Relay Selection for Decode-and-Forward Cooperative Network with Multiple Antennas

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***Abstract*—In this paper, a new relay selection scheme for decode-and-forward (DF) relay cooperative network with multiple antennas is proposed based on both channel state information (CSI) and transmission scheme by deriving the upper bound on the pairwise error probability (PEP) of the near-maximum-likelihood (near-ML) decoder. It is also proved that the proposed relay selection which selects m $(1 \leq m \leq N)$ relays from N relays achieves full diversity $M_S M_D + N M_R \min[M_S, M_D]$ regardless of the value of m in the DF relay network consisting of one source, one destination, and N relays with M_S, M_D, and M_R antennas, respectively. Through Monte Carlo simulation, the error correction performance of the proposed relay selection for various m is shown for the uncoded single-antenna, Alamouti coded, and multiple-input multiple-output (MIMO) DF relay networks.**

***Index Terms*—Decode-and-forward (DF), diversity, maximum-likelihood (ML), multiple antennas, pairwise error probability (PEP), relay selection.**

I. Introduction

IN the wireless communication networks, deep fading often causes failure of reliable data transmission. In this case, relays can be used to cooperatively assist the data transmission. Such a system is called a cooperative communication network, where the cooperative diversity can be achieved [1]-[3].

In [1] and [2], Sendonaris, Erkip, and Aazhang presented an information theoretic model for cooperative communication network and analyzed the achievable rate region and outage probability in the code division multiple access (CDMA) system. Laneman and Wornell [3] developed various cooperative diversity algorithms for a source and destination pair based on relays amplifying or fully decoding and forwarding their received signals. These algorithms are referred as amplify-and-forward (AF) and decode-and-forward (DF) relaying, respectively. Even though the relay operation for the AF relay is simple, their transceivers require expensive radio frequency amplifiers [4].

Manuscript received January 9, 2010; revised August 5, 2010 and January 12, 2011; accepted August 15, 2011. The associate editor coordinating the review of this paper and approving it for publication was D. Michelson.

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This work was partly supported by the KCC (Korea Communications Commission), Korea, under the R&D program supervised by the KCA (Korea Communications Agency) (KCA-2011-08913-04003), a Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MEST) (No. 2011-0000328), and the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2011-0005909).

Digital Object Identifier 10.1109/TWC.2011.092911.100030

For the DF relay network, a maximum-likelihood (ML) decoder has been introduced and a suboptimal low-complexity decoder, called λ maximum ratio combining (λ-MRC), was proposed in [2] for a single-antenna system with binary phase shift keying (BPSK). In [4], a cooperative MRC (C-MRC) was proposed and it was proven that C-MRC for the uncoded single-antenna DF relay network can achieve full diversity. For many DF relay networks with multiple antennas, ML decoder becomes very complicated and thus cannot be used for the most cases, for example, multiple-input multiple-output (MIMO) DF relay network. In [5], Jin *et al.* proposed a near-ML decoder and proved that the near-ML decoder for the DF relay network with multiple antennas achieves full diversity.

Recently, relay networks with relay selection have been widely investigated [6]-[14]. In [6], a relay selection criterion from the information theoretic aspect was proposed for DF relay cooperative network, in which the relays are allowed to cooperate if their source-relay (SR) channel coefficient magnitudes exceed a threshold. In [7], a nearest relay selection criterion, that is, selecting relays nearest to the source or to the destination, was proposed. Some other works for relay selection have been focused on maximizing the received signal to noise ratio (SNR) at the destination [8]-[10], maximizing the minimum coefficient magnitude of SR and relay-destination (RD) channels [11], [12], and minimizing the upper bound on average symbol error probability [13]. All of these works were performed under the single-antenna assumption.

For the multiple-antenna case, Fan and Thompson [14] used a selection criterion of choosing a relay which achieves the highest network capacity for the optimal selective routing of a two-hop network. However, this relay selection criterion of maximizing the network capacity may not achieve the best error correction performance for most of the multiple-antenna cases because it only considers the channel state information (CSI) but not the transmission scheme. Therefore, in this paper, we focus on deriving a new relay selection criterion to minimize the pairwise error probability (PEP) of decoding the received signal into $\tilde{\mathrm{x}}$ when x is transmitted from the source by assuming that there are only two symbols x and $\tilde{\mathrm{x}}$ based on both CSI and transmission scheme for the DF relay cooperative network with multiple antennas.

In this paper, one source, one destination, and N relays with the direct link between the source and destination are considered with M_S, M_D, and M_R antennas, respectively. For simplicity, it is assumed that all of the relays use the same number of antennas and the same transmission scheme, and all of the SR channels experience the independent and identically distributed (i.i.d.) fading and so do all of the RD channels. We first extend the results of the near-ML decoder in [5] to the multiple-relay cases and derive an upper bound on PEP for the DF relay network in high SNR region. Using this upper bound, a new relay selection scheme based on both CSI and transmission scheme is proposed. We also prove that the proposed relay selection which selects m $(1 \leq m \leq N)$ relays from N relays achieves full diversity $M_S M_D + N M_R \min[M_S, M_D]$ regardless of the value of m. Through simulation, compared with the relay selection in [14] we show that the proposed relay selection has the similar error performance for uncoded single-antenna DF relay network and better error performance for Alamouti coded and MIMO DF relay networks.

It is also interesting to find how many relays should be selected to satisfy the requirement of the networks. It is clear that the proposed relay selection with $m = 1$ is the best in terms of bandwidth efficiency. However, it is not easy to determine analytically the number of selected relays to achieve the best error performance. Thus, the proposed relay selection for various m is evaluated through simulation for the uncoded single-antenna, Alamouti coded, and MIMO DF relay networks.

This paper is organized as follows. In Section II, the system model and the near-ML decoder are introduced. An upper bound on PEP for the DF relay network with multiple antennas is derived in Section III. Using the upper bound on PEP, a new relay selection scheme is proposed and its diversity is derived in Section IV. The discussion and simulation results are provided in Section V and the conclusion is given in Section VI.

The following notations are used in this paper: the capital letter denotes a matrix; I_n denotes the $n \times n$ identity matrix; $\mathbb{C}^{n\times m}$ denotes a set of $n\times m$ complex matrices; $\|\cdot\|$ and tr$(\cdot)$ represent the Frobenius norm and the trace of a matrix, respectively; $E[\cdot]$ is the expectation; the superscript $(\cdot)^\dagger$ denotes the complex conjugate transpose; Re$(\cdot)$ means the real part of a complex number. For $A \in \mathbb{C}^{n\times m}$, $A \sim \mathcal{CN}(0, \sigma^2 I_{nm})$ denotes that the elements of A are i.i.d. circularly symmetric Gaussian random variables with zero mean and variance σ^2. $P(a = b)$ in probability, i.e., $\lim_{\sigma^2\to 0} P(a = b) = 1$, is denoted by $a \overset{P}{=} b$ and similarly the notations $\overset{P}{\leq}$, $\overset{P}{\geq}$, $\overset{P}{\approx}$, $\overset{P}{\lessapprox}$, and $\overset{P}{\gtrapprox}$ are also used.

II. Near-ML Decoder for Decode-and-Forward Relay Cooperative Network with Multiple Relays

A. System Model

A DF relay network with one source, one destination, and multiple relays using half-duplex transmission is shown in Fig. 1. It is also assumed that the channels are frequency-flat quasi-static fading channels, the relays know the CSI of the corresponding SR channel, and the destination knows the CSIs of all SR, source-destination (SD), and RD channels.

In the first phase, the source with M_S antennas broadcasts $M_S \times T_1$ codeword $X_S(\mathrm{x})$ constructed from L-tuple message vector $\mathrm{x} = (x_1, x_2, \ldots, x_L) \in \mathcal{A}^L$ to the N relays and the destination, where $\mathcal{A}$ is the set of message symbols from the M-ary signal constellation. Then, the received signals at the ith relay with M_R antennas in the first phase can be written as

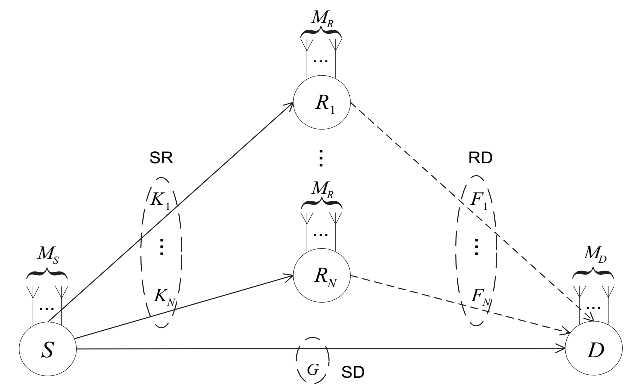

Fig. 1. The DF relay network using multiple relays. The solid line denotes the first phase transmission and the dashed line denotes the second phase transmission.

$$Y_{SR_i} = \sqrt{P_S} K_i X_S(\mathrm{x}) + N_{SR_i} \tag{1}$$

where P_S is the average transmit power at the source, $K_i \in \mathbb{C}^{M_R\times M_S}$ is the channel coefficient matrix of the ith SR channel distributed as $K_i \sim \mathcal{CN}(0, \sigma_{SR}^2 I_{M_R M_S})$, and $N_{SR_i} \in \mathbb{C}^{M_R\times T_1}$ is the noise matrix with distribution $N_{SR_i} \sim \mathcal{CN}(0, \sigma^2 I_{M_R T_1})$. At the same time, the destination also receives the signal transmitted from the source as

$$Y_{SD} = \sqrt{P_S} G X_S(\mathrm{x}) + N_{SD} \tag{2}$$

where $G \in \mathbb{C}^{M_D\times M_S}$ is the channel coefficient matrix of the SD channel distributed as $G \sim \mathcal{CN}(0, \sigma_{SD}^2 I_{M_D M_S})$ and $N_{SD} \in \mathbb{C}^{M_D\times T_1}$ represents the noise matrix at the destination with distribution $N_{SD} \sim \mathcal{CN}(0, \sigma^2 I_{M_D T_1})$.

In the second phase, N relays transmit the codewords constructed for their decoded symbols through N orthogonal RD channels, where "orthogonal" means that the relays use the independent channels, e.g., time division or frequency division channel. Thus, the received signal at the destination through the ith orthogonal channel for the ith relay is given as

$$Y_{R_i D} = \sqrt{P_R} F_i X_R(\mathrm{x}_{R_i}) + N_{R_i D} \tag{3}$$

where $X_R(\mathrm{x}_{R_i}) \in \mathbb{C}^{M_R\times T_2}$ is the codeword constructed from the L-tuple message vector $\mathrm{x}_{R_i} = (x_1^{R_i}, x_2^{R_i}, \ldots, x_L^{R_i}) \in \mathcal{A}^L$ decoded by the ith relay in the first phase. P_R is the average transmit power at each relay, $F_i \in \mathbb{C}^{M_D\times M_R}$ is the channel coefficient matrix of the ith RD channel distributed as $F_i \sim \mathcal{CN}(0, \sigma_{RD}^2 I_{M_D M_R})$, and $N_{R_i D} \in \mathbb{C}^{M_D\times T_2}$ is the noise matrix at the destination in the ith orthogonal channel with $N_{R_i D} \sim \mathcal{CN}(0, \sigma^2 I_{M_D T_2})$.

B. Near-ML Decoder

Let $P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})$ be the probability that the ith relay R_i decodes the received signal to $\hat{\mathrm{x}}_{R_i}$ when the source transmits the codeword corresponding to the message vector x in the first phase. Considering the decoding error at the relay as in

[5], the ML decoder for DF relay network with N relays can be written as

$$\begin{aligned}\hat{\mathrm{x}} &= \arg\max_{\mathrm{x}\in\mathcal{A}^L} p\left(Y_{SD}, Y_{R_1D}, \ldots, Y_{R_ND}|\mathrm{x}\right)\\ &= \arg\max_{\mathrm{x}\in\mathcal{A}^L} p\left(Y_{SD}|X_S(\mathrm{x})\right)\prod_{i=1}^{N}\sum_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L} p\left(Y_{R_iD}|X_R(\hat{\mathrm{x}}_{R_i})\right)P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})\\ &= \arg\max_{\mathrm{x}\in\mathcal{A}^L}\Bigg[-\frac{\left\|Y_{SD}-\sqrt{P_S}GX_S(\mathrm{x})\right\|^2}{\sigma^2}\\ &+\sum_{i=1}^{N}\ln\sum_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\exp\left(\frac{-\left\|Y_{R_iD}-\sqrt{P_R}F_iX_R(\hat{\mathrm{x}}_{R_i})\right\|^2+\sigma^2\ln P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})}{\sigma^2}\right)\Bigg]. \end{aligned} \tag{4}$$

Since it is very difficult to derive $P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})$ for the codeword $X_S(\mathrm{x})$, the PEP at the ith relay $P_{SR_i}(\mathrm{x}\rightarrow\hat{\mathrm{x}}_{R_i})$ will be used instead of $P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})$ in (4). Although the PEP $P_{SR_i}(\mathrm{x}\rightarrow\hat{\mathrm{x}}_{R_i})$ is not equal to $P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})$, it can be a good substitution for $P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})$ to find the solution of (4) as in [5].[1] The widely-used max-log approximation $\ln\sum_i e^{x_i}\approx\max_i x_i$ [15], [16], [17] is also used. Through these two steps, the ML decoder can be simplified to the so-called near-ML decoder as in [5]. Then, the near-ML decoder for (4) can be written as

$$\begin{aligned}\hat{\mathrm{x}} &= \arg\min_{\mathrm{x}\in\mathcal{A}^L}\Bigg\{\left\|Y_{SD}-\sqrt{P_S}GX_S(\mathrm{x})\right\|^2\\ &+\sum_{i=1}^{N}\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\left[\left\|Y_{R_iD}-\sqrt{P_R}F_iX_R(\hat{\mathrm{x}}_{R_i})\right\|^2-\sigma^2\ln P_{SR_i}(\mathrm{x}\rightarrow\hat{\mathrm{x}}_{R_i})\right]\Bigg\}.\end{aligned}\tag{5}$$

Unlike the ML decoder, the near-ML decoder can be applied to most of the DF relay networks with multiple antennas, such as space-time code (STC) and MIMO DF relay networks.

In order to derive the relay selection criterion, the PEP of the near-ML decoder will be derived in the next section.

III. Pairwise Error Probability for DF Relay Network with N Relays

The following theorem is used to derive the PEP of the near-ML decoder.

Theorem 1: [5] Let A and B be complex matrices satisfying $\|B\|^2 > \|A\|^2$ and C be a random matrix of the entries with complex Gaussian distribution $\mathcal{CN}(0,\sigma^2)$. Then, for $\sigma^2\rightarrow 0$, $\|B+C\|^2 \geq \|A+C\|^2$ in probability, i.e.,

$$\lim_{\sigma^2\rightarrow 0} P\left(\|B+C\|^2\geq\|A+C\|^2\right)=1.$$

□

Since a relay may transmit any vector in the signal set $\mathcal{A}^L$ due to the decoding error, the PEP between x and $\tilde{\mathrm{x}}$ at the destination should be written as

$$\begin{aligned}&P(\mathrm{x}\rightarrow\tilde{\mathrm{x}})\\ &=\sum_{\mathrm{x}_{R_1}\in\mathcal{A}^L}\cdots\sum_{\mathrm{x}_{R_N}\in\mathcal{A}^L}P(\mathrm{x}\rightarrow\tilde{\mathrm{x}}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N})\prod_{i=1}^{N}P_{SR_i}(\mathrm{x}_{R_i}|\mathrm{x})\end{aligned}\tag{6}$$

[1] $P_{SR_i}(\mathrm{x}\rightarrow\hat{\mathrm{x}}_{R_i})$ is equal to $P_{SR_i}(\hat{\mathrm{x}}_{R_i}|\mathrm{x})$ for a single-antenna system with BPSK modulation.

where $P(\mathrm{x}\rightarrow\tilde{\mathrm{x}}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N})$ denotes the conditional PEP of decoding the received signals into $\tilde{\mathrm{x}}$ at the destination when x and x_{R_i} for $i=1,\ldots,N$ are transmitted from the source and the relays, respectively. The condition x will be omitted to simplify the expression as $P(\mathrm{x}\rightarrow\tilde{\mathrm{x}}|\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N})$.

Then, the conditional PEP in (6) can be written as

$$\begin{aligned}&P(\mathrm{x}\rightarrow\tilde{\mathrm{x}}|\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N})\\ &=P\Big(m\big([Y_{SD},Y_{R_1D},\ldots,Y_{R_ND}],\mathrm{x}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N}\big)\\ &\qquad> m\big([Y_{SD},Y_{R_1D},\ldots,Y_{R_ND}],\tilde{\mathrm{x}}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N}\big)\Big)\end{aligned}\tag{7}$$

where

$$\begin{aligned}&m\big([Y_{SD},Y_{R_1D},\ldots,Y_{R_ND}],\mathrm{x}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N}\big)=\left\|Y_{SD}-\sqrt{P_S}GX_S(\mathrm{x})\right\|^2\\ &+\sum_{i=1}^{N}\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\left[\left\|Y_{R_iD}-\sqrt{P_R}F_iX_R(\hat{\mathrm{x}}_{R_i})\right\|^2-\sigma^2\ln P_{SR_i}(\mathrm{x}\rightarrow\hat{\mathrm{x}}_{R_i})\right]\end{aligned}$$

and

$$\begin{aligned}&m\big([Y_{SD},Y_{R_1D},\ldots,Y_{R_ND}],\tilde{\mathrm{x}}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N}\big)=\left\|Y_{SD}-\sqrt{P_S}GX_S(\tilde{\mathrm{x}})\right\|^2\\ &+\sum_{i=1}^{N}\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\left[\left\|Y_{R_iD}-\sqrt{P_R}F_iX_R(\hat{\mathrm{x}}_{R_i})\right\|^2-\sigma^2\ln P_{SR_i}(\tilde{\mathrm{x}}\rightarrow\hat{\mathrm{x}}_{R_i})\right]\end{aligned}$$

are the metrics in (5) to decide x and $\tilde{\mathrm{x}}$ for the given x and $\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N}$ transmitted from the source and relays, respectively.

As derived in [18], the PEPs for the ith SR channel with the given K_i can be written as

$$P_{SR_i}(\mathrm{x}\rightarrow\hat{\mathrm{x}}_{R_i})=Q\left(\sqrt{\frac{P_S}{2\sigma^2}\left\|K_i(X_S(\mathrm{x})-X_S(\hat{\mathrm{x}}_{R_i}))\right\|^2}\right)\tag{8}$$

and

$$P_{SR_i}(\tilde{\mathrm{x}}\rightarrow\hat{\mathrm{x}}_{R_i})=Q\left(\sqrt{\frac{P_S}{2\sigma^2}\left\|K_i(X_S(\tilde{\mathrm{x}})-X_S(\hat{\mathrm{x}}_{R_i}))\right\|^2}\right).$$

Using the above PEPs, (2), and (3), the metrics in (7) can be written as

$$\begin{aligned}&m\big([Y_{SD},Y_{R_1D},\ldots,Y_{R_ND}],\mathrm{x}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N}\big)\\ &=\left\|N_{SD}\right\|^2+\sum_{i=1}^{N}\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\Bigg[\left\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}_{R_i})-X_R(\hat{\mathrm{x}}_{R_i})\big)+N_{R_iD}\right\|^2\\ &\qquad-\sigma^2\ln Q\left(\sqrt{\frac{P_S}{2\sigma^2}\left\|K_i\big(X_S(\mathrm{x})-X_S(\hat{\mathrm{x}}_{R_i})\big)\right\|^2}\right)\Bigg]\end{aligned}\tag{9}$$

and

$$\begin{aligned}&m\big([Y_{SD},Y_{R_1D},\ldots,Y_{R_ND}],\tilde{\mathrm{x}}|\mathrm{x},\mathrm{x}_{R_1},\ldots,\mathrm{x}_{R_N}\big)\\ &=\left\|\sqrt{P_S}G\big(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}})\big)+N_{SD}\right\|^2\\ &+\sum_{i=1}^{N}\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\Bigg[\left\|\sqrt{P_S}F_i\left(X_R(\mathrm{x}_{R_i})-X_R(\hat{\mathrm{x}}_{R_i})\right)+N_{R_iD}\right\|^2\\ &\qquad-\sigma^2\ln Q\left(\sqrt{\frac{P_S}{2\sigma^2}\left\|K_i(X_S(\tilde{\mathrm{x}})-X_S(\hat{\mathrm{x}}_{R_i}))\right\|^2}\right)\Bigg].\end{aligned}\tag{10}$$

The conditional PEP in (7) is very difficult to simplify due to the Q function. However, by using $Q(x)\approx$

$$P(\mathrm{x} \to \tilde{\mathrm{x}}) = \sum_{n_R=0}^{N} \sum_{S\in S(n_R)} \sum_{\substack{\mathrm{x}_{R_i}\in\mathcal{A}^L \\ \mathrm{x}_{R_i}\neq \mathrm{x}, i\in S_N\setminus S}} P\Big(\mathrm{x}\to\tilde{\mathrm{x}}\big|\mathrm{x}_{R_j}=\mathrm{x}, j\in S \text{ and } \mathrm{x}_{R_i}, i\in S_N\setminus S\Big) \cdot \prod_{j\in S} P_{SR_j}(\mathrm{x}|\mathrm{x}) \prod_{i\in S_N\setminus S} P_{SR_i}(\mathrm{x}_{R_i}|\mathrm{x}) \quad (13)$$

$$\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L} \Big[\big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}_{R_i}) - X_R(\hat{\mathrm{x}}_{R_i})\big) + N_{R_iD}\big\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\tilde{\mathrm{x}}) - X_S(\hat{\mathrm{x}}_{R_i})\big)\big\|^2\Big]$$
$$\overset{P}{\geq} \min\Big[\|N_{R_iD}\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\tilde{\mathrm{x}}) - X_S(\mathrm{x})\big)\big\|^2, \big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}) - X_R(\mathrm{x}_{F_i}^{\min})\big) + N_{R_iD}\big\|^2\Big] \quad (15)$$

$\frac{\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)}{(1-\frac{1}{\pi})x + \frac{1}{\pi}\sqrt{x^2+2\pi}}$, $0 < x < \infty$, in [19], the following approximation can be obtained as

$$\lim_{\sigma^2\to 0} \sigma^2 \ln Q\Big(\sqrt{\frac{P_S}{2\sigma^2}\big\|K_i(X_S(\mathrm{x}) - X_S(\check{\mathrm{x}}))\big\|^2}\Big) \approx -\frac{P_S}{4}\big\|K_i(X_S(\mathrm{x}) - X_S(\check{\mathrm{x}}))\big\|^2.$$

Thus, for high SNR, the metrics (9) and (10) can be approximated as

$$m\big([Y_{SD}, Y_{R_1D}, \ldots, Y_{R_ND}], \mathrm{x}|\mathrm{x}, \mathrm{x}_{R_1}, \ldots, \mathrm{x}_{R_N}\big) \approx \big\|N_{SD}\big\|^2 + \sum_{i=1}^{N} \min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\Big[\big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}_{R_i}) - X_R(\hat{\mathrm{x}}_{R_i})\big) + N_{R_iD}\big\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\mathrm{x}) - X_S(\hat{\mathrm{x}}_{R_i})\big)\big\|^2\Big] \quad (11)$$

and

$$m\big([Y_{SD}, Y_{R_1D}, \ldots, Y_{R_ND}], \tilde{\mathrm{x}}|\mathrm{x}, \mathrm{x}_{R_1}, \ldots, \mathrm{x}_{R_N}\big) \approx \big\|\sqrt{P_S}G\big(X_S(\mathrm{x}) - X_S(\tilde{\mathrm{x}})\big) + N_{SD}\big\|^2 + \sum_{i=1}^{N} \min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\Big[\big\|\sqrt{P_S}F_i\big(X_R(\mathrm{x}_{R_i}) - X_R(\hat{\mathrm{x}}_{R_i})\big) + N_{R_iD}\big\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\tilde{\mathrm{x}}) - X_S(\hat{\mathrm{x}}_{R_i})\big)\big\|^2\Big]. \quad (12)$$

Since the minimization in (11) and (12) still makes it difficult to use, we further simplify two metrics (11) and (12) by splitting x_{R_i} in (6) into two cases, $\mathrm{x}_{R_i} = \mathrm{x}$ and $\mathrm{x}_{R_i} \neq \mathrm{x}$, for high SNR region. Let $S_N = \{1, \ldots, N\}$ and $S(n_R) = \{S' | S' \subseteq S_N \text{ with } |S'| = n_R\}$, where $0 \leq n_R \leq N$. We assume that $\mathrm{x}_{R_i} = \mathrm{x}$ for n_R relays in $S \in S(n_R)$ and $\mathrm{x}_{R_i} \neq \mathrm{x}$ for $N - n_R$ relays in $S_N \setminus S$, where $S_N \setminus S$ means the complement of S in S_N, i.e., n_R relays have no decoding error and $N - n_R$ relays have decoding error. Then, the PEP in (6) can be rewritten as (13). Then, the PEP $P(\mathrm{x} \to \tilde{\mathrm{x}})$ can be obtained by deriving the summand in (13).

First, we consider the metrics (11) and (12) for the case of $\mathrm{x}_{R_i} = \mathrm{x}, i \in S$ and $\mathrm{x}_{R_i} \neq \mathrm{x}, i \in S_N\setminus S$. For $i \in S$, i.e., $\mathrm{x}_{R_i} = \mathrm{x}$, the minimization in (11) results in $\|N_{R_iD}\|^2$ in probability from Theorem 1. For $i \in S_N\setminus S$, i.e., $\mathrm{x}_{R_i} \neq \mathrm{x}$, it is not easy to determine the value of the minimization term. However, we know that the value for any $\hat{\mathrm{x}}_{R_i} \in \mathcal{A}^L$ is greater than the minimization term. Thus, we have the upper bound on the minimization term when $\hat{\mathrm{x}}_{R_i} = \mathrm{x}_{R_i}$. The minimization term in (11) can be derived as

$$\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\Big[\big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}_{R_i}) - X_R(\hat{\mathrm{x}}_{R_i})\big) + N_{R_iD}\big\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\mathrm{x}) - X_S(\hat{\mathrm{x}}_{R_i})\big)\big\|^2\Big] \begin{cases} \overset{P}{=} \|N_{R_iD}\|^2 & \text{for } i\in S \\ \overset{P}{\leq} \|N_{R_iD}\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\mathrm{x}) - X_S(\mathrm{x}_{R_i})\big)\big\|^2 & \text{for } i\in S_N\setminus S. \end{cases}$$

Thus, an upper bound on the metric in (11) can be obtained as

$$m\big([Y_{SD}, Y_{R_1D}, \ldots, Y_{R_ND}], \mathrm{x}|\mathrm{x}, \mathrm{x}_{R_1}, \ldots, \mathrm{x}_{R_N}\big) \overset{P}{\lessapprox} \|N_{SD}\|^2 + \sum_{i\in S_N}\|N_{R_iD}\|^2 + \sum_{i\in S_N\setminus S}\frac{P_S}{4}\big\|K_i\big(X_S(\mathrm{x}) - X_S(\mathrm{x}_{R_i})\big)\big\|^2. \quad (14)$$

Similarly, the metric $m\big([Y_{SD}, Y_{R_1D}, \ldots, Y_{R_ND}], \tilde{\mathrm{x}}|\mathrm{x}, \mathrm{x}_{R_1}, \ldots, \mathrm{x}_{R_N}\big)$ in (12) can also be derived. For $i \in S$, using Theorem 1, we have

$$\|\sqrt{P_R}F_i(X_R(\mathrm{x}_{R_i}) - X_R(\hat{\mathrm{x}}_{R_i})) + N_{R_iD}\|^2 + \frac{P_S}{4}\|K_i(X_S(\tilde{\mathrm{x}}) - X_S(\hat{\mathrm{x}}_{R_i}))\|^2 \begin{cases} = \|N_{R_iD}\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\tilde{\mathrm{x}}) - X_S(\mathrm{x})\big)\big\|^2 & \text{if } \hat{\mathrm{x}}_{R_i} = \mathrm{x} \\ \overset{P}{\geq} \big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}) - X_R(\mathrm{x}_{F_i}^{\min})\big) + N_{R_iD}\big\|^2 & \text{if } \hat{\mathrm{x}}_{R_i} \neq \mathrm{x} \end{cases}$$

where $\mathrm{x}_{F_i}^{\min} = \arg\min_{\hat{\mathrm{x}}_{R_i}\neq\mathrm{x}} \big\|F_i\big(X_R(\mathrm{x}) - X_R(\hat{\mathrm{x}}_{R_i})\big)\big\|^2$, and thus we have an inequality (15). For $i \in S_N\setminus S$, from Theorem 1, we also have

$$\min_{\hat{\mathrm{x}}_{R_i}\in\mathcal{A}^L}\Big[\big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}_{R_i}) - X_R(\hat{\mathrm{x}}_{R_i})\big) + N_{R_iD}\big\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\tilde{\mathrm{x}}) - X_S(\hat{\mathrm{x}}_{R_i})\big)\big\|^2\Big] \overset{P}{\geq} \|N_{R_iD}\|^2. \quad (16)$$

Combining (15) and (16), (12) can be lower bounded as

$$m\big([Y_{SD}, Y_{R_1D}, \ldots, Y_{R_ND}], \tilde{\mathrm{x}}|\mathrm{x}, \mathrm{x}_{R_1}, \ldots, \mathrm{x}_{R_N}\big) \overset{P}{\gtrapprox} \big\|\sqrt{P_S}G(X_S(\mathrm{x}) - X_S(\tilde{\mathrm{x}})) + N_{SD}\big\|^2 + \sum_{i\in S_N\setminus S}\|N_{R_iD}\|^2 + \sum_{i\in S}\min\Big[\big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x}) - X_R(\mathrm{x}_{F_i}^{\min})\big) + N_{R_iD}\big\|^2, \|N_{R_iD}\|^2 + \frac{P_S}{4}\big\|K_i\big(X_S(\tilde{\mathrm{x}}) - X_S(\mathrm{x})\big)\big\|^2\Big]. \quad (17)$$

Using $P_{SR_i}(\mathrm{x}|\mathrm{x}) \leq 1$ for $i \in S$, $P_{SR_i}(\mathrm{x}_{R_i}|\mathrm{x}) \leq P_{SR_i}(\mathrm{x} \to \mathrm{x}_{R_i})$ for $i \in S_N\setminus S$, (8), and $Q(x) \leq \exp(-x^2/2), x > 0$, the

$$
\begin{aligned}
&P\Big(\mathrm{x}\rightarrow\tilde{\mathrm{x}}\big|\mathrm{x}_{R_i}=\mathrm{x}, i\in S \text{ and } \mathrm{x}_{R_i}\neq \mathrm{x}, i\in S_N\backslash S\Big)\prod_{i\in S}P_{SR_i}(\mathrm{x}|\mathrm{x})\prod_{i\in S_N\backslash S}P_{SR_i}(\mathrm{x}_{R_i}|\mathrm{x})\\
&\overset{P}{\lessapprox} P\Bigg(\big\|N_{SD}\big\|^2+\sum_{i=1}^{N}\big\|N_{R_iD}\big\|^2+\sum_{i\in S_N\backslash S}\frac{P_S}{4}\big\|K_i(X_S(\mathrm{x})-X_S(\mathrm{x}_{R_i}))\big\|^2\\
&\qquad>\sum_{i\in S}\min\Big[\big\|\sqrt{P_R}F_i\big(X_R(\mathrm{x})-X_R(\mathrm{x}_{F_i}^{\min})\big)+N_{R_iD}\big\|^2, \big\|N_{R_iD}\big\|^2+\frac{P_S}{4}\big\|K_i\big(X_S(\tilde{\mathrm{x}})-X_S(\mathrm{x})\big)\big\|^2\Big]\\
&\qquad+\big\|\sqrt{P_S}G\big(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}})\big)+N_{SD}\big\|^2+\sum_{i\in S_N\backslash S}\big\|N_{R_iD}\big\|^2\Bigg)\exp\Big(-\sum_{i\in S_N\backslash S}\frac{P_S}{4\sigma^2}\big\|K_i\big(X_S(\mathrm{x})-X_S(\mathrm{x}_{R_i})\big)\big\|^2\Big) \qquad (19)
\end{aligned}
$$

product term in (13) is upper bounded as

$$
\begin{aligned}
&\prod_{i\in S}P_{SR_i}(\mathrm{x}|\mathrm{x})\prod_{i\in S_N\backslash S}P_{SR_i}(\mathrm{x}_{R_i}|\mathrm{x})\\
&\leq \exp\Bigg(-\sum_{i\in S_N\backslash S}\frac{P_S}{4\sigma^2}\big\|K_i\big(X_S(\mathrm{x})-X_S(\mathrm{x}_{R_i})\big)\big\|^2\Bigg). \qquad (18)
\end{aligned}
$$

Thus, by using (14), (17), and (18), the summand in (13) can be upper bounded as (19). Plugging (19) into (13), the following theorem can be derived.

Theorem 2: For the DF relay network with N relays and multiple antennas, the PEP of near-ML decoder between L-tuple message vectors x and $\tilde{\mathrm{x}}$ from the M-ary signal constellation can be upper bounded as

$$
\begin{aligned}
P(\mathrm{x}\rightarrow\tilde{\mathrm{x}})&\overset{P}{\lessapprox}2(M^L+1)^N\exp\Big(-\frac{1}{4\sigma^2}P_S\|G(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}}))\|^2\Big)\\
&\exp\Bigg(-\frac{1}{4\sigma^2}\sum_{i\in S_N}\min\Big[\frac{P_S}{2}\big\|K_i\big(X_S(\mathrm{x})-X_S(\mathrm{x}_{K_i}^{\min})\big)\big\|^2,\\
&\qquad P_R\big\|F_i\big(X_R(\mathrm{x})-X_R(\mathrm{x}_{F_i}^{\min})\big)\big\|^2\Big]\Bigg) \qquad (20)
\end{aligned}
$$

where $\mathrm{x}_{K_i}^{\min}=\arg\min_{\check{\mathrm{x}}\neq\mathrm{x}}\big\|K_i\big(X_S(\mathrm{x})-X_S(\check{\mathrm{x}})\big)\big\|^2$ and $\mathrm{x}_{F_i}^{\min}=\arg\min_{\hat{\mathrm{x}}_{R_i}\neq\mathrm{x}}\big\|F_i\big(X_R(\mathrm{x})-X_R(\hat{\mathrm{x}}_{R_i})\big)\big\|^2$. P_S, P_R, G, K_i, F_i, X_S, X_R, and σ^2 are introduced in Section II.

Proof: See the Appendix A. □

In the next section, a relay selection criterion for DF relay network with multiple antennas is derived from Theorem 2.

IV. Relay Selection for Decode-and-Forward Relay Network

A. A New Relay Selection Scheme

In general, it is very difficult to derive the optimal criterion for the relay selection in the DF relay networks with multiple antennas. However, the maximum PEP can be used for deriving a criterion of selecting good relays because the maximum PEP is a dominant term for the union bound on bit error probability (BEP) [20]. It is also difficult to derive the exact PEP in the DF relay network with multiple antennas as shown in Section III. Thus the upper bound on PEP is derived in Theorem 2, which can be separated into two parts, one is the SD direct path and the other is the source-relay-destination (SRD) path. The SRD path is also separated into N SRD paths corresponding to N relays. The upper bound on PEP is very useful for finding a new relay ordering.

From the upper bound on the PEP in (20), we define a relay path metric γ_i as

$$
\begin{aligned}
\gamma_i=\min\Big[&\frac{P_S}{2}\min_{\mathrm{x},\check{\mathrm{x}}\neq\mathrm{x}}\big\|K_i\big(X_S(\mathrm{x})-X_S(\check{\mathrm{x}})\big)\big\|^2,\\
&P_R\min_{\mathrm{x},\check{\mathrm{x}}\neq\mathrm{x}}\big\|F_i\big(X_R(\mathrm{x})-X_R(\check{\mathrm{x}})\big)\big\|^2\Big]. \qquad (21)
\end{aligned}
$$

Based on the fact that the upper bound on PEP in (20) decreases as γ_i increases, we propose the following selection scheme of m relays ($1\leq m\leq N$) for the DF relay network:

1) The destination finds m relays which have m largest relay path metrics γ_i;
2) The destination sends the indices of the selected m relays to N relays;
3) The selected relays transmit the signal.

Next, we discuss the complexity and overhead for the proposed relay selection scheme.

i) Complexity:

The DF relay network considered in this paper includes one SD and multiple SRD links and thus the error probability is related to the channel coefficients and transmission schemes of SD link and SRD links. To select m relays among N relays, we have to consider the total error probability, i.e., all of the SD and SRD links and compare $\binom{N}{m}$ possible sets of the relays. However, our relay selection scheme considers each relay separately through the metric γ_i (SRD link), but not SD link and thus only needs to compare N SRD links. Therefore, our new relay selection scheme has less complexity.

ii) Overhead:

To feedback the indices of the selected relays, the following two methods for the feedback message can be used:

- N bits, where each bit indicates whether the corresponding relay transmits signal or not;
- $m\lceil\log_2 N\rceil$ bits for the indices of the m selected relays.

Since the first and second methods need total N bits and $m\lceil\log_2 N\rceil$ bits, respectively, the number of required bits for the feedback message is $\min\big[N, m\lceil\log_2 N\rceil\big]$.

As mentioned in Section I, the relay selection with $m=1$ is the best in terms of bandwidth efficiency. However, it is not easy to determine how many relays should be selected to achieve the best error correction performance. To compare the error performance for various m, we set up the relay selection for arbitrary m ($1\leq m\leq N$). The diversity for various m is derived in the next subsection and the simulated BEP is given in Section V.

B. Diversity Analysis for the New Relay Selection Scheme

Clearly, there are $\binom{N}{m}$ sets of the selected relays. Let $S_n, n = 1,\ldots,\binom{N}{m}$, be the sets of possible selected relay indices with $|S_n| = m$. Then, using (20), the PEP for DF relay network with m relay selection out of N relays can be derived as

$$E[P_{RSelect}(\mathrm{x}\to\tilde{\mathrm{x}})] \overset{P}{\lessapprox} 2(M^L+1)^m E\left[\exp\left(-\frac{P_S\|G(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}}))\|^2}{4\sigma^2}\right)\right] E\left[\exp\left(-\frac{1}{4\sigma^2}\max_{n\in\{1,\ldots,\binom{N}{m}\}}\sum_{i\in S_n}\gamma_i\right)\right]. \quad (22)$$

Let r_S and r_R be the minimum ranks among the ranks of $\big(X_S(\mathrm{x}) - X_S(\check{\mathrm{x}})\big)\big(X_S(\mathrm{x}) - X_S(\check{\mathrm{x}})\big)^\dagger$ and $\big(X_R(\mathrm{x}) - X_R(\check{\mathrm{x}})\big)\big(X_R(\mathrm{x}) - X_R(\check{\mathrm{x}})\big)^\dagger$ for all $\check{\mathrm{x}} \neq \mathrm{x}$, respectively. We define $M_D\times r_S$ matrix G', $M_R\times r_S$ matrix K_i', and $M_D\times r_R$ matrix $F_i', i = 1,\ldots,N$ with $[G']_l = [GU]_l$ for $l = 1,\ldots,r_S$, $[K_i']_l = [K_iU]_l$ for $l = 1,\ldots,r_S$, and $[F_i']_l = [F_iV]_l$ for $l = 1,\ldots,r_R$, respectively, where $[A]_l$ means the lth column of the matrix A, U and V are the unitary matrices whose columns are the eigenvectors of $(X_S(\mathrm{x})–X_S(\check{\mathrm{x}}))(X_S(\mathrm{x})–X_S(\check{\mathrm{x}}))^\dagger$ and $(X_R(\mathrm{x})-X_R(\check{\mathrm{x}}))(X_R(\mathrm{x})-X_R(\check{\mathrm{x}}))^\dagger$ for any $\check{\mathrm{x}} \neq \mathrm{x}$, respectively. Since the multiplication of the unitary matrix does not change the statistical distribution of the matrix with circularly symmetric complex Gaussian entries, the entries of G', K_i', and F_i' have the same distribution as the entries of G, K_i, and F_i, respectively. Therefore, by Fact 1 in Appendix B, the upper bound on the average PEP in (22) can be rewritten as

$$E[P_{RSelect}(\mathrm{x}\to\tilde{\mathrm{x}})] \overset{P}{\lessapprox} 2(M^L+1)^m E\left[\exp\left(-\frac{P_S\omega_{\min}\|G'\|^2}{4\sigma^2}\right)\right] E\left[\exp\left(-\frac{1}{4\sigma^2}\max_{n\in\{1,\ldots,\binom{N}{m}\}}\sum_{i\in S_n}\min\left[\frac{P_S}{2}\omega_{\min}\|K_i'\|^2, P_R\mu_{\min}\|F_i'\|^2\right]\right)\right] \quad (23)$$

where $\omega_{\min}$ and $\mu_{\min}$ are the minimum values among nonzero eigenvalues of $(X_S(\mathrm{x})-X_S(\check{\mathrm{x}}))(X_S(\mathrm{x})-X_S(\check{\mathrm{x}}))^\dagger$ and $(X_R(\mathrm{x})-X_R(\check{\mathrm{x}}))(X_R(\mathrm{x})-X_R(\check{\mathrm{x}}))^\dagger$ for all $\check{\mathrm{x}} \neq \mathrm{x}$, respectively. Let $\gamma_i' = \min\left[\frac{P_S}{2}\omega_{\min}\|K_i'\|^2,\ P_R\mu_{\min}\|F_i'\|^2\right]$, $y_{\max} = \max_{n\in\{1,\ldots,\binom{N}{m}\}}\sum_{i\in S_n}\gamma_i'$, and $y_G = P_S\omega_{\min}\|G'\|^2$. Then, we need to know the distribution of the random variables $y_{\max}$ and y_G. Note that y_G is an r_SM_D-Erlang random variable with rate parameter $P_S\omega_{\min}\sigma_{SD}^2$. However, the cumulative distribution function (CDF) or the probability density function (PDF) of the random variable $y_{\max}$ is very difficult to derive and thus we have to find another way to derive the upper bound on the average PEP in (23). Since

$$E\left[\exp\left(-\frac{y_{\max}}{4\sigma^2}\right)\right] = \frac{1}{4\sigma^2}\int_0^\infty \exp\left(-\frac{y}{4\sigma^2}\right)P_{y_{\max}}(y)dy \quad (24)$$

in the right-hand side (RHS) of (23) where $P_{y_{\max}}(y)$ is the CDF of the random variable $y_{\max}$, the upper bound on the average PEP in (23) can be derived by calculating the upper bound on the CDF. The upper bound on the CDF of $y_{\max}$ is derived as in the following theorem.

Theorem 3: The CDF $P_{y_{\max}}(y)$ of $y_{\max}$ can be upper bounded as

$$P_{y_{\max}}(y) \le \left[1-\exp\left(-\left(\frac{2}{P_S\omega_{\min}\sigma_{SR}^2}+\frac{1}{P_R\mu_{\min}\sigma_{RD}^2}\right)y\right)\right]^{N\min[r_SM_R, r_RM_D]}. \quad (25)$$

Proof: See the Appendix C. □

Using the PDF of y_G and the upper bound on the CDF of $y_{\max}$ derived in Theorem 3, the following theorem for the achievable diversity can be established.

Theorem 4: The new relay selection scheme with the near-ML decoder achieves the diversity $r_SM_D + N\min[r_SM_R, r_RM_D]$ regardless of the number m of the selected relays in the DF relay network consisting of one source, one destination, and N relays with M_S, M_D, and M_R antennas, respectively. The full diversity $M_SM_D+NM_R\min[M_S, M_D]$ is achieved when $r_S = M_S$ and $r_R = M_R$.

Proof: See the Appendix D. □

V. Discussion and Simulation Results

In this section, the application of the proposed relay selection to the following three cases are considered and evaluated. For simplicity, it is assumed that M-ary symbols are normalized to the unit power.

i) Uncoded single-antenna DF relay network: $M_S = M_R = M_D = L = 1$,

$$X_S(\mathrm{x}) = x \qquad \text{and} \qquad X_R(\mathrm{x}_{R_i}) = x_{R_i},$$

and $\omega_{\min} = \mu_{\min} = \min_{x,\check{x}\neq x}|x-\check{x}|^2$. This is a special case of the multiple-antenna systems. Since the source and relays use the same modulation, the relay path metric in (21) becomes

$$\gamma_i = \min_{x,\check{x}\neq x}|x-\check{x}|^2\min\left[\frac{P_S}{2}|k_i|^2, P_R|f_i|^2\right]. \quad (26)$$

The maximum achievable diversity order using the proposed relay selection scheme for total N relays is $N+1$ for the case of $M_S = M_R = M_D = 1$ from Theorem 4.

ii) Alamouti coded DF relay network: Alamouti scheme [21] is used at the source and relay, i.e., $M_S = M_R = M_D = 2$,

$$X_S(\mathrm{x}) = \frac{1}{\sqrt{2}}\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \text{ and } X_R(\mathrm{x}_{R_i}) = \frac{1}{\sqrt{2}}\begin{bmatrix} x_1^{R_i} & -x_2^{R_i*} \\ x_2^{R_i} & x_1^{R_i*} \end{bmatrix},$$

and $\omega_{\min} = \mu_{\min} = \frac{1}{2}\min_{x_1,\check{x}_1\neq x_1}|x_1-\check{x}_1|^2$. Similarly to the uncoded single-antenna case, the source and relays use the same modulation, and thus, the relay path metric becomes

$$\gamma_i = \frac{1}{2}\min_{x_1,\check{x}_1\neq x_1}|x_1-\check{x}_1|^2\min\left[\frac{P_S}{2}\|K_i\|^2, P_R\|F_i\|^2\right]. \quad (27)$$

The maximum achievable diversity order using the proposed relay selection scheme for total N relays is $4(N+1)$ for the case of $M_S = M_R = M_D = 2$ from Theorem 4.

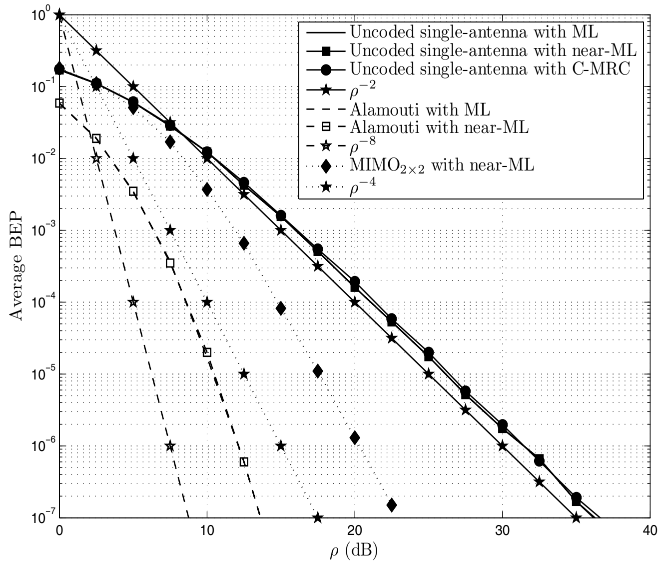

Fig. 2. Average BEP comparison among the ML, near-ML, C-MRC decoders for uncoded single-antenna, Alamouti coded, and MIMO DF relay network with $(\sigma^2_{SR}, \sigma^2_{SD}, \sigma^2_{RD}) = (1, 1, 1)$.

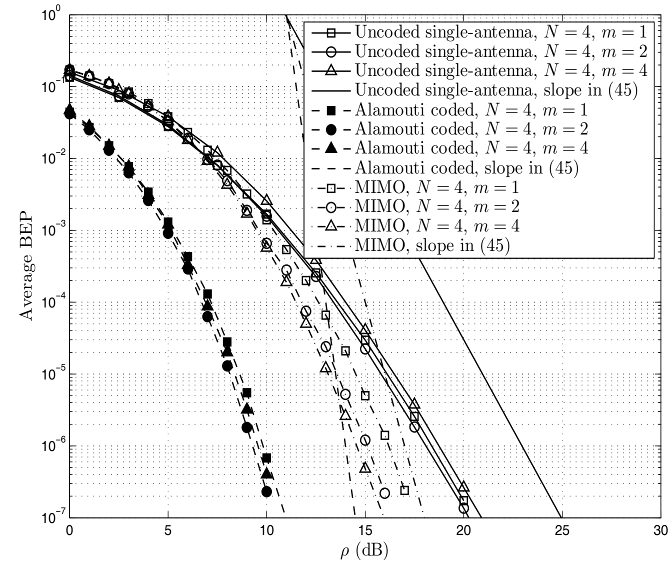

Fig. 4. Average BEP for DF relay network with the proposed relay selection scheme for $(\sigma^2_{SR}, \sigma^2_{SD}, \sigma^2_{RD}) = (1, 1, 1)$.

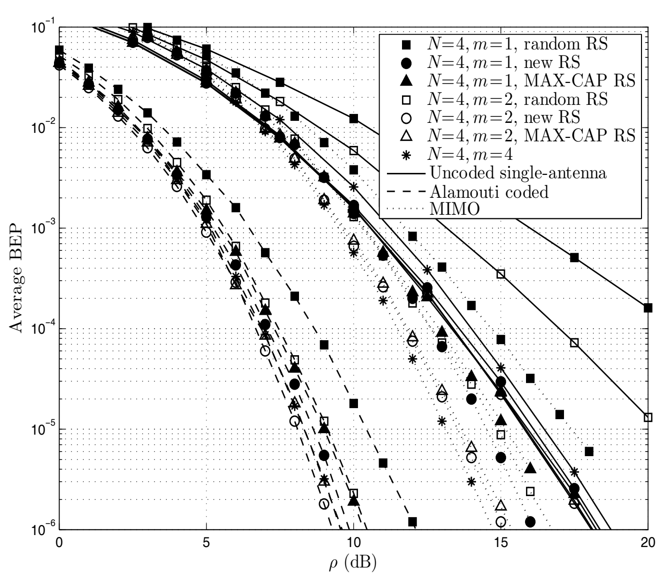

Fig. 3. Average BEP comparison between the proposed relay selection and MAX-CAP relay selection for DF relay network with $(\sigma^2_{SR}, \sigma^2_{SD}, \sigma^2_{RD}) = (1, 1, 1)$.

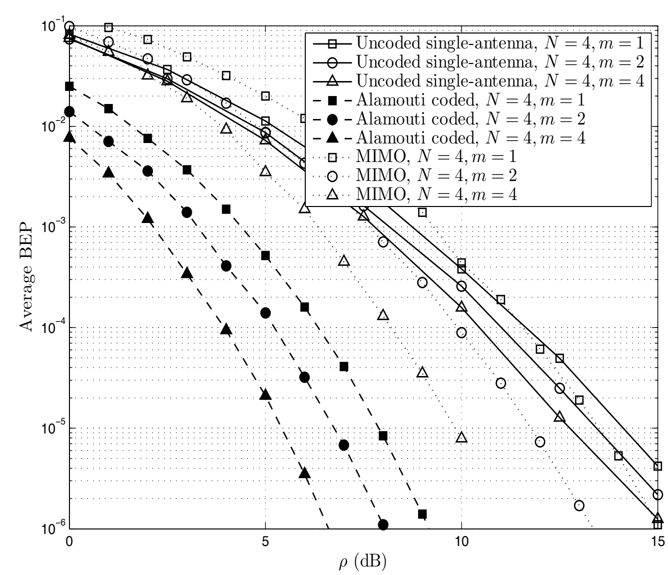

Fig. 5. Average BEP for DF relay network with the proposed relay selection scheme for $(\sigma^2_{SR}, \sigma^2_{SD}, \sigma^2_{RD}) = (1, 1, 10)$.

iii) MIMO DF relay network: The MIMO system is used at the source and relay, i.e., the case of $T_1 = T_2 = 1$ and $M_S = M_R = L$. In this paper, we consider the case of $M_S = M_R = M_D = L = 2$. Then,

$$X_S(\mathrm{x}) = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } X_R(\mathrm{x}_{R_i}) = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1^{R_i} \\ x_2^{R_i} \end{bmatrix},$$

and $\omega_{\min} = \mu_{\min} = \frac{1}{2}\min_{x_1, \check{x}_1 \neq x_1} |x_1 - \check{x}_1|^2$. Even if the source and relays use the same modulation, the relay path metric cannot be simplified and the original relay path metric in (21) should be used. The MIMO DF relay network with the proposed relay selection for total N relays achieves the diversity order $2(N+1)$ from Theorem 4 .

Next, we evaluate the above three cases by Monte Carlo simulation and compare their error correction performance. For the simulation, quadrature phase shift keying (QPSK) and power allocation of $P_S = 1$ and $P_R = 1/m$ are used.

First, we compare the average BEPs of ML, near-ML, and C-MRC decoders for the DF relay network with one relay in Fig. 2. For uncoded single-antenna DF relaying, the ML, near-ML, and C-MRC decoders have similar BEP performance and the same diversity order 2; for Alamouti coded DF relay network, the ML and near-ML decoder have almost the same BEP performance and achieve the same diversity order 8; for MIMO DF relay network, the near-ML decoder achieves the diversity order 4.

Second, we compare the proposed relay selection with the conventional relay selection scheme. Unlike the proposed relay

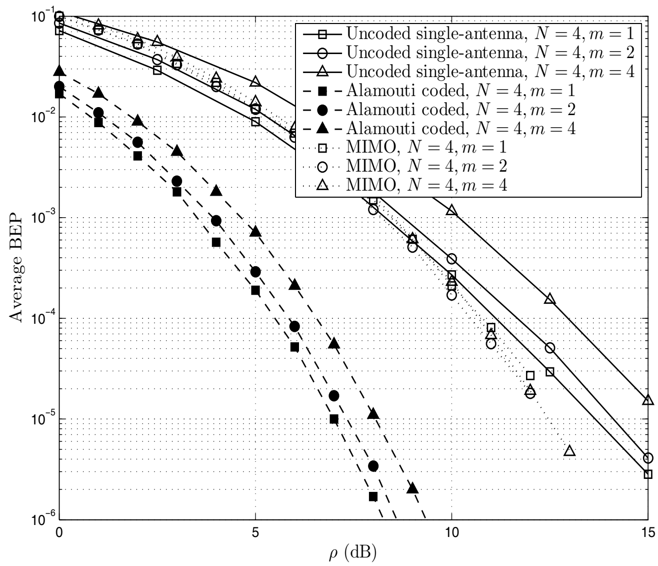

Fig. 6. Average BEP for DF relay network with the proposed relay selection scheme for $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (10, 1, 1)$.

selection scheme, the conventional scheme does not consider the transmission scheme such as the optimal selective routing scheme [14] of selecting single relay which achieves the highest two-hop network capacity (without considering the direct link between the source and destination), that is called MAX-CAP relay selection in this paper. In Fig. 3, we compare the BEPs of the proposed relay selection and MAX-CAP relay selection for the uncoded single-antenna, Alamouti coded, and MIMO DF relay networks. We compare the above two relay selection schemes for $m = 1$ and $m = 2$ and note that the MAX-CAP scheme selects single relay as explained in [14]. Fig. 3 shows that compared to MAX-CAP relay selection, the proposed relay selection has similar performance for uncoded single-antenna DF relay network, and better performance for Alamouti coded and MIMO DF relay networks. Therefore, the proposed relay selection scheme has better BEP performance than MAX-CAP relay selection even though the complexity increases for high order modulations because the proposed relay selection scheme is required to search the minimum values of $\left\|K_i\big(X_S(\mathrm{x})-X_S(\check{\mathrm{x}})\big)\right\|^2$ and $\left\|F_i\big(X_R(\mathrm{x})-X_R(\check{\mathrm{x}})\big)\right\|^2$ for different set of x and $\check{\mathrm{x}}$.

Third, to show the achievable diversity derived in Theorem 4, we compare the slope of the upper bound on the average PEP of (45) in the proof of Theorem 4 and Monte Carlo simulated BEP for the DF relay network with relay selection in Fig. 4. As shown in Fig. 4, the slope of the upper bound on the average PEP in (45) is almost the same as that of the simulated BEP for uncoded single-antenna DF relay network in high SNR region. For the Alamouti coded and MIMO DF relay networks, the slopes of the upper bound on the average PEP in (45) become closer to those of the simulated BEP as the SNR increases, and those slopes seem to become the same in very high SNR region. This means that the DF relay network with relay selection achieves the same diversity as the upper bound in (45) regardless of the value of m.

Finally, we discuss and compare the performance of the proposed relay selection for various m. It is clear that the proposed relay selection has better bandwidth efficiency as the number of selected relays m decreases. However, it is not easy to determine the error performance and power efficiency for various m. Since we use the same total transmit power, i.e., $P_S = 1$ and $P_R = 1/m$, good BEP performance means good power efficiency. For $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (1, 1, 1)$, Fig. 4 compares the BEP of the relay selection with $m = 1, 2, 4$ for $N = 4$. for uncoded single-antenna and Alamouti coded DF relay networks, the case of $m = 2$ has the best error correction performance. That is, the required power for the case of $m = 2$ is about 0.3dB and 0.6dB less than that for the case of $m = 1$ at BEP= 10^{-5} for uncoded single-antenna and Alamouti coded DF relay networks, respectively. For MIMO DF relay network, the case of $m = 4$ has the best average BEP and the required power for the case of $m = 4$ is about 1.4dB less than that for the case of $m = 1$ at BEP= 10^{-5}. We also consider different channels $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (1, 1, 10)$ and $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (10, 1, 1)$ in Figs 5 and 6, respectively. In the case of $(1, 1, 10)$, the BEP improves as m increases, i.e., $m = 4$ has the best error performance and $m = 1$ has the worst error performance. As an example, we focus on the dotted line, i.e., for MIMO DF network, the required power for the case of $m = 4$ is about 2dB and 3.6dB less than that for the cases of $m = 2$ and $m = 1$ at BEP= 10^{-5}, respectively. In the case of $(10, 1, 1)$, the above three DF relay networks have different tendencies from one another. For the uncoded single-antenna and Alamouti coded DF relay networks, the BEP degrades as m increases but for the MIMO DF relay network, the BEP has no clear tendency. These results can be explained by considering the extreme cases for the channel states. In the case of $\sigma_{SR}^2 << \sigma_{RD}^2$, the error performance is determined by the SR channels and since $P_S = 1$, the BEP clearly improves as the number of the selected relays m increases. In the case of $\sigma_{SR}^2 >> \sigma_{RD}^2$, the error performance is determined by the RD channels and since $P_R = 1/m$, the equivalent average total power for the system is the same regardless of the value of m. Thus, the error performance depends on both channels and codewords but it is difficult to determine which one has the best error performance through analysis.

VI. Conclusion

In this paper, we proposed a new relay selection scheme for arbitrary m $(1 \leq m \leq N)$ based on both CSI and transmission scheme by deriving the upper bound on the PEP of the near-ML decoder for the DF relay network with multiple antennas. We also proved that the proposed relay selection scheme which selects m relays from N relays can achieve full diversity $M_S M_D + N M_R \min[M_S, M_D]$ regardless of the value of m in the DF relay network consisting of one source, one destination, and N relays with M_S, M_D, and M_R antennas, respectively. We compared the error performance of the proposed relay selection with that of the MAX-CAP relay selection. The simulation results show that the proposed relay selection has better error performance than the MAX-CAP relay selection for the multiple-antenna cases. We also discussed and simulated the error performance of the proposed relay selection scheme for various m in the uncoded single-antenna, Alamouti coded, and MIMO DF relay networks. As a further work, the study on the relay selection for the non-orthogonal DF relay network with multiple antennas is also very interesting.

Appendix A
Proof of Theorem 2

Let $s=2\sqrt{P_S}\mathrm{Re}\{\mathrm{tr}(G(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}}))N_{SD}^{\dagger})\}$, $t_i=2\sqrt{P_R}\,\mathrm{Re}\{\mathrm{tr}(F_i(X_{R_i}(\mathrm{x})-X_{R_i}(\mathrm{x}_{F_i}^{\min}))N_{R_iD}^{\dagger})\}$, $q_i=P_S\|K_i(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}}))\|^2$, $q_i'=P_S\|K_i(X_S(\mathrm{x})-X_S(\mathrm{x}_{R_i}))\|^2$, $w=P_S\|G(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}}))\|^2$, and $h_i=P_R\left\|F_i(X_{R_i}(\mathrm{x})-X_{R_i}(\mathrm{x}_{F_i}^{\min}))\right\|^2$. Then, $s\sim\mathcal{N}(0,2w\sigma^2)$ and $t_i\sim\mathcal{N}(0,2h_i\sigma^2)$. The RHS of (19) can be rewritten as

$$P\Big(\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S}\min\big[h_i+t_i,\frac{q_i}{4}\big]\Big)\exp\Big(-\sum_{i\in S_N\setminus S}\frac{q_i'}{4\sigma^2}\Big). \tag{28}$$

Let $S'(p)=\{S'|S'\subseteq S \text{ with } |S'|=p\}$, where $0\le p\le n_R$. We assume that $t_i<\frac{q_i}{4}-h_i$ for p relays in $S'\in S'(p)$ and $t_i>\frac{q_i}{4}-h_i$ for n_R-p relays in $S\setminus S'$. Then the event of $\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S}\min\big[h_i+t_i,\frac{q_i}{4}\big]$ can be written as

$$\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S\setminus S'}\frac{q_i}{4}+\sum_{i\in S'}(h_i+t_i).$$

Since t_i's and s are independent, the probability in (28) can be rewritten as

$$\begin{aligned}&P\Big(\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S}\min\big[h_i+t_i,\frac{q_i}{4}\big]\Big)\\&=\sum_{p=0}^{n_R}\sum_{S'\in S'(p)}P\Big(t_i>\frac{q_i}{4}-h_i,i\in S\setminus S'\Big)\\&\qquad P\Big(t_i<\frac{q_i}{4}-h_i,i\in S',\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S\setminus S'}\frac{q_i}{4}+\sum_{i\in S'}h_i+\sum_{i\in S'}t_i\Big)\\&\le\sum_{p=0}^{n_R}\sum_{S'\in S'(p)}P\Big(t_i<\frac{q_i}{4}-h_i,i\in S',\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S\setminus S'}\frac{q_i}{4}+\sum_{i\in S'}h_i+\sum_{i\in S'}t_i\Big).\end{aligned} \tag{29}$$

In addition, when $t_i<\frac{q_i}{4}-h_i$ for $i\in S'$ is true, $\sum_{i\in S'}t_i<\sum_{i\in S'}\frac{q_i}{4}-\sum_{i\in S'}h_i$ must be true. Therefore, (28) can be upper bounded as

$$\begin{aligned}&P\Big(\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S}\min\big[h_i+t_i,\frac{q_i}{4}\big]\Big)\exp\Big(-\sum_{i\in S_N\setminus S}\frac{q_i'}{4\sigma^2}\Big)\\&\le\sum_{p=0}^{n_R}\sum_{S'\in S'(p)}P\Big(\sum_{i\in S'}t_i<\sum_{i\in S'}\frac{q_i}{4}-\sum_{i\in S'}h_i,\\&\qquad\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S\setminus S'}\frac{q_i}{4}+\sum_{i\in S'}h_i+\sum_{i\in S'}t_i\Big)\exp\Big(-\sum_{i\in S_N\setminus S}\frac{q_i'}{4\sigma^2}\Big).\end{aligned} \tag{30}$$

Then, for $p=0$, i.e., S' is an empty set, the summand in (30) can be upper bounded as

$$\begin{aligned}&P\Big(\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S}\frac{q_i}{4}\Big)\exp\Big(-\sum_{i\in S_N\setminus S}\frac{q_i'}{4\sigma^2}\Big)\\&\le\begin{cases}\exp\Big(-\frac{\big(w+\sum_{i\in S}\frac{q_i}{4}-\sum_{i\in S_N\setminus S}\frac{q_i'}{4}\big)^2}{4w\sigma^2}-\frac{\sum_{i\in S_N\setminus S}q_i'}{4\sigma^2}\Big) & \text{if } w+\sum_{i\in S}\frac{q_i}{4}>\sum_{i\in S_N\setminus S}\frac{q_i'}{4}\\ \exp\Big(-\frac{\sum_{i\in S_N\setminus S}q_i'}{4\sigma^2}\Big) & \text{if } w+\sum_{i\in S}\frac{q_i}{4}\le\sum_{i\in S_N\setminus S}\frac{q_i'}{4}\end{cases}\\&\le\exp\Big(-\frac{w+\sum_{i\in S}\frac{q_i}{2}+\sum_{i\in S_N\setminus S}\frac{q_i'}{2}}{4\sigma^2}\Big).\end{aligned} \tag{31}$$

Let $q_x=\sum_{i\in S'}\frac{q_i}{4}$, $q_y=\sum_{i\in S\setminus S'}\frac{q_i}{4}$, $q_z=\sum_{i\in S_N\setminus S}\frac{q_i'}{4}$, $t=\sum_{i\in S'}t_i$, and $h=\sum_{i\in S'}h_i$. Then, for $p=1,\ldots,n_R$, the summand in (30) can be rewritten as

$$P\Big(t<q_x-h,s<-w-h-t-q_y+q_z\Big)\exp\Big(-\frac{q_z}{\sigma^2}\Big). \tag{32}$$

Since $s\sim\mathcal{N}(0,2w\sigma^2)$ and $t\sim\mathcal{N}(0,2h\sigma^2)$, the probability in (32) can be derived as

$$\begin{aligned}&P\Big(t<q_x-h,s<-w-h-t-q_y+q_z\Big)\\&=\int_{-\infty}^{q_x-h}Q\Big(\frac{w+h+t+q_y-q_z}{\sqrt{2w\sigma^2}}\Big)\frac{\exp(-\frac{t^2}{4\sigma^2h})}{\sqrt{4\pi\sigma^2h}}dt\\&\le\begin{cases}\int_{-\infty}^{-w-h-q_y+q_z}\frac{\exp(-\frac{t^2}{4\sigma^2h})}{\sqrt{4\pi\sigma^2h}}dt\\ \quad+\int_{-w-h-q_y+q_z}^{q_x-h}\exp\big(-\frac{(w+h+t+q_y-q_z)^2}{4w\sigma^2}\big)\frac{\exp(-\frac{t^2}{4\sigma^2h})}{\sqrt{4\pi\sigma^2h}}dt & \text{if } w+q_x+q_y>q_z\\ \int_{-\infty}^{q_x-h}\frac{\exp(-\frac{t^2}{4\sigma^2h})}{\sqrt{4\pi\sigma^2h}}dt & \text{if } w+q_x+q_y\le q_z\end{cases}\\&\le\begin{cases}1+\exp\big(-\frac{(w+h+q_y-q_z)^2}{4(w+h)\sigma^2}\big) & \text{if } w+q_x+q_y>q_z>w+h+q_y\\ \exp\big(-\frac{(w+h+q_y-q_z)^2}{4h\sigma^2}\big)\\ \quad+\exp\big(-\frac{(w+h+q_y-q_z)^2}{4(w+h)\sigma^2}\big) & \text{if } w+q_x+q_y>q_z \text{ and } w+h+q_y>q_z\\ 1 & \text{if } w+q_x+q_y\le q_z \text{ and } h\le q_x\\ \exp\big(-\frac{(h-q_x)^2}{4\sigma^2h}\big) & \text{if } w+q_x+q_y\le q_z \text{ and } h>q_x.\end{cases}\end{aligned} \tag{33}$$

Then (32) can be upper bounded as

$$\begin{aligned}&P\Big(t<q_x-h,s<-w-h-t-q_y+q_z\Big)\exp\Big(-\frac{q_z}{\sigma^2}\Big)\\&\le 2\exp\Big(-\frac{w+h+2q_y+2q_z}{4\sigma^2}\Big).\end{aligned} \tag{34}$$

Plugging (31) and (34) into (30), the upper bound on (28) can be derived as (35) at the top of the next page.

Since $h_i=P_R\left\|F_i(X_{R_i}(\mathrm{x})-X_{R_i}(\mathrm{x}_{F_i}^{\min}))\right\|^2$, $q_i=P_S\|K_i(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}}))\|^2\ge P_S\left\|K_i(X_S(\mathrm{x})-X_S(\mathrm{x}_{K_i}^{\min}))\right\|^2$, and $q_i'=P_S\|K_i(X_S(\mathrm{x})-X_S(\mathrm{x}_{R_i}))\|^2\ge P_S\left\|K_i(X_S(\mathrm{x})-X_S(\mathrm{x}_{K_i}^{\min}))\right\|^2$, we have

$$\begin{aligned}&\sum_{i\in S'}h_i+\sum_{i\in S\setminus S'}\frac{q_i}{2}+\sum_{i\in S_N\setminus S}\frac{q_i'}{2}\\&\ge\sum_{i\in S_N}\min\Big[\frac{P_S}{2}\left\|K_i(X_S(\mathrm{x})-X_S(\mathrm{x}_{K_i}^{\min}))\right\|^2,P_R\left\|F_i(X_{R_i}(\mathrm{x})-X_{R_i}(\mathrm{x}_{F_i}^{\min}))\right\|^2\Big].\end{aligned} \tag{36}$$

Plugging (35) and (36) into (19), the summand in (13) can be upper bounded as (37). Then, the PEP in (13) can be derived as (20).

$$P\Big(\sum_{i\in S_N\setminus S}\frac{q_i'}{4}>w+s+\sum_{i\in S}\min\left[h_i+t_i,\frac{q_i}{4}\right]\Big)\exp\left(-\sum_{i\in S_N\setminus S}\frac{q_i'}{4\sigma^2}\right)$$
$$\le\exp\left(-\frac{w+\sum_{i\in S}\frac{q_i}{2}+\sum_{i\in S_N\setminus S}\frac{q_i'}{2}}{4\sigma^2}\right)+\sum_{p=1}^{n_R}\sum_{S'\in S'(p)}2\exp\Big(-\frac{w+\sum_{i\in S'}h_i+\sum_{i\in S\setminus S'}\frac{q_i}{2}+\sum_{i\in S_N\setminus S}\frac{q_i'}{2}}{4\sigma^2}\Big)$$
$$\le 2\exp\Big(-\frac{w}{4\sigma^2}\Big)\sum_{p=0}^{n_R}\sum_{S'\in S'(p)}\exp\left(-\frac{\sum_{i\in S'}h_i+\sum_{i\in S\setminus S'}\frac{q_i}{2}+\sum_{i\in S_N\setminus S}\frac{q_i'}{2}}{4\sigma^2}\right) \tag{35}$$

$$P\Big(\mathrm{x}\to\tilde{\mathrm{x}}\big|\mathrm{x}_{R_i}=\mathrm{x},i\in S\text{ and }\mathrm{x}_{R_i}\ne\mathrm{x},i\in S_N\setminus S\Big)\prod_{i\in S}P_{SR_i}(\mathrm{x}|\mathrm{x})\prod_{i\in S_N\setminus S}P_{SR_i}(\mathrm{x}_{R_i}|\mathrm{x})$$
$$\overset{P}{\lessapprox}2\exp\Big(-\frac{P_S\|G(X_S(\mathrm{x})-X_S(\tilde{\mathrm{x}}))\|^2}{4\sigma^2}\Big)\sum_{p=0}^{n_R}\sum_{S'\in S'(p)}\exp\Big(-\frac{\sum_{i\in S_N}\min\Big[\frac{P_S}{2}\big\|K_i\big(X_S(\mathrm{x})-X_S(\mathrm{x}_{K_i}^{\min})\big)\big\|^2,P_R\big\|F_i\big(X_{R_i}(\mathrm{x})-X_{R_i}(\mathrm{x}_{F_i}^{\min})\big)\big\|^2\Big]}{4\sigma^2}\Big) \tag{37}$$

Appendix B
Fact 1

Fact 1: [5] For an $n\times m$ matrix A, there exist a unitary matrix U and a real diagonal matrix Λ such that $AA^\dagger=U\Lambda U^\dagger$, where $\Lambda=\mathrm{diag}(\lambda_1,\ldots,\lambda_n)$, $\lambda_1,\ldots,\lambda_n$ are the eigenvalues of $AA^\dagger$, and the columns of U are the corresponding eigenvectors. Suppose that $\lambda_1,\ldots,\lambda_r\ne 0$, $\lambda_{r+1}=\cdots=\lambda_n=0$, and $\lambda_{\min}$ is the minimum nonzero eigenvalue. Then, the following inequality holds for any $l\times n$ matrix B as

$$\|BA\|^2\ge\lambda_{\min}\|B'\|^2$$

where B' is an $l\times r$ matrix constructed by using the ith column of BU as its ith column, $i=1,\ldots,r$. □

Appendix C
Proof of Theorem 3

The CDF of $y_{\max}$ can be written as

$$P_{y_{\max}}(y) = P\Big(\sum_{i\in S_1}\gamma_i'\le y,\ldots,\sum_{i\in S_{\binom{N}{m}}}\gamma_i'\le y\Big). \tag{38}$$

Since the random variables $\sum_{i\in S_j}\gamma_i', j=1,\ldots,\binom{N}{m}$ are not statistically independent, the CDF of $y_{\max}$ is very difficult to be derived. However, we can find that $\max_{i\in S_j}\gamma_i'\le\sum_{i\in S_j}\gamma_i'$ and $\gamma_i', i=1,\ldots,N$ are i.i.d., and thus, the CDF of $y_{\max}$ is upper bounded as

$$P\Big(\sum_{i\in S_1}\gamma_i'\le y,\ldots,\sum_{i\in S_{\binom{N}{m}}}\gamma_i'\le y\Big)$$
$$\le P\Big(\max_{i\in S_1}\gamma_i'\le y,\ldots,\max_{i\in S_{\binom{N}{m}}}\gamma_i'\le y\Big)=\Big[P(\gamma_1'\le y)\Big]^N. \tag{39}$$

Plugging (39) into (24), the upper bound on the RHS in (24) can be derived. Since $\|K_1'\|^2$ and $\|F_1'\|^2$ are r_SM_R-Erlang and r_RM_D-Erlang random variables with rate parameters σ_{SR}^2 and σ_{RD}^2, the expression of $P(\gamma_1'\le y)$ is very complicated and thus, the expectation in (24) cannot be upper bounded by a closed-form expression. Let $n_u=r_SM_R$ and $n_v=r_RM_D$. Using the fact that the summation of n i.i.d. exponential random variables is an n-Erlang random variable, $\frac{1}{2}P_S\omega_{\min}\|K_1'\|^2$ and $P_R\mu_{\min}\|F_1'\|^2$ can be rewritten as

$$\frac{1}{2}P_S\omega_{\min}\|K_1'\|^2=\sum_{j=1}^{n_u}u_j\quad\text{and}\quad P_R\mu_{\min}\|F_1'\|^2=\sum_{j=1}^{n_v}v_j,$$

respectively, where $u_j, j=1,\ldots,n_u$, are i.i.d. exponential random variables with parameter $\lambda_u=2/P_S\omega_{\min}\sigma_{SR}^2$, and $v_j, j=1,\ldots,n_v$, are i.i.d exponential random variables with parameter $\lambda_v=1/P_R\mu_{\min}\sigma_{RD}^2$. In addition, we have

$$\min\Big[\sum_{j=1}^{n_u}u_j,\sum_{j=1}^{n_v}v_j\Big]\ge\sum_{j=1}^{\min[n_u,n_v]}[u_j,v_j]$$
$$\ge\max_{j\in\{1,\ldots,\min[n_u,n_v]\}}\min[u_j,v_j]$$

from the nonnegativity of u_j and v_j. Therefore, the upper bound on the CDF of $y_{\max}$ can be rewritten as

$$P_{y_{\max}}(y)$$
$$\le\Big[P\Big(\max_{j\in\{1,\ldots,\min[n_u,n_v]\}}\min[u_j,v_j]<y\Big)\Big]^N$$
$$=\big[P\big(\min[u_1,v_1]<y\big)\big]^{N\min[n_u,n_v]}$$
$$=\big[1-P\big(\min[u_1,v_1]\ge y\big)\big]^{N\min[n_u,n_v]}$$
$$=\Big[1-\exp\Big(-\Big(\frac{2}{P_S\omega_{\min}\sigma_{SR}^2}+\frac{1}{P_R\mu_{\min}\sigma_{RD}^2}\Big)y\Big)\Big]^{N\min[r_SM_R,r_RM_D]}.$$

Appendix D
Proof of Theorem 4

Using the upper bound on the CDF of $y_{\max}$ in (25), the upper bound on the second expectation in the RHS of (23) is derived. Let $a=1/4\sigma^2$, $b=2/P_S\omega_{\min}\sigma_{SR}^2+1/P_R\mu_{\min}\sigma_{RD}^2$, and $N_p=N\min[r_SM_R,r_RM_D]$. From (24), the upper bound on the second expectation in the RHS of (23) can be rewritten

$$E\Big[\exp\big(-\frac{y_{\max}}{4\sigma^2}\big)\Big] \le \frac{(N\min[r_SM_R, r_RM_D])!\big(\frac{2}{P_S\omega_{\min}\sigma_{SR}^2}+\frac{1}{P_R\mu_{\min}\sigma_{RD}^2}\big)^{N\min[r_SM_R,r_RM_D]}}{\prod_{i=1}^{N\min[r_SM_R,r_RM_D]}\big(\frac{1}{4\sigma^2}+(\frac{2}{P_S\omega_{\min}\sigma_{SR}^2}+\frac{1}{P_R\mu_{\min}\sigma_{RD}^2})i\big)}$$
$$\le (N\min[r_SM_R, r_RM_D])!\Big(\frac{8\sigma^2}{P_S\omega_{\min}\sigma_{SR}^2}+\frac{4\sigma^2}{P_R\mu_{\min}\sigma_{RD}^2}\Big)^{N\min[r_SM_R,r_RM_D]} \quad (43)$$

as

$$E\Big[\exp\big(-\frac{y_{\max}}{4\sigma^2}\big)\Big]$$
$$\le a\int_0^\infty \exp(-ay)[1-\exp(-by)]^{N_p}dy$$
$$= a\int_0^\infty \exp(-ay)\sum_{k=0}^{N_p}\binom{N_p}{k}(-1)^k\exp(-kby)dy$$
$$= a\sum_{k=0}^{N_p}\binom{N_p}{k}(-1)^k\frac{1}{a+kb}$$
$$= \frac{\sum_{k=0}^{N_p}\binom{N_p}{k}(-1)^k\Big[\sum_{n=0}^{N_p}a^{N_p-n}b^n\sum_{\substack{0\le l_1<\cdots<l_n\le N_p\\ l_1,\ldots,l_n\neq k}}l_1\cdots l_n\Big]}{\prod_{i=1}^{N_p}(a+bi)}.$$

Let $P_n = \sum_{l_1<\cdots<l_n} l_1\cdots l_n$ for $n\ge 1$ and $P_0 = 1$. Then we have

$$\sum_{\substack{l_1<\cdots<l_n\\ l_1,\ldots,l_n\neq k}} l_1\cdots l_n = P_n - k\sum_{\substack{l_1<\cdots<l_{n-1}\\ l_1,\ldots,l_{n-1}\neq k}} l_1\ldots l_{n-1}$$
$$= \sum_{s=0}^{n}(-1)^s k^s P_{n-s}\ . \quad (40)$$

Using the result in (40), we have

$$E\Big[\exp\big(-\frac{y_{\max}}{4\sigma^2}\big)\Big]$$
$$\le \frac{\sum_{k=0}^{N_p}\binom{N_p}{k}(-1)^k\sum_{n=0}^{N_p}a^{N_p-n}b^n\sum_{s=0}^{n}(-1)^sk^sP_{n-s}}{\prod_{i=1}^{N_p}(a+bi)}$$
$$\overset{(b)}{=} \frac{(-1)^{N_p}N_p!\,b^{N_p}(-1)^{N_p}P_0}{\prod_{i=1}^{N_p}(a+bi)}$$
$$= \frac{N_p!b^{N_p}}{\prod_{i=1}^{N_p}(a+bi)} \quad (41)$$

where (b) is established from the following equality in [22]

$$\sum_{k=0}^{N}(-1)^k\binom{N}{k}k^n = \begin{cases}0 & \text{if } 0\le n<N\\ (-1)^N N! & \text{if } n=N.\end{cases}$$

Therefore, the second expectation in the RHS of (23) can be upper bounded as (43). Since $y_G = P_S\omega_{\min}\|G'\|^2$ is an r_SM_D-Erlang random variable with rate parameter $P_S\omega_{\min}\sigma_{SD}^2$, the first expectation in the RHS of (23) can be rewritten as

$$E\Big[\exp\Big(-\frac{P_S\omega_{\min}\|G'\|^2}{4\sigma^2}\Big)\Big] = \left(\frac{\frac{4\sigma^2}{P_S\omega_{\min}\sigma_{SD}^2}}{\frac{4\sigma^2}{P_S\omega_{\min}\sigma_{SD}^2}+1}\right)^{r_SM_D}$$
$$\le \Big(\frac{4\sigma^2}{P_S\omega_{\min}\sigma_{SD}^2}\Big)^{r_SM_D}. \quad (44)$$

Plugging (43) and (44) into (23), the average PEP of the new relay selection scheme can be upper bounded as

$$E[P_{RSelect}(\mathrm{x}\to\tilde{\mathrm{x}})]$$
$$\overset{P}{\lessapprox} 2\big(M^L+1\big)^m\big(N\min[r_SM_R, r_RM_D]\big)!$$
$$\Big(\frac{4\sigma^2}{P_S\omega_{\min}\sigma_{SD}^2}\Big)^{r_SM_D}\Big(\frac{8\sigma^2}{P_S\omega_{\min}\sigma_{SR}^2}+\frac{4\sigma^2}{P_R\mu_{\min}\sigma_{RD}^2}\Big)^{N\min[r_SM_R,r_RM_D]}. \quad (45)$$

From the upper bound on the average PEP, the theorem is proved.

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