proposed using the inverse Gray mapping and binary sequence families of odd period N. Family size and the maximum absolute value of correlation spectrum of the proposed quaternary sequence families are also derived. If we use binary sequence families which are optimal with respect to the Welch bound, the constructed sequence family has about $\sqrt{2}$ times of the optimal maximum correlation. This value is smaller than the case of a sequence family $\mathcal{L}$, but it is rather larger than the cases of optimal sequence families $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{S}$, $\mathcal{U}$, and $\mathcal{E}$. However, the proposed quaternary sequence family has much simpler construction process than $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{S}$, $\mathcal{U}$, and $\mathcal{E}$.

2. Preliminaries

Let $g(t)$ be a q-ary sequence of period N for positive integers q and N. The sequence $g(t)$ is said to be balanced if all the differences among numbers of occurrences of each symbol in a period are less than or equal to one.

The autocorrelation function $R_g(\tau)$ of $g(t)$ is defined as

$$R_g(\tau) = \sum_{t=0}^{N-1} \omega_q^{g(t)-g(t+\tau)}$$

where $0 \leq \tau < N$ and ω_q is the complex primitive qth root of unity, e.g., $\omega_4 = \sqrt{-1}$. The cross-correlation function $R_{g_1 g_2}(\tau)$ of $g_1(t)$ and $g_2(t)$ is defined as

$$R_{g_1 g_2}(\tau) = \sum_{t=0}^{N-1} \omega_q^{g_1(t)-g_2(t+\tau)}.$$

It is well known that a binary sequence of odd period N with the ideal autocorrelation has the distribution of autocorrelation values as

$$R_g(\tau) = \begin{cases} N, & \text{once} \\ -1, & N-1 \text{ times.} \end{cases}$$

In addition, the autocorrelation distribution of a quaternary sequence of even period N with the ideal autocorrelation and balance properties is given as

$$R_g(\tau) = \begin{cases} N, & \text{once} \\ 0, & \frac{N}{2} \text{ times} \\ -2, & \frac{N}{2}-1 \text{ times} \end{cases}$$

or

$$R_g(\tau) = \begin{cases} N, & \text{once} \\ 0, & \frac{N}{2}-1 \text{ times} \\ -2, & \frac{N}{2} \text{ times.} \end{cases}$$

For a family $\mathcal{F}$ of sequences, C_{max} is defined as the maximum absolute value of correlation among all pairs of sequences in $\mathcal{F}$ except for the inphase autocorrelation. Welch [1] has established the well-known lower bounds on the smallest possible C_{max} for a given family of size M and sequence period N as

$$(C_{max})^{2k} \geq \frac{1}{(MN-1)} \left\{ \frac{MN^{2k+1}}{\binom{k+N-1}{N-1}} - N^{2k} \right\}$$

where k is a positive integer. When C_{max} of a given family $\mathcal{F}$ achieves the above bound, $\mathcal{F}$ is said to be optimal with respect to the Welch bound. Roughly speaking, C_{max} is very close to $\sqrt{N}$ when $\mathcal{F}$ is optimal with respect to the Welch bound.

Let $\phi[s_0, s_1]$ be the inverse Gray mapping defined by

$$\phi[s_0, s_1] = \begin{cases} 0, & \text{if } (s_0, s_1) = (0,0) \\ 1, & \text{if } (s_0, s_1) = (0,1) \\ 2, & \text{if } (s_0, s_1) = (1,1) \\ 3, & \text{if } (s_0, s_1) = (1,0). \end{cases}$$

Given two binary sequences $s_0(t)$ and $s_1(t)$ of period N, a quaternary sequence of period N defined by $q(t) = \phi[s_0(t), s_1(t)]$ can be equivalently expressed as [11]

$$\omega_4^{q(t)} = \frac{1+\omega_4}{2}(-1)^{s_0(t)} + \frac{1-\omega_4}{2}(-1)^{s_1(t)}. \tag{1}$$

Krone and Sarwate derived the relation between the correlations of the binary sequences and those of the quaternary sequences in (1) as follows.

Theorem 1: *[11] Let $s_0(t)$, $s_1(t)$, $s_2(t)$, and $s_3(t)$ be binary sequences of the same period. Let $q_0(t)$ and $q_1(t)$ be quaternary sequences defined by $q_0(t) = \phi[s_0(t), s_1(t)]$ and $q_1(t) = \phi[s_2(t), s_3(t)]$, respectively. Then the cross-correlation function $R_{q_0 q_1}(\tau)$ between $q_0(t)$ and $q_1(t)$ is given as*

$$R_{q_0 q_1}(\tau) = \frac{1}{2}\{R_{s_0 s_2}(\tau) + R_{s_1 s_3}(\tau) + \omega_4 (R_{s_0 s_3}(\tau) - R_{s_1 s_2}(\tau))\}$$

where $R_{s_i s_j}(\tau)$ is the cross-correlation function of $s_i(t)$ and $s_j(t)$. □

3. New Construction of Quaternary Sequences with the Ideal Autocorrelation and Balance Properties Using Binary Sequences with the Ideal Autocorrelation Property

In this section, by using binary sequences of period N with the ideal autocorrelation and the inverse Gray mapping, we propose a new construction method of quaternary sequences of period $2N$ with the ideal autocorrelation and balance properties. The autocorrelation distribution of the proposed quaternary sequences is also derived.

Let $a(t)$ and $b(t)$ be two binary sequences of odd period N with the ideal autocorrelation. Then, a new quaternary sequence $q(t)$ of period $2N$ is defined as

$$q(t) = \phi[s_0(t), s_1(t)] \tag{2}$$

where $s_0(t)$ and $s_1(t)$ are two binary sequences of period $2N$ defined by

$$s_0(t) = \begin{cases} a(t), & \text{for } t \equiv 0 \bmod 2 \\ a(t), & \text{for } t \equiv 1 \bmod 2 \end{cases}$$

$$s_1(t) = \begin{cases} b(t), & \text{for } t \equiv 0 \bmod 2 \\ b(t) \oplus 1, & \text{for } t \equiv 1 \bmod 2. \end{cases}$$

First of all, we investigate into the condition which makes $q(t)$ be balanced as in the following lemma.

Lemma 2: *Let $q(t)$ be the quaternary sequence defined in (2). If $a(t)$ has the balance property, then $q(t)$ also has the balance property, i.e.,*

$$q(t) = \begin{cases} 0, & \frac{N-1}{2} \text{ times} \\ 1, & \frac{N-1}{2} \text{ times} \\ 2, & \frac{N+1}{2} \text{ times} \\ 3, & \frac{N+1}{2} \text{ times.} \end{cases}$$

Proof: Let $B_i, i = 0, 1, 2, 3$, be the numbers defined by

$$B_i = |\{t | q(t) = i, 0 \le t < 2N\}|.$$

If we define N_0, N_1, N_2, and N_3 as

$$\begin{aligned} N_0 &= |\{t | a(t) = 0 \text{ and } b(t) = 0, 0 \le t < N\}| \\ N_1 &= |\{t | a(t) = 0 \text{ and } b(t) = 1, 0 \le t < N\}| \\ N_2 &= |\{t | a(t) = 1 \text{ and } b(t) = 1, 0 \le t < N\}| \\ N_3 &= |\{t | a(t) = 1 \text{ and } b(t) = 0, 0 \le t < N\}| \end{aligned} \quad (3)$$

then, by using the definition in (2), we have

$$\begin{aligned} B_0 = B_1 &= N_0 + N_1 = |\{t | a(t) = 0, 0 \le t < N\}| \\ B_2 = B_3 &= N_2 + N_3 = |\{t | a(t) = 1, 0 \le t < N\}|. \end{aligned}$$

Since $a(t)$ has the balance property, it is clear that

$$\begin{aligned} N_0 + N_1 &= \tfrac{N-1}{2} \\ N_2 + N_3 &= \tfrac{N+1}{2}. \end{aligned}$$

□

Next, by using the autocorrelation functions of $a(t)$ and $b(t)$, the autocorrelation function of $q(t)$ can be derived as follows.

Lemma 3: *Let $q(t)$ be the quaternary sequence defined in (2). Then autocorrelation distribution of $q(t)$ can be expressed as*

$$R_q(\tau) = \begin{cases} R_a(\tau) + R_b(\tau), & \text{for } \tau \equiv 0 \bmod 2 \\ R_a(\tau) - R_b(\tau), & \text{for } \tau \equiv 1 \bmod 2. \end{cases}$$

Proof: See the proof of Lemma 6.

□

Finally, the quaternary sequences with the ideal autocorrelation and balance properties can be constructed as follows.

Theorem 4: *Let $a(t)$ and $b(t)$ be two binary sequences of odd period N with the ideal autocorrelation. Then, a quaternary sequence $q(t)$ in (2) of period $2N$ has the ideal autocorrelation and balance properties with the following distribution*

$$R_q(\tau) = \begin{cases} 2N, & \text{for } \tau = 0 \\ 0, & \text{for } \tau \equiv 1 \bmod 2 \\ -2, & \text{for } \tau \equiv 0 \bmod 2 \text{ and } \tau \ne 0. \end{cases}$$

Proof: Since all binary sequences with the ideal autocorrelation have the balance property, $a(t)$ is balanced. By Lemma 2, we know that $q(t)$ also has the balance property.

When $a(t)$ and $b(t)$ have the ideal autocorrelation property, it is clear that

$$R_a(\tau) = R_b(\tau) = \begin{cases} N, & \text{for } \tau = 0 \\ -1, & \text{otherwise.} \end{cases}$$

By Lemma 3, we can easily derive the distribution of $R_q(\tau)$.

□

Note that if $a(t) = b(t)$, the constructed quaternary sequence is the same as the one proposed by Jang, Kim, Kim, and No [5]. In this case, the resulting quaternary sequence takes the symbols 0 and 2 at the even indices and the symbols 1 and 3 at the odd indices. This characteristic reduces the randomness of the sequences. This problem can be resolved by selecting two different binary sequences $a(t)$ and $b(t)$.

An example of the above theorem using a binary m-sequence and a GMW sequence is given as follows.

Example 5: *Let $a(t)$ be the binary m-sequence of period 63 given by*

$$\begin{aligned} a(t) = &000001000011000101001111010001110 \\ &010010110111011001101010111111. \end{aligned}$$

And let $b(t)$ be the binary GMW sequence of period 63 given by

$$\begin{aligned} b(t) = &011110111001111010010110111101000 \\ &110101100110100011101000100000 0. \end{aligned}$$

From the definition of $q(t)$ defined in (2), $q(t)$ of period 126 can be generated as

$$\begin{aligned} q(t) = &0010121011331012120033221311 1232 \\ &1030031230222132102212021232 3231 \\ &1010301002201030311223302000 3230 \\ &1211203213330230133031303 23232. \end{aligned}$$

The autocorrelation of $q(t)$ is calculated as

$$R_q(\tau) = \begin{cases} 126, & \text{for } \tau = 0 \\ 0, & \text{for } \tau \equiv 1 \bmod 2 \\ -2, & \text{for } \tau \equiv 0 \bmod 2 \text{ and } \tau \ne 0. \end{cases}$$

□

4. New Construction of Quaternary Sequence Families Using Binary Sequence Families

In this section, we propose a new construction method of quaternary sequence families using existing binary sequence families. The C_{max} of the proposed quaternary sequence family is derived.

Let $\mathcal{F}_a = \{a_i(t) | 0 \le t < N, 0 \le i < M_a\}$ and $\mathcal{F}_b = \{b_i(t) | 0 \le t < N, 0 \le i < M_b\}$ be two binary sequence families of odd period N. Then, a new quaternary sequence family $\mathcal{F}_q = \{q_i(t) | 0 \le t < 2N, 0 \le i < M_q\}$, where $M_q = \min(M_a, M_b)$, is defined as

$$q_i(t) = \phi[s_{i0}(t), s_{i1}(t)] \quad (4)$$

where $s_{i0}(t)$ and $s_{i1}(t)$ are the binary sequences of period $2N$ defined by

$$s_{i0}(t) = \begin{cases} a_i(t), & \text{for } t \equiv 0 \bmod 2 \\ a_i(t), & \text{for } t \equiv 1 \bmod 2 \end{cases}$$

$$s_{i1}(t) = \begin{cases} b_i(t), & \text{for } t \equiv 0 \bmod 2 \\ b_i(t) \oplus 1, & \text{for } t \equiv 1 \bmod 2. \end{cases}$$

In order to derive C_{max} of the proposed quaternary sequence family $\mathcal{F}_q$, the following lemma is needed.

Lemma 6: *Let $q_i(t)$ and $q_j(t)$ be two quaternary sequences defined in (4). Then cross-correlation function between $q_i(t)$ and $q_j(t)$ can be expressed as*

$$R_{q_i q_j}(\tau) = \begin{cases} R_{a_i a_j}(\tau) + R_{b_i b_j}(\tau), & \text{for } \tau \equiv 0 \bmod 2 \\ R_{a_i a_j}(\tau) - R_{b_i b_j}(\tau), & \text{for } \tau \equiv 1 \bmod 2. \end{cases}$$

Proof: From Theorem 1, $R_{q_i q_j}(\tau)$ can be rewritten as

$$R_{q_i q_j}(\tau) = \frac{1}{2}\{R_{s_{i0} s_{j0}}(\tau) + R_{s_{i1} s_{j1}}(\tau) + \omega_4 (R_{s_{i0} s_{j1}}(\tau) - R_{s_{i1} s_{j0}}(\tau))\}.$$

From the definition of $s_{i0}(t)$, $s_{i1}(t)$, $s_{j0}(t)$, and $s_{j1}(t)$, the cross-correlation functions $R_{s_{i0} s_{j0}}(\tau)$ and $R_{s_{i1} s_{j1}}(\tau)$ are expressed as

$$R_{s_{i0} s_{j0}}(\tau) = 2R_{a_i a_j}(\tau)$$

$$R_{s_{i1} s_{j1}}(\tau) = \begin{cases} 2R_{b_i b_j}(\tau), & \text{for } \tau \equiv 0 \bmod 2 \\ -2R_{b_i b_j}(\tau), & \text{for } \tau \equiv 1 \bmod 2. \end{cases}$$

Also, it is easy to check that

$$s_{i0}(t) - s_{j1}(t+\tau) = s_{i0}(t+N) - s_{j1}(t+N+\tau) \oplus 1$$
$$s_{i1}(t) - s_{j0}(t+\tau) = s_{i1}(t+N) - s_{j0}(t+N+\tau) \oplus 1$$

where $0 \le t < N$ and thus we have

$$R_{s_{i0} s_{j1}}(\tau) = R_{s_{i1} s_{j0}}(\tau) = 0.$$

□

Note that Lemma 3 is considered as a corollary of Lemma 6 in the case when $q_i(t) = q_j(t)$.

Finally, from Lemma 6, we derive C_{max} of the quaternary sequence family $\mathcal{F}_q$ as follows.

Theorem 7: *Let $\mathcal{F}_a = \{a_i(t) | 0 \le t < N, 0 \le i < M_a\}$ and $\mathcal{F}_b = \{b_i(t) | 0 \le t < N, 0 \le i < M_b\}$ be two binary sequence families of odd period N. Then C_{max} of the quaternary sequence family $\mathcal{F}_q = \{q_i(t) | 0 \le t < 2N, 0 \le i < M_q, M_q = \min(M_a, M_b)\}$ in (4) is less than or equal to the sum of C_{max} of $\mathcal{F}_a$ and C_{max} of $\mathcal{F}_b$.*

Proof: Let C_{max} of $\mathcal{F}_q$, $\mathcal{F}_a$, and $\mathcal{F}_b$ be $C_{q,max}$, $C_{a,max}$, and $C_{b,max}$, respectively. From Lemma 6, it is easy to check that

$$C_{q,max} \le C_{a,max} + C_{b,max}.$$

□

If both $\mathcal{F}_a$ and $\mathcal{F}_b$ are optimal with respect to the Welch bound, $C_{a,max}$ and $C_{b,max}$ are about $\sqrt{N}$. In this case, from Theorem 7, we can know that the upper bound of $C_{q,max}$ is about $2\sqrt{N}$. Since the Welch bound of the quaternary sequence family $\mathcal{F}_q$ is $\sqrt{2N}$, the upper bound of $C_{q,max}$ is about $\sqrt{2}$ times of the Welch bound for $\mathcal{F}_q$.

Using a small set of Kasami sequences and a set of No sequences, an example of the above theorem is given as follows.

Example 8: *Let $\mathcal{F}_a$ be a small set of Kasami sequences of period N and family size $M = \sqrt{N+1}$. The spectrum of correlation values is given by*

$$\{-1, -1-M, -1+M\}.$$

Let $\mathcal{F}_b$ be a set of No sequences of period N and family size $M = \sqrt{N+1}$. The spectrum of correlation values is the same as that of a small set of Kasami sequences.

Let $\mathcal{F}_q$ be the quaternary sequence family defined in (4). Then, $\mathcal{F}_q$ is a set of $M = \sqrt{N+1}$ quaternary sequences of period $2N$. In addition, the spectrum of correlation values is given as

$$\{-2 \pm 2M, \pm 2M, -2 \pm M, \pm M, -2, 0\}.$$

Therefore,

$$C_{q,max} = 2 + 2M = 2 + 2\sqrt{N+1} \sim 2\sqrt{N} = \sqrt{2}\sqrt{2N}$$

which is about $\sqrt{2}$ times of the Welch bound $\sqrt{2N}$ of $\mathcal{F}_q$.

□

5. Conclusion

In this paper, we proposed a new construction method of quaternary sequences of even period having the ideal autocorrelation and balance properties. Also, we constructed quaternary sequence families of even period having relatively small maximal correlation and a simple construction method. In both cases, quaternary sequences are constructed by applying the inverse Gray mapping to two binary sequences. These two binary sequences can be chosen from any binary sequence having good correlation property. Therefore, the proposed construction methods have a great amount of flexibility.

Since the higher order modulations are widely preferred in the wireless communication systems such as 3GPP, the proposed quaternary sequences can be used in such wireless communication systems as spreading codes.

Acknowledgments

This work was supported by the IT R&D program of MKE/KEIT[KI001809, Intelligent Wireless Communication Systems in 3 Dimensional Environment] and the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No.2010-0000867).

References

[1] T. Helleseth and P. Vijay Kumar, "Sequences with low correlation," in Handbok of Coding Theory vol.2, ed. V.S. Pless and W.C. Huffman, pp.1765–1854, Elsevier Science B.V, 1998.
[2] R.A. Scholtz and L.R. Welch, "GMW sequences," IEEE Trans. Inf. Theory, vol.IT-30, pp.548–553, May 1984.
[3] J.-S. No, H.-K. Lee, H. Chung, H.-Y. Song, and K. Yang, "Trace representation of Legendre sequences of Mersenne prime period," IEEE Trans. Inf. Theory, vol.42, no.6, pp.2254–2255, Nov. 1996.
[4] J.-S. No and P.V. Kumar, "A new family of binary pseudorandom sequences having optimal periodic correlation properties and large linear span," IEEE Trans. Inf. Theory, vol.35, no.2, pp.371–379, March 1989.
[5] J.-W. Jang, Y.-S. Kim, S.-H. Kim, and J.-S. No, "New quaternary sequences with ideal autocorrelation constructed from binary sequences with ideal autocorrelation," ISIT 2009, pp.278–281, Seoul, Korea, June 2009.
[6] Y.-S. Kim, J.-W. Jang, S.-H. Kim, and J.-S. No, "New construction of quaternary sequences with ideal autocorrelation from Legendre sequences," ISIT 2009, pp.282–285, Seoul, Korea, June 2009.
[7] H. Dieter Luke, H.D. Schotten, and H. Hadinejad-Mahram, "Binary and quadriphase sequences with optimal autocorrelation properties: A survey," IEEE Trans. Inf. Theory, vol.49, no.12, pp.3271–3282, Dec. 2003.
[8] J.-H. Chung, Y.K. Han, and K. Yang, "New quaternary sequences with even period and three-valued autocorrelation," IEICE Trans. Fundamentals, vol.E93-A, no.1, pp.309–315, Jan. 2010.
[9] X. Tang, T. Helleseth, and P. Fan, "A new optimal quaternary sequence family of length $2(2^n - 1)$ obtained from the orthogonal transformation of Families $\mathcal{B}$ and $\mathcal{C}$," Des. Codes Cryptogr., vol.53, pp.137–148, Dec. 2009.
[10] Y.-S. Kim, J.-S. Chung, J.-S. No, and H. Chung, "New families of M-ary sequences with low correlation constructed from Sidel'nikov sequences," IEEE Trans. Inf. Theory, vol.54, no.8, pp.3768–3774, Aug. 2008.
[11] S.M. Krone and D.V. Sarwate, "Quadriphase sequences for spread spectrum multiple-access communication," IEEE Trans. Inf. Theory, vol.IT-30, no.3, pp.520–529, May 1984.
[12] T. Kasami, "Weight distribution formula for some class of cyclic codes," Technical Report R-285 (AD 632574), Coordinated Science Laboratory, University of Illinois, Urbana, April 1966.
[13] B. Gordon, W.H. Mills, and L.R. Welch, "Some new difference sets," Canadian J. Math., vol.14, no.4, pp.614–625, 1962.
[14] R. Lidl and H. Niederreiter, Finite fields, Reading, Addison-Wesley, MA, 1983.
[15] L.R. Welch, "Lower bounds on the maximum cross correlation of signals," IEEE Trans. Inf. Theory, vol.IT-20, pp.397–399, May 1974.

Taehyung Lim received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 2004 and 2006, respectively. Currently, he is a senior researcher in the Department of Electrical Engineering and Computer Science, INMC, Seoul National University. His research interests include pseudo random sequences, error correcting codes, and communication theory.

Jong-Seon No received the B.S. and M.S. degrees in electronics engineering from Seoul National University, Seoul, Korea, in 1981 and 1984, respectively, and the Ph.D. degree in electrical engineering from the University of Southern California, Los Angeles, in 1988. He was a Senior MTS at Hughes Network Systems from February 1988 to July 1990. He was also an Associate Professor in the Department of Electronic Engineering, Konkuk University, Seoul, Korea, from September 1990 to July 1999. He joined the faculty of the Department of Electrical Engineering and Computer Science, Seoul National University, in August 1999, where he is currently a Professor. His area of research interests includes error-correcting codes, sequences, cryptography, space-times codes, LDPC codes, and wireless communication systems.

Habong Chung received the B.S. degree from Seoul National University, Seoul, in 1981 and the M.S. and the Ph.D. degrees from the University of Southern California, Los Angeles, in 1985 and 1988, respectively. From 1988 to 1991, he was an Assistant Professor in the Department of Electrical and Computer Engineering, the State University of New York at Buffalo. Since 1991, he has been with the School of Electronic and Electrical Engineering, Hongik University, Seoul, where he is a Professor. His research interests include coding theory, combinatorics, and sequence design.