PAPER

Analysis on Soft-Decision-and-Forward Cooperative Networks with Multiple Relays

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SUMMARY In this paper, we analyze the best relay selection scheme for the soft-decision-and-forward (SDF) cooperative networks with multiple relays. The term 'best relay selection' implies that the relay having the largest end-to-end signal-to-noise ratio is selected to transmit in the second phase transmission. The approximate performances in terms of pairwise error probability (PEP) and bit error rate (BER) are analyzed and compared with the conventional multiple-relay transmission scheme where all the relays participate in the second phase transmission. Using the asymptotics of the Fox's H-function, the diversity orders of the best relay selection and conventional relay scheme for the SDF cooperative networks are derived. It is shown that both have the same full diversity order. The numerical results show that the best relay selection scheme outperforms the conventional one in terms of bit error rate.

key words: *cooperative diversity, Fox's H-function, moment generating function (MGF), relay selection, soft-decision-and-forward (SDF)*

1. Introduction

Next-generation wireless communication systems such as IMT-Advanced [1] require higher spectral efficiency and data rate, that is, 100 Mbps for high-speed mobility and 1 Gbps for low-speed mobility. These demands might be achieved by using multiple-antenna technique [2] which increases the channel capacity. However, due to the limitation on implementation, standardizations such as IEEE 802.16j [3] and Long-Term Evolution (LTE)-Advanced [4] for IMT-Advanced recommend the virtual multiple-antenna technique in a distributed sense.

Sendonaris, Erkip, and Aazhang [5], [6] proposed the cooperative diversity using the cooperation between source and relay. In [7], Zhao, Adve, and Lim analyzed outage behaviors such as outage capacity and outage probability of two different amplify-and-forward (AF) relay schemes with single antenna, namely, all-participated relay transmission and relay selection. They also proposed the power allocation strategies for those two schemes. Similarly, Ikki, and Ahmed proposed and analyzed the best relay selection scheme based on AF protocol with single antenna at each node in [8]. Bletsas et al. [9] described the forward channel estimation for the opportunistic relay selection in case of multiple relays. Yang, Song, No, and Shin [10] proposed the maximum-likelihood (ML) decoding for AF and soft-decision-and-forward (SDF) protocols with multiple antennas using Alamouti codes [11]. Since the relay nodes can separate the signals in SDF protocol while they simply amplify the received signals according to the power constraint in AF protocol, SDF becomes substantially different from AF in the case of multiple antennas. In [12], performance on SDF with single relay where each node is equipped with two antennas was analyzed in terms of bit error rate (BER). Furthermore, power allocation scheme between source and relay nodes was proposed so as to maximize the instantaneous end-to-end signal-to-noise ratio (SNR).

In this paper, the best relay selection scheme using Alamouti code for the SDF cooperative networks with multiple relays is analyzed. The indications of the performance such as the pairwise-error probability (PEP) and the diversity order of the best relay selection scheme are compared with those of the conventional multiple-relay transmission scheme where all the relays participate in the second phase transmission. For these two schemes, we express the end-to-end SNRs and derive the PEPs under the ML decoding proposed in [10]. From the derived PEPs for the multiple-relay cooperative networks with SDF protocol, the diversity orders and the approximate BERs are obtained. The best relay selection scheme has an advantage over the conventional one in terms of BER and throughput.

This paper is organized as follows. Section 2 describes the SDF protocol and reviews the results in [12]. Sections 3 and 4 address the conventional multiple-relay transmission and the 'best relay selection' schemes, respectively. The analytical and simulation results are shown in Sect. 5. Finally, the concluding remarks are given in Sect. 6.

Throughout this paper, the following notations are used. $\mathbb{E}[\cdot]$ denotes the expectation of a random variable. $X \sim \mathcal{CN}(0,\sigma^2)$ means that X is a complex normal random variable with zero mean and variance $\sigma^2/2$ in both real and imaginary parts, respectively. $\Re\{\cdot\}$ denotes the real part of a complex number. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\dagger}$, and $\|\cdot\|$ denote the transpose of a matrix, the conjugate transpose of a matrix, and the Frobenius norm of a matrix or a vector, respectively. Bold-face uppercase and lowercase letters denote matrices and vectors, respectively. If X is a sum of K

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independent exponential random variables, each of which has the same mean Ω, then, X is called a gamma random variable whose probability density function (PDF) is expressed in terms of a shape parameter K and a scale parameter Ω. We denote it by $X \sim \mathcal{G}(K, \Omega)$. The Alamouti code $\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$ is denoted by $\mathbf{A}(a, b)$. Also, for any 2×2 matrix $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, the 4×2 matrix $\mathbf{B}'$ and the vector $cv(\mathbf{B})$ are defined as $\mathbf{B}' = \begin{bmatrix} b_{11} & b_{21}^* & b_{12} & b_{22}^* \\ b_{21} & -b_{11}^* & b_{22} & -b_{12}^* \end{bmatrix}^{\mathrm{T}}$ and $cv(\mathbf{B}) = \begin{bmatrix} b_{11} & b_{21}^* & b_{12} & b_{22}^* \end{bmatrix}^{\mathrm{T}}$.

2. Soft-Decision-and-Forward Protocol

The system model of SDF protocol [10] with multiple relays where each node is equipped with two antennas is depicted in Fig. 1. This cooperative communication system is composed of one source (S), one destination (D), and M relays (R_m, $m = 1, \cdots, M$). In the second phase transmission, the following two transmission methods for the multiple relays are considered: conventional multiple-relay transmission and the best relay selection.

The total transmit power P in the network is defined as the sum of the source power P_1 at S and the total relay power P_2 at R_m's. The channel gains of each link $\mathsf{S} \to \mathsf{D}$, $\mathsf{S} \to \mathsf{R}_m$, and $\mathsf{R}_m \to \mathsf{D}$ are assumed to be Rayleigh-faded, i.e., $f_0^{ij} \sim \mathcal{CN}(0, \sigma^2_{\mathsf{SD}})$, $f_m^{ij} \sim \mathcal{CN}(0, \sigma^2_{\mathsf{SR}_m})$, and $g_m^{ij} \sim \mathcal{CN}(0, \sigma^2_{\mathsf{R}_m\mathsf{D}})$, where f_0^{ij}, f_m^{ij}, and g_m^{ij}, $i, j = 1, 2$, $m = 1, \cdots, M$, denote the path gain from the ith transmit antenna at S to the jth receive antenna at D, from the ith transmit antenna at S to the jth receive antenna at R_m, and from the ith transmit antenna at R_m to the jth receive antenna at D, respectively. These path gains are represented as the channel matrices $\mathbf{F}_0 = [f_0^{ij}]$, $\mathbf{F}_m = [f_m^{ij}]$, and $\mathbf{G}_m = [g_m^{ij}]$.

To help understand the SDF protocol, we briefly review the SDF protocol with single relay, where each node has two antennas. The transmission is composed of two phases. In the first phase, S transmits the signal using Alamouti code to R and D. Thus, the received signals at R and D are represented, respectively, as

$$\mathbf{Y}_{\mathsf{R}} = \sqrt{\frac{P_1}{2}} \mathbf{X} \mathbf{F}_1 + \mathbf{N}_{\mathsf{R}}$$

$$\mathbf{Y}_{\mathsf{D1}} = \sqrt{\frac{P_1}{2}} \mathbf{X} \mathbf{F}_0 + \mathbf{N}_{\mathsf{D1}}$$

where $\mathbf{X} = \mathbf{A}(x_1, x_2)$ is the transmit codeword at S in the first phase, $\mathbf{F}_0$ and $\mathbf{F}_1$ denote the channel matrices of $\mathsf{S} \to \mathsf{D}$ and $\mathsf{S} \to \mathsf{R}$, respectively, and $\mathbf{N}_{\mathsf{R}}$ and $\mathbf{N}_{\mathsf{D1}}$ are the 2×2 additive white Gaussian noise (AWGN) matrices with zero-mean and unit-variance entries. During the intermediate decoding at R, R obtains the soft-decision values from the received signals using maximal ratio combining as

$$\tilde{\mathbf{x}} \triangleq \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 \end{bmatrix}^{\mathrm{T}} = \lambda \mathbf{F}_1'^{\dagger} cv(\mathbf{Y}_{\mathsf{R}})$$

where

$$cv(\mathbf{Y}_{\mathsf{R}}) = \begin{bmatrix} y_{11}^{(\mathsf{R})} & y_{21}^{(\mathsf{R})*} & y_{12}^{(\mathsf{R})} & y_{22}^{(\mathsf{R})*} \end{bmatrix}^{\mathrm{T}} = \sqrt{\frac{P_1}{2}} \mathbf{F}_1' \mathbf{x} + cv(\mathbf{N}_{\mathsf{R}})$$

$$\lambda = \sqrt{\frac{2}{\|\mathbf{F}_1\|^2 (P_1 \|\mathbf{F}_1\|^2 + 2)}}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}}$ is the transmitted signal vector at S. And then, R transmits the following codeword into D

$$\mathbf{X}_{\mathsf{R}} = \mathbf{A}(\tilde{x}_1, \tilde{x}_2) = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 \\ -\tilde{x}_2^* & \tilde{x}_1^* \end{bmatrix}.$$

In the second phase, the received signal at D is expressed as

$$\mathbf{Y}_{\mathsf{D2}} = \sqrt{\frac{P_2}{2}} \mathbf{X}_{\mathsf{R}} \mathbf{G}_1 + \mathbf{N}_{\mathsf{D2}}$$

where $\mathbf{G}_1$ is the channel matrix of $\mathsf{R} \to \mathsf{D}$ and $\mathbf{N}_{\mathsf{D2}}$ denotes the 2×2 AWGN matrix with zero-mean and unit-variance entries. Converting the matrix form into the vector form gives the following alternative expression

$$cv(\mathbf{Y}_{\mathsf{D2}}) = \frac{\sqrt{P_1 P_2}}{2} \lambda \|\mathbf{F}_1\|^2 \mathbf{G}_1' \mathbf{x} + \sqrt{\frac{P_2}{2}} \lambda \mathbf{G}_1' \mathbf{F}_1'^{\dagger} cv(\mathbf{N}_{\mathsf{R}}) + cv(\mathbf{N}_{\mathsf{D2}}).$$

The received signal at D during two phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D1}}) \\ cv(\mathbf{Y}_{\mathsf{D2}}) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}} \begin{bmatrix} \mathbf{F}_0' \\ \sqrt{\frac{P_2}{2}} \lambda \|\mathbf{F}_1\|^2 \mathbf{G}_1' \end{bmatrix}}_{\mathbf{H}} \mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}_{\mathsf{D1}}) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix}}_{\mathbf{n}} \quad (1)$$

where $cv(\mathbf{N}_{\mathsf{D}})$ means the equivalent noise at D in the vector form, given by

$$cv(\mathbf{N}_{\mathsf{D}}) = \sqrt{\frac{P_2}{2}} \lambda \mathbf{G}_1' \mathbf{F}_1'^{\dagger} cv(\mathbf{N}_{\mathsf{R}}) + cv(\mathbf{N}_{\mathsf{D2}}).$$

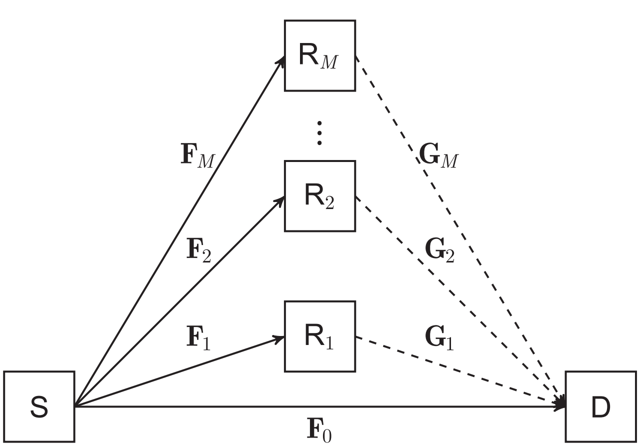

Fig. 1 Cooperative communication network composed of one source (S), M relays (R_m), and one destination (D) with two antennas in each node.

$$\hat{x}_i = \arg\min_{x_i} \left[\left(\frac{P_1}{2} \|\mathbf{F}_0\|^2 + \frac{P_1 P_2 \|\mathbf{F}_1\|^2 \|\mathbf{G}_1\|^2}{2(P_1\|\mathbf{F}_1\|^2 + P_2\|\mathbf{G}_1\|^2 + 2)} \right) |x_i|^2 - 2\Re\{\eta_i x_i\} \right] \tag{2}$$

The ML decoding rule for the SDF protocol can be written as

$$\begin{aligned} \hat{\mathbf{x}} &= \arg\min_{\mathbf{x}} [(\mathbf{y} - \mathbf{H}\mathbf{x})^\dagger \mathcal{K}_\mathbf{n}^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x})] \\ &= \arg\min_{\mathbf{x}} \left[\mathbf{x}^\dagger \mathbf{H}^\dagger \mathcal{K}_\mathbf{n}^{-1} \mathbf{H}\mathbf{x} - 2\Re\{\mathbf{y}^\dagger \mathcal{K}_\mathbf{n}^{-1} \mathbf{H}\mathbf{x}\} \right] \end{aligned}$$

where $\mathcal{K}_\mathbf{n} = \mathcal{E}\left[\mathbf{n}\mathbf{n}^\dagger\right] = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \mathcal{K}_{cv(\mathsf{N}_\mathsf{D})} \end{bmatrix}$ with $\mathcal{K}_{cv(\mathsf{N}_\mathsf{D})} = \mathbf{I}_4 + P_2/(P_1\|\mathbf{F}_1\|^2 + 2) \cdot \mathbf{G}_1' \mathbf{G}_1'^\dagger$. The ML decoder for SDF protocol [10] chooses $\hat{x}_i$ such as (2) at the top in this page, where $\begin{bmatrix} \eta_1 & \eta_2 \end{bmatrix} = \mathbf{y}^\dagger \mathcal{K}_\mathbf{n}^{-1} \mathbf{H}$. It is important to estimate $\mathcal{K}_\mathbf{n}$ as exact as possible. In this paper, we assume that the exact estimation on the covariance matrix at D is possible.

Using (1), the conditional PEP can be written as

$$\begin{aligned} \Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) &= Q\left(\sqrt{\frac{1}{2} \left\| \mathcal{K}_\mathbf{n}^{-\frac{1}{2}} \mathbf{H}(\hat{\mathbf{x}} - \mathbf{x}) \right\|^2} \right) \\ &= Q\left(\sqrt{\frac{\delta_\mathbf{x}^2}{2} \left\{ \frac{P_1}{2}\|\mathbf{F}_0\|^2 + \frac{P_1 P_2 \|\mathbf{F}_1\|^2 \|\mathbf{G}_1\|^2}{2(P_1\|\mathbf{F}_1\|^2 + P_2\|\mathbf{G}_1\|^2 + 2)} \right\}} \right) \\ &= Q\left(\sqrt{\frac{\delta_\mathbf{x}^2}{2} \left(\gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1} \right)} \right) \end{aligned} \tag{3}$$

where $Q(x) = \int_x^\infty e^{-u^2/2} / \sqrt{2\pi} du$, $\delta_\mathbf{x} = \|\hat{\mathbf{x}} - \mathbf{x}\|$, and

$$\begin{aligned} \gamma_0 &= \frac{P_1}{2}\|\mathbf{F}_0\|^2 \sim \mathcal{G}\left(4, \sigma_{\mathsf{SD}}^2 P_1/2\right) \\ \gamma_1 &= \frac{P_1}{2}\|\mathbf{F}_1\|^2 \sim \mathcal{G}\left(4, \sigma_{\mathsf{SR}}^2 P_1/2\right) \\ \gamma_2 &= \frac{P_2}{2}\|\mathbf{G}_1\|^2 \sim \mathcal{G}\left(4, \sigma_{\mathsf{RD}}^2 P_2/2\right). \end{aligned}$$

It is shown in [12] that the PEP depends on the instantaneous end-to-end SNR, γ_{eq}, which is the sum of two SNRs, i.e.,

$$\gamma_{\text{eq}} = \gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1}. \tag{4}$$

And also, (4) can be upper and lower bounded as

$$\gamma_0 + \frac{\gamma_1\gamma_2}{c(\gamma_1 + \gamma_2)} \le \gamma_{\text{eq}} \le \gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} \tag{5}$$

where $c > 1 + (\gamma_1 + \gamma_2)^{-1}$. Since $Q(x)$ is monotonically decreasing for $x \ge 0$, substitution of (5) into (3) leads to the following inequalities as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \le Q\left(\sqrt{\frac{1}{2} \left\{ \gamma_0 + \frac{\gamma_1\gamma_2}{c(\gamma_1 + \gamma_2)} \right\} \delta_\mathbf{x}^2} \right)$$

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \ge Q\left(\sqrt{\frac{1}{2} \left\{ \gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} \right\} \delta_\mathbf{x}^2} \right). \tag{6}$$

Furthermore, the Q-function is bounded as

$$\sum_{n=1}^{N} a_n \exp(-b_{n-1}u) \le Q(\sqrt{u}) \le \sum_{n=1}^{N} a_n \exp(-b_n u) \tag{7}$$

for $a_n = (\theta_n - \theta_{n-1})/\pi$ and $b_n = 1/(2\sin^2\theta_n)$ for $n = 1, \cdots, N$ with $\theta_0 = 0$ and $\theta_N = \pi/2$. Then, we can bound the PEP by averaging the conditional PEP in (6) over $\mathbf{H}$ using the Q-function inequality in (7). It gives the upper and lower bounds of the average PEP for the SDF protocol [12] as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \le \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n\delta_\mathbf{x}^2}{2}\right) \mathcal{M}_{H(\gamma_1,\gamma_2)}\left(\frac{b_n\delta_\mathbf{x}^2}{4c}\right) \tag{8}$$

and

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \ge \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_{n-1}\delta_\mathbf{x}^2}{2}\right) \mathcal{M}_{H(\gamma_1,\gamma_2)}\left(\frac{b_{n-1}\delta_\mathbf{x}^2}{4}\right) \tag{9}$$

where $\mathcal{M}_X(\cdot)$ is the moment generating function(MGF) for random variable X and $H(a, b) \triangleq 2ab/(a + b)$.

3. SDF Protocol with the Conventional Multiple-Relay Transmission

In this section, the PEP and diversity order of the 'conventional' SDF cooperative networks with multiple relays are derived, where all relays participate in the second phase transmission.

3.1 System Model

The signal transmission in the cooperative networks is composed of two phases. In the first phase, S transmits the signals using Alamouti code to R_m, $m = 1, \cdots, M$ and D. The received signals at R_m and D are represented, respectively, as

$$\begin{aligned} \mathbf{Y}_{\mathsf{R}_m} &= \sqrt{\frac{P_1}{2}} \mathbf{X}\mathbf{F}_m + \mathbf{N}_{\mathsf{R}_m}, \quad m = 1, \cdots, M \\ \mathbf{Y}_{\mathsf{D}1} &= \sqrt{\frac{P_1}{2}} \mathbf{X}\mathbf{F}_0 + \mathbf{N}_{\mathsf{D}1} \end{aligned} \tag{10}$$

where $\mathbf{X} = \mathbf{A}(x_1, x_2)$ is the transmit codeword for the message vector $\mathbf{x} = [x_1\ x_2]^\mathrm{T}$ at S in the first phase, $\mathbf{F}_0$ and $\mathbf{F}_m$ denote the channel matrices of S → D and S → R_m, respectively, and $\mathbf{N}_{\mathsf{R}_m}$ and $\mathbf{N}_{\mathsf{D}1}$ are the 2×2 AWGN matrices with zero-mean and unit-variance entries. Transforming matrix

$$\hat{x}_i = \arg\min_{x_i}\left[\left(\frac{P_1}{2}\|\mathbf{F}_0\|^2 + \sum_{m=1}^{M}\frac{P_1P_2\|\mathbf{F}_m\|^2\|\mathbf{G}_m\|^2}{2(P_1\|\mathbf{F}_m\|^2 + P_2\|\mathbf{G}_m\|^2 + 2)}\right)|x_i|^2 - 2\Re\{\eta_i x_i\}\right]. \tag{13}$$

form into vector form, (10) can be rewritten as

$$cv(\mathbf{Y}_{\mathsf{R}_m}) = \left[y_{11}^{(\mathsf{R}_m)} \quad y_{21}^{(\mathsf{R}_m)*} \quad y_{12}^{(\mathsf{R}_m)} \quad y_{22}^{(\mathsf{R}_m)*}\right]^{\mathrm{T}} = \sqrt{\frac{P_1}{2}}\mathbf{F}'_m\mathbf{x} + cv(\mathbf{N}_{\mathsf{R}_m})$$

$$cv(\mathbf{Y}_{\mathsf{D1}}) = \left[y_{11}^{(\mathsf{D1})} \quad y_{21}^{(\mathsf{D1})*} \quad y_{12}^{(\mathsf{D1})} \quad y_{22}^{(\mathsf{D1})*}\right]^{\mathrm{T}} = \sqrt{\frac{P_1}{2}}\mathbf{F}'_0\mathbf{x} + cv(\mathbf{N}_{\mathsf{D1}}).$$

During the intermediate decoding at R_m, the soft-decision values are obtained from the received signals using the maximal ratio combining as

$$\tilde{\mathbf{x}}_m \triangleq \left[\tilde{x}_{m,1} \quad \tilde{x}_{m,2}\right]^{\mathrm{T}} = \lambda_m \mathbf{F}'^{\dagger}_m cv(\mathbf{Y}_{\mathsf{R}_m})$$

where

$$\lambda_m = \sqrt{\frac{2}{\|\mathbf{F}_m\|^2(P_1\|\mathbf{F}_m\|^2 + 2)}}.$$

And then, R_m transmits the following codeword to D

$$\mathbf{X}_{\mathsf{R}_m} = \mathbf{A}(\tilde{x}_{m,1}, \tilde{x}_{m,2}) = \begin{bmatrix}\tilde{x}_{m,1} & \tilde{x}_{m,2}\\ -\tilde{x}^*_{m,2} & \tilde{x}^*_{m,1}\end{bmatrix}.$$

In the second phase, the re-encoded codewords of M relays are transmitted during M time slots and the received signal at D in the mth time slot is expressed as

$$\mathbf{Y}^{(m)}_{\mathsf{D2}} = \sqrt{\frac{P_2}{2}}\mathbf{X}_{\mathsf{R}_m}\mathbf{G}_m + \mathbf{N}^{(m)}_{\mathsf{D2}}$$

where $\mathbf{G}_m$ is the channel matrix of $\mathsf{R}_m \to \mathsf{D}$ and $\mathbf{N}^{(m)}_{\mathsf{D2}}$ denotes the 2×2 AWGN matrix with each element having zero-mean and unit-variance. The signals during the M time slots in the second phase are transmitted from M relays in the orthogonal transmission manner. Thus, the received signals at D during the second phase can be written as

$$\mathbf{Y}^{(m)}_{\mathsf{D2}} = \sqrt{\frac{P_2}{2}}\mathbf{X}_{\mathsf{R}_m}\mathbf{G}_m + \mathbf{N}^{(m)}_{\mathsf{D2}}, \quad m = 1,\cdots,M.$$

Converting the matrix form into the vector form gives the following alternative expression

$$cv\left(\mathbf{Y}^{(m)}_{\mathsf{D2}}\right) = \frac{\sqrt{P_1P_2}}{2}\lambda_m\|\mathbf{F}_m\|^2\mathbf{G}'_m\mathbf{x} + \underbrace{\sqrt{\frac{P_2}{2}}\lambda_m\mathbf{G}'_m\mathbf{F}'^{\dagger}_m cv(\mathbf{N}_{\mathsf{R}_m}) + cv\left(\mathbf{N}^{(m)}_{\mathsf{D2}}\right)}_{cv(\mathbf{N}^{(m)}_{\mathsf{D}})}$$

where $cv(\mathbf{N}^{(m)}_{\mathsf{D}})$ means the equivalent noise at D in the second phase transmission.

The received signals at D in the both phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix}cv(\mathbf{Y}_{\mathsf{D1}})\\ cv(\mathbf{Y}^{(1)}_{\mathsf{D2}})\\ \vdots\\ cv(\mathbf{Y}^{(M)}_{\mathsf{D2}})\end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}}\begin{bmatrix}\mathbf{F}'_0\\ \sqrt{\frac{P_2}{2}}\lambda_1\|\mathbf{F}_1\|^2\mathbf{G}'_1\\ \vdots\\ \sqrt{\frac{P_2}{2}}\lambda_M\|\mathbf{F}_M\|^2\mathbf{G}'_M\end{bmatrix}}_{\mathbf{H}}\mathbf{x} + \underbrace{\begin{bmatrix}cv(\mathbf{N}_{\mathsf{D1}})\\ cv(\mathbf{N}^{(1)}_{\mathsf{D}})\\ \vdots\\ cv(\mathbf{N}^{(M)}_{\mathsf{D}})\end{bmatrix}}_{\mathbf{n}}. \tag{11}$$

The ML decoding rule for (11) of the SDF protocol is as follows:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}}[(\mathbf{y}-\mathbf{H}\mathbf{x})^{\dagger}\mathcal{K}^{-1}_{\mathbf{n}}(\mathbf{y}-\mathbf{H}\mathbf{x})] = \arg\min_{\mathbf{x}}\left[\mathbf{x}^{\dagger}\mathbf{H}^{\dagger}\mathcal{K}^{-1}_{\mathbf{n}}\mathbf{H}\mathbf{x} - 2\Re\{\mathbf{y}^{\dagger}\mathcal{K}^{-1}_{\mathbf{n}}\mathbf{H}\mathbf{x}\}\right] \tag{12}$$

where

$$\mathcal{K}_{\mathbf{n}} = \mathcal{E}\left[\mathbf{n}\mathbf{n}^{\dagger}\right] = \begin{bmatrix}\mathbf{I}_4 & \mathbf{0}_4 & \cdots & \mathbf{0}_4\\ \mathbf{0}_4 & \mathcal{K}_{cv\left(\mathbf{N}^{(1)}_{\mathsf{D}}\right)} & \mathbf{0}_4 & \vdots\\ \mathbf{0}_4 & \mathbf{0}_4 & \ddots & \mathbf{0}_4\\ \mathbf{0}_4 & \cdots & \mathbf{0}_4 & \mathcal{K}_{cv\left(\mathbf{N}^{(M)}_{\mathsf{D}}\right)}\end{bmatrix}$$

with

$$\mathcal{K}_{cv(\mathbf{N}_{\mathsf{D}}{}^{(m)})} = \mathbf{I}_4 + \frac{P_2}{P_1\|\mathbf{F}_m\|^2 + 2}\mathbf{G}'_m\mathbf{G}'^{\dagger}_m.$$

The ML decoder in (12) can be restated as (13) at the top in this page, where $\mathbf{y}^{\dagger}\mathcal{K}^{-1}_{\mathbf{n}}\mathbf{H} = \left[\eta_1 \quad \eta_2\right]$.

3.2 PEP and Diversity Order

Let γ_0, $\gamma_{m,1}$, and $\gamma_{m,2}$ be the SNRs of $\mathsf{S} \to \mathsf{D}$, $\mathsf{S} \to \mathsf{R}_m$, and $\mathsf{R}_m \to \mathsf{D}$ links defined by $\gamma_0 = P_1\|\mathbf{F}_0\|^2/2$, $\gamma_{m,1} = P_1\|\mathbf{F}_m\|^2/2$, and $\gamma_{m,2} = P_2\|\mathbf{G}_m\|^2/2$, respectively. Then, the instantaneous end-to-end SNR for the conventional multiple-relay transmission under ML decoder can be written as

$$\gamma_{\mathrm{eq}} = \gamma_0 + \sum_{m=1}^{M}\frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1}+\gamma_{m,2}+1} \tag{14}$$

where $\gamma_0 \sim \mathcal{G}\left(4, \frac{\sigma^2_{\mathsf{SD}}P_1}{2}\right)$, $\gamma_{m,1} \sim \mathcal{G}\left(4, \frac{\sigma^2_{\mathsf{SR}_m}P_1}{2}\right)$, and $\gamma_{m,2} \sim \mathcal{G}\left(4, \frac{\sigma^2_{\mathsf{R}_m\mathsf{D}}P_2}{2}\right)$, respectively.

Applying the similar approach in Sect. 2 to the conventional scheme, the upper and lower bounds on the conditional PEP can be expressed as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \le Q\left(\sqrt{\frac{\delta_{\mathbf{x}}^2}{2}\left\{\gamma_0+\sum_{m=1}^{M}\left(\frac{\gamma_{m,1}\gamma_{m,2}}{c_m(\gamma_{m,1}+\gamma_{m,2})}\right)\right\}}\right)$$
$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \ge Q\left(\sqrt{\frac{\delta_{\mathbf{x}}^2}{2}\left\{\gamma_0+\sum_{m=1}^{M}\left(\frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1}+\gamma_{m,2}}\right)\right\}}\right) \quad (15)$$

where $c_m > 1+(\gamma_{m,1}+\gamma_{m,2})^{-1}$. By averaging the conditional PEP over $\mathbf{H}$, the upper and lower bounds on the PEP for the conventional multiple-relay transmission under ML decoder are derived as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \le \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n}{2}\delta_{\mathbf{x}}^2\right)\prod_{m=1}^{M}\left[\mathcal{M}_{\gamma_m}\left(\frac{b_n}{4c_m}\delta_{\mathbf{x}}^2\right)\right]$$
$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \ge \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n}{2}\delta_{\mathbf{x}}^2\right)\prod_{m=1}^{M}\left[\mathcal{M}_{\gamma_m}\left(\frac{b_n}{4}\delta_{\mathbf{x}}^2\right)\right] \quad (16)$$

where $\gamma_m = H(\gamma_{m,1}, \gamma_{m,2})$.

For the sake of tractability, let us assume that the uniform power allocation is used between the source and relays, i.e., $P_1 = P_2 = P/(M+1)$. Using the results in [12], the MGFs of γ_0 and γ_m are expressed as

$$\mathcal{M}_{\gamma_0}(s) = \left(1+\frac{P}{2(M+1)}s\right)^{-4}$$
$$\mathcal{M}_{\gamma_m}(s) = {}_2F_1\left(4, 8; \frac{9}{2}; -\frac{P}{4(M+1)}s\right). \quad (17)$$

where

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}dt$$

for $\Re\{c\} > \Re\{b\} > 0$ and $|\arg(1-z)| < \pi$. Note that the diversity order for $\mathsf{S} \to \mathsf{R}_m \to \mathsf{D}$ link in (17) is four [12]. From the above results, we conclude that the diversity order of SDF cooperative communication with conventional multiple-relay transmission is $4(M+1)$, which is the maximum achievable since $4(M+1)$ is the total number of distinct paths from S to D.

4. SDF Protocol with the Best Relay Selection

In contrast to the conventional multiple-relay transmission, we consider the case when the only one relay participates in the second phase. For this case, it is important to select the relay so as to improve the system performance such as error rate or capacity. It is clear that from (3), the instantaneous end-to-end SNR is a good criterion for the relay selection. In this section, the PEP and diversity order of the SDF cooperative networks with the best relay selection are derived.

4.1 System Model

The signal transmission of the best relay selection scheme in the first phase is the same as the conventional scheme. In contrast to the conventional multiple-relay transmission, the signal in the second phase of the best relay selection scheme is transmitted from only one relay $\mathsf{R}_{\hat{m}}$ according to the relay selection criterion

$$\hat{m} = \arg\max_{m}\left\{\frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1}+\gamma_{m,2}+1}\right\}. \quad (18)$$

This criterion assumes that information on $\gamma_{m,1}$ for each relay has to be notified to the destination node. Here, information is only the channel norm between S and R_m. Note that this criterion guarantees the maximum channel capacity as well as the minimum PEP.

Then, the selected $\hat{m}$th relay $\mathsf{R}_{\hat{m}}$ transmits the following codeword to D

$$\mathbf{X}_{\mathsf{R}_{\hat{m}}} = \mathbf{A}(\tilde{x}_{\hat{m},1}, \tilde{x}_{\hat{m},2}) = \begin{bmatrix} \tilde{x}_{\hat{m},1} & \tilde{x}_{\hat{m},2} \\ -\tilde{x}_{\hat{m},2}^* & \tilde{x}_{\hat{m},1}^* \end{bmatrix}.$$

In the second phase, the destination D receives the signal from the $\hat{m}$th relay as

$$\mathbf{Y}_{\mathsf{D}2} = \sqrt{\frac{P_2}{2}}\mathbf{X}_{\mathsf{R}_{\hat{m}}}\mathbf{G}_{\hat{m}} + \mathbf{N}_{\mathsf{D}2}$$

where $\mathbf{G}_{\hat{m}}$ is the channel matrix of $\mathsf{R}_{\hat{m}} \to \mathsf{D}$ and $\mathbf{N}_{\mathsf{D}2}$ denotes the 2×2 AWGN matrix with zero-mean and unit-variance entries. Converting the matrix equation into the vector form gives us the following alternative expression

$$cv(\mathbf{Y}_{\mathsf{D}2}) = \frac{\sqrt{P_1P_2}}{2}\lambda_{\hat{m}}\|\mathbf{F}_{\hat{m}}\|^2\mathbf{G}'_{\hat{m}}\mathbf{x} + \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\mathbf{G}'_{\hat{m}}\mathbf{F}'^{\dagger}_{\hat{m}}cv(\mathbf{N}_{\mathsf{R}_{\hat{m}}}) + cv(\mathbf{N}_{\mathsf{D}2}).$$

The received signal at D in both phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}_{\mathsf{D}1}) \\ cv(\mathbf{Y}_{\mathsf{D}2}) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}}\begin{bmatrix} \mathbf{F}'_0 \\ \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\|\mathbf{F}_{\hat{m}}\|^2\mathbf{G}'_{\hat{m}} \end{bmatrix}}_{\mathbf{H}}\mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}_{\mathsf{D}1}) \\ cv(\mathbf{N}_{\mathsf{D}}) \end{bmatrix}}_{\mathbf{n}} \quad (19)$$

where $cv(\mathbf{N}_{\mathsf{D}})$ means the equivalent noise at D in the vector form given by

$$cv(\mathbf{N}_{\mathsf{D}}) = \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\mathbf{G}'_{\hat{m}}\mathbf{F}'^{\dagger}_{\hat{m}}cv(\mathbf{N}_{\mathsf{R}_{\hat{m}}}) + cv(\mathbf{N}_{\mathsf{D}2}).$$

The ML decoder for the best relay selection is the same as the one for a single relay case.

4.2 PEP and Diversity Order

In the best relay selection scheme, the relay $\mathsf{R}_{\hat{m}}$ selected according to the selection criterion in (18) transmits the signals with power P_2 in the second phase. For the easy derivation of PEP, it is assumed that the uniform power allocation is used between S and $\mathsf{R}_{\hat{m}}$, i.e., $P_1 = P_2 = P/2$. Let

γ_0, $\gamma_{\hat{m},1}$, and $\gamma_{\hat{m},2}$ be the SNRs of S → D, S → $R_{\hat{m}}$, and $R_{\hat{m}}$ → D links defined by $\gamma_0 = P\|\mathbf{F}_0\|^2/4$, $\gamma_{\hat{m},1} = P\|\mathbf{F}_{\hat{m}}\|^2/4$, and $\gamma_{\hat{m},2} = P\|\mathbf{G}_{\hat{m}}\|^2/4$, respectively. Then, the instantaneous end-to-end SNR for best relay selection can be expressed as

$$\begin{aligned}\gamma_{\mathrm{eq}} &= \gamma_0 + \max_m \frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1}+\gamma_{m,2}+1}\\ &= \gamma_0 + \frac{\gamma_{\hat{m},1}\gamma_{\hat{m},2}}{\gamma_{\hat{m},1}+\gamma_{\hat{m},2}+1} \end{aligned}\tag{20}$$

where $\gamma_0 \sim \mathcal{G}\left(4, \sigma^2_{\mathrm{SD}}P/4\right)$, $\gamma_{\hat{m},1} \sim \mathcal{G}\left(4, \sigma^2_{\mathrm{SR}_{\hat{m}}}P/4\right)$, and $\gamma_{\hat{m},2} \sim \mathcal{G}\left(4, \sigma^2_{\mathrm{R}_{\hat{m}}\mathrm{D}}P/4\right)$, respectively.

Using the similar approach to the single-relay transmission, the conditional PEP of the SDF protocol with the best relay selection can be bounded by

$$\begin{aligned}\Pr(\mathbf{x}\to\hat{\mathbf{x}}|\mathbf{H}) &\le Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_{\hat{m},1}\gamma_{\hat{m},2}}{c_{\hat{m}}(\gamma_{\hat{m},1}+\gamma_{\hat{m},2})}\right\}\delta^2_{\mathbf{x}}}\right)\\ \Pr(\mathbf{x}\to\hat{\mathbf{x}}|\mathbf{H}) &\ge Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_{\hat{m},1}\gamma_{\hat{m},2}}{\gamma_{\hat{m},1}+\gamma_{\hat{m},2}}\right\}\delta^2_{\mathbf{x}}}\right)\end{aligned}\tag{21}$$

for $c_{\hat{m}} > 1 + (\gamma_{\hat{m},1} + \gamma_{\hat{m},2})^{-1}$.

Thus, the average PEP for the SDF cooperative networks with the best relay selection scheme is bounded as

$$\Pr(\mathbf{x}\to\hat{\mathbf{x}}) \le \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n\delta^2_{\mathbf{x}}}{2}\right)\mathcal{M}_{\gamma_{\max}}\left(\frac{b_n\delta^2_{\mathbf{x}}}{4c_{\hat{m}}}\right)\tag{22}$$

$$\Pr(\mathbf{x}\to\hat{\mathbf{x}}) \ge \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_{n-1}\delta^2_{\mathbf{x}}}{2}\right)\mathcal{M}_{\gamma_{\max}}\left(\frac{b_{n-1}\delta^2_{\mathbf{x}}}{4}\right)\tag{23}$$

where $\gamma_{\max} \triangleq H(\gamma_{\hat{m},1}, \gamma_{\hat{m},2})$.

Deriving the diversity order directly from the above inequalities is not an easy task. Instead, we will use the following relation:

$$\begin{aligned}\Pr(\gamma_{\max} \le \gamma) &= \Pr(\gamma_1 \le \gamma, \cdots, \gamma_M \le \gamma)\\ &= \Pr\left(\lim_{r\to\infty}\left[\gamma_1^r + \cdots + \gamma_M^r \le \gamma^r\right]\right).\end{aligned}$$

Since γ_m's are independent, the MGF of $\gamma_{\max}$ equals to the Mth power of the MGF of γ_m^r, i.e.,

$$\mathcal{M}_{\gamma_{\max}}(s) = \mathcal{M}_{\sum_{m=1}^M \gamma_m^r}(s) = \left[\mathcal{M}_{\gamma_m^r}(s)\right]^M$$

as $r \to \infty$. As γ_m has Fox's H-function distribution [15], so does γ_m^r. From the property of Fox's H-function, the MGF of γ_m^r can be written as in the following theorem.

Theorem 1. *The asymptotic MGF of $\gamma_{\max}$ is represented as*

$$\mathcal{M}_{\gamma_{\max}}(s) \approx \left\{\frac{1}{2^{10}3^2}\left(\frac{\Omega}{2}\right)^{-4}\left[3 + \frac{95}{2^7}\left(\frac{\Omega}{2}\right)^{-4}\log\left(\frac{\Omega}{2}\right)\right]\right\}^M.$$

Proof. See the Appendix. □

From the above result, we can conclude that the diversity order of S → $R_{\hat{m}}$ → D is $4M$. Since the diversity order of S → D is four, the diversity order of the cooperative network with the best relay selection under ML decoder becomes $4(M+1)$, which is the same as that of the conventional multiple-relay scheme.

5. Simulation Results

It is assumed that the channel is Rayleigh-faded and frequency-flat quasi-static, i.e., the channel state does not change within one phase but varies independently from phase to phase. Quadrature phase-shift keying (QPSK) is used. For the sake of simplicity, the symmetric channel is considered, i.e., $\sigma^2_{\mathrm{SD}} = \sigma^2_{\mathrm{SR}_m} = \sigma^2_{\mathrm{R}_m\mathrm{D}} = 1$. The total transmit power during two phases is set to P. The cooperative networks with M relays equipped with two antennas both at the transmitter and receiver are considered. The analytical results for BERs of both the conventional and the best relay selection schemes under ML decoder can be obtained from the PEPs by using the relation between BER and PEP in [19].

Figures 2 and 3 depict the BER performance of the conventional multiple-relay transmission. In Fig. 2, the analytical and numerical results for the conventional multiple-relay SDF cooperative network are shown when the uniform power allocation between S and R_m is used. In this figure, the analytical results match well with the simulation results in the high SNR region where the discrepancy due to using the bounds of (14) becomes negligible. Furthermore, in the low SNR region, the larger the number of the relays used, the worse the BERs. This can be explained from the fact that the effect of S → R_m → D becomes more dominant than that of S → D as the number of the relays increases. Figure 3 describes the effect of the BER performance with respect to the power allocations, where the following cases are considered;

$$\text{Uniform}: \; P_1 = P_2 = \frac{P}{M+1}$$

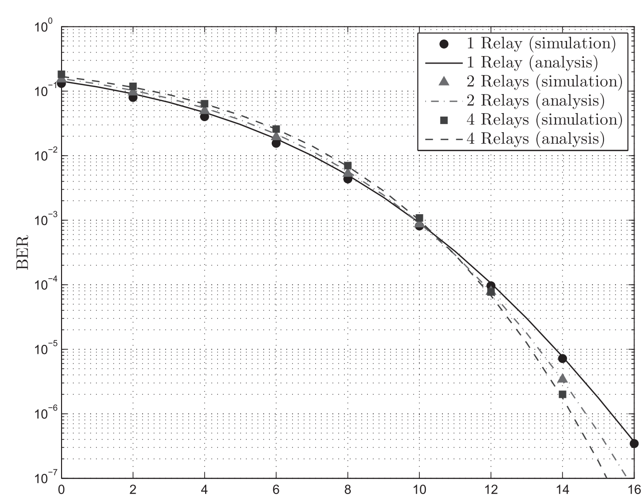

Fig. 2 BERs of SDF cooperative network with the conventional multiple-relay transmission.

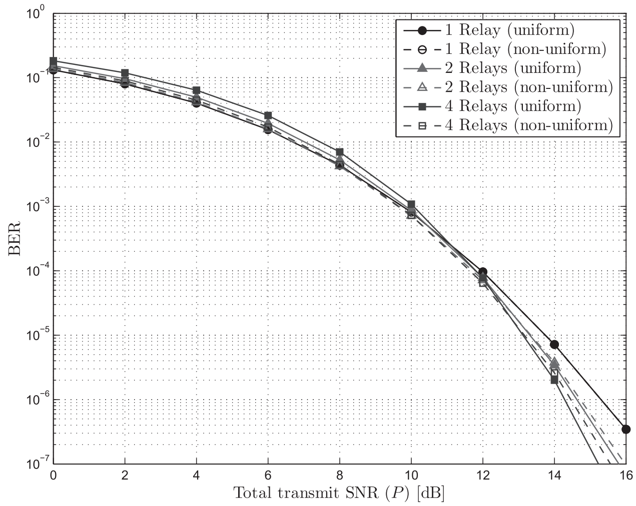

Fig. 3 Comparison of BERs of SDF for the conventional multiple-relay transmission between uniform and non-uniform power allocation.

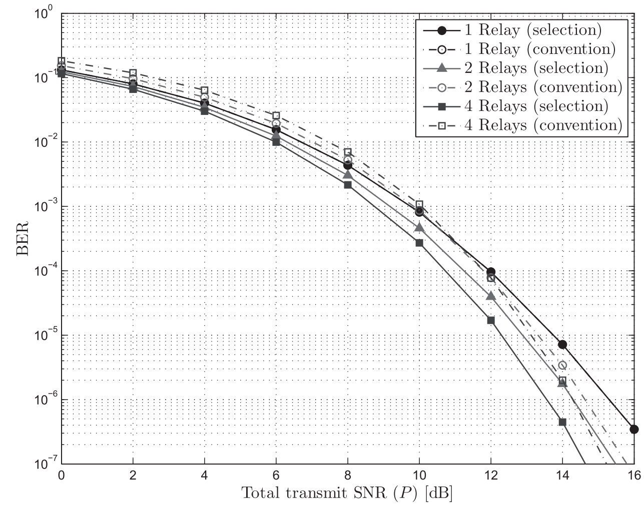

Fig. 5 Comparison of BERs of SDF with the conventional multiple-relay transmission vs. best relay selection.

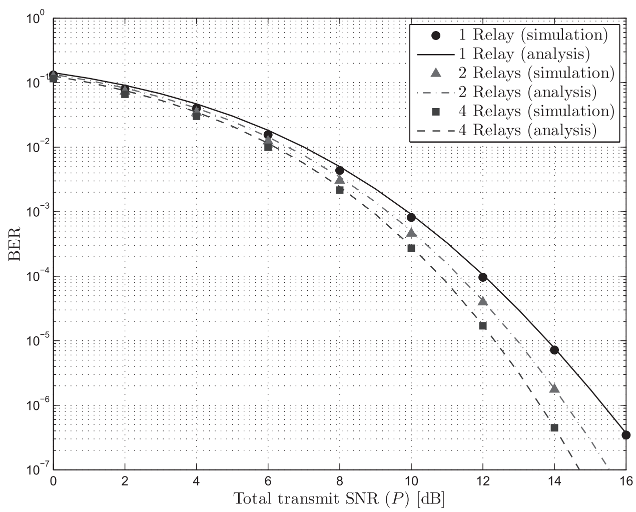

Fig. 4 BERs of SDF cooperative network with the best relay selection.

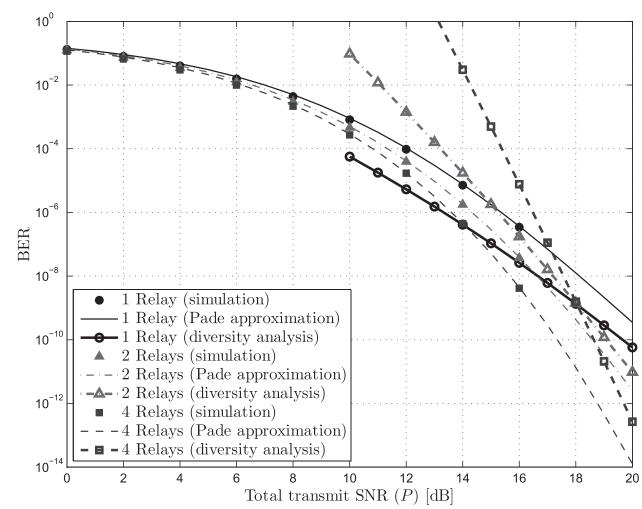

Fig. 6 BERs and diversity order of SDF cooperative network with the best relay selection (The diversity plots are scaled for the clarity of view).

$$\text{Non-uniform}: \; P_1 = \frac{P}{2}, \; P_2 = \frac{P}{2M}.$$

From the numerical results, the conventional SDF cooperative network with uniform power allocation outperforms that with non-uniform power allocation in the relatively large SNR region.

Figure 4 shows the analytical and numerical results for the SDF cooperative network with the best relay selection when the uniform power allocation between S and $\mathsf{R}_{\hat{m}}$ is used, i.e., $P_1 = P_2 = P/2$. Unlike the conventional scheme, as the number of the relays increases, the BER performance of the best relay selection scheme is always enhanced.

In Fig. 5, two schemes with uniform power allocation are compared in terms of BER. From the numerical results, the best relay selection scheme outperforms the conventional multiple-relay transmission in the all SNR region. Furthermore, since less time slots are used for the transmission of the best relay selection scheme in the second phase, the best relay selection scheme has also an advantage of the throughput (spectral efficiency) against the conventional one.

Finally, Fig. 6 shows the analytical and numerical results for the cooperative network with the best relay selection when the uniform power allocation between S and $\mathsf{R}_{\hat{m}}$ is used, i.e., $P_1 = P_2 = P/2$. The 'Pade approximation' results are obtained using Padé approximation technique [13]. And also, diversity orders from PEP are plotted. From this result, we can confirm that the proposed scheme has full diversity order.

6. Conclusion

In this paper, the performance of the SDF cooperative networks with two different relay-assisted transmission schemes has been analyzed. The PEP of the SDF cooperative networks with the conventional multiple-relay transmission scheme is derived by using Gauss' hypergeometric function. And it has been shown that the conventional scheme has full diversity. Also, The diversity order for the SDF cooperative networks with the best relay selection scheme under ML decoder is obtained by using Fox's H-function. From the numerical results, it has been shown that the best relay selection scheme outperforms the conventional multiple-relay transmission scheme in terms of BER and throughput.

Acknowledgement

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Appendix: Proof of Theorem 1

The distribution of a general non-negative random variable, i.e., $\Pr(X < 0) = 0$, can be expressed in terms of Fox's H-function [15]. Then, the random variable X is said to be H-function distributed [14] and its probability density function(PDF) is given as

$$f_X(x) = \begin{cases} k\mathcal{H}^{m,n}_{p,q}\left[cx \middle| \begin{matrix} (a_1,\alpha_1),\cdots,(a_p,\alpha_p) \\ (b_1,\beta_1),\cdots,(b_q,\beta_q) \end{matrix}\right], & x > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{A$\cdot$1}$$

The advantage of introducing H-function distribution is the fact that the PDFs of product, rational power, and quotient of independent random variables can be also in terms of H-function. Furthermore, the MGF of H function distribution can be expressed as H function and is given as

$$\mathcal{M}_X(s) = \frac{k}{c}\mathcal{H}^{n+1,m}_{q,p+1}\left[\frac{s}{c} \middle| \begin{matrix} (1-b_1-\beta_1,\beta_1),\cdots,(1-b_q-\beta_q,\beta_q) \\ (0,1),(1-a_1-\alpha_1,\alpha_1),\cdots,(1-a_q-\alpha_p,\alpha_p) \end{matrix}\right]. \tag{A$\cdot$2}$$

This fact will be used in the derivation of the diversity order for the best relay selection scheme.

For the calculation of the asymptotic behavior of H-function, the following lemma is used:

Lemma 1 ([15]). *Let $\Delta \leq 0$. Suppose that some poles of the gamma functions $\Gamma(1-a_i-\alpha_i s)$ $(i = 1,\cdots,n)$ coincide. Let N_{ik} be the orders of these poles if some poles a_{ik} coincide. Then, the principal term of the asymptotic expansion of $\mathcal{H}^{m,n}_{p,q}[z]$ at the near infinity is given as*

$$\mathcal{H}^{m,n}_{p,q}[z] \approx \sum_i{}' h_i z^{(a_i-1)/\alpha_i} + \sum_i{}'' H_{i,k} z^{(a_i-1)/\alpha_i}[\log z]^{N_i-1}$$

where h_i and $H_{i,k}$ are defined at the top in the next page.

Proof of Theorem: Let $Z_m = Y_m^r$, where $Y_m = \mu_H(\gamma_{m,1},\gamma_{m,2})$ for $\gamma_{m,1},\gamma_{m,2} \sim \mathcal{G}(K,\Omega)$. From the result of

$$h_i = \frac{1}{\alpha_i} \frac{\prod_{j=1}^{m} \Gamma\left(b_j + (1-a_i)\frac{\beta_j}{\alpha_i}\right) \prod_{\substack{j=1 \\ j\neq i}}^{n} \Gamma\left(1 - a_j - (1-a_i)\frac{\alpha_j}{\alpha_i}\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j + (1-a_i)\frac{\alpha_j}{\alpha_i}\right) \prod_{j=m+1}^{q} \Gamma\left(1 - b_j - (1-a_i)\frac{\beta_j}{\alpha_i}\right)}$$

and

$$H_{i,k} = \frac{(-1)^{N_i-1}}{(N_i-1)!}\left(\prod_{k=1}^{N_i} \frac{(-1)^{i_k-1}}{i_k!\alpha_{i_k}}\right) \frac{\prod_{j=1}^{m} \Gamma\left(b_j + (1-a_i)\frac{\beta_j}{\alpha_i}\right) \prod_{\substack{j=1 \\ j\neq i_1,\cdots,i_{N_i}}}^{n} \Gamma\left(1 - a_j - (1-a_i)\frac{\alpha_j}{\alpha_i}\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j + (1-a_i)\frac{\alpha_j}{\alpha_i}\right) \prod_{j=m+1}^{q} \Gamma\left(1 - b_j - (1-a_i)\frac{\beta_j}{\alpha_i}\right)}.$$

H-function distribution in (A· 1), the PDF of Z_m is given as

$$f_{Z_m}(z) = \frac{\sqrt{\pi}}{2^{2K-2}\Gamma^2(K)\Omega}\left(\frac{2}{\Omega}\right)^{r-1} \mathcal{H}^{2,0}_{1,2}\left[\left(\frac{2}{\Omega}\right)^{r} z \middle| \begin{matrix} (K-r+\frac{1}{2}, r) \\ (K-r, r), (2K-r, r) \end{matrix}\right].$$

Using (A· 2), the MGF of Z_m can be expressed as

$$\mathcal{M}_{Z_m}(s) = \frac{\sqrt{\pi}}{2^{2K-1}\Gamma^2(K)} \mathcal{H}^{1,2}_{2,2}\left[cs \middle| \begin{matrix} (1-K, r), (1-2K, r) \\ (0,1), (\frac{1}{2}-K, r) \end{matrix}\right] \tag{A· 3}$$

where $c = (\frac{\Omega}{2})^r$ and parameters are $a^* = 1 + r$ and $\Delta = 1 - r$ in [15]. In this case, $K = 4$ is considered. According to Lemma 1, the following relation is held at the infinity for s:

$$\begin{aligned} &\mathcal{H}^{1,2}_{2,2}\left[cs \middle| \begin{matrix} (-3, r), (-7, r) \\ (0,1), (-\frac{7}{2}, r) \end{matrix}\right] \\ &\approx \sum_i{}' h_i (cs)^{\frac{a_i-1}{\alpha_i}} + \sum_i{}'' H_{i,k} (cs)^{\frac{a_i-1}{\alpha_i}} (\log cs) \\ &= h_1 (cs)^{-\frac{4}{r}} + \left\{H_{1,4}(cs)^{-\frac{4}{r}} + H_{2,0}(cs)^{-\frac{8}{r}}\right\} \log(cs). \end{aligned} \tag{A· 4}$$

The constants h_1, H_{1,k_1}, and H_{2,k_2} are expressed as

$$\begin{aligned} h_1 &= \frac{1}{r}\frac{\Gamma(\frac{4}{r})\Gamma(4)}{\Gamma(\frac{1}{2})} \\ H_{1,k_1} &= \frac{1}{2r^2}\frac{\Gamma(\frac{4+k_1}{r})}{\Gamma(\frac{1}{2}-k_1)} \\ H_{2,k_2} &= \frac{1}{2r^2}\frac{\Gamma(\frac{8+k_2}{r})}{\Gamma(\frac{1}{2}-k_2)}. \end{aligned}$$

Thus, using the relation

$$\lim_{r\to\infty} \frac{\Gamma(\frac{x}{r})}{r} = \frac{1}{x} \text{ and } \lim_{r\to\infty} s^{-\frac{k}{r}} = 1,$$

the limits of each term in (A· 4) are calculated at the top in the next page.

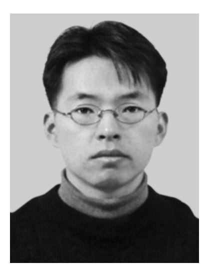
Summing up the above results, (A· 3) is approximated as

$$\lim_{r\to\infty} \mathcal{M}_{Z_m}(s) \approx \frac{1}{2^{10}3^2}\left(\frac{\Omega}{2}\right)^{-4}\left[3 + \frac{95}{2^7}\left(\frac{\Omega}{2}\right)^{-4} \log\left(\frac{\Omega}{2}\right)\right].$$

Let $\gamma_{\max} = \sum_{m=1}^{M} Z_m$, where Z_m's are i.i.d. random variables for $m = 1, \cdots, M$. Then, the MGF for $\gamma_{\max}$ can be obtained as

$$\mathcal{M}_{\gamma_{\max}}(s) \approx \left\{\frac{1}{2^{10}3^2}\left(\frac{\Omega}{2}\right)^{-4}\left[3 + \frac{95}{2^7}\left(\frac{\Omega}{2}\right)^{-4} \log\left(\frac{\Omega}{2}\right)\right]\right\}^{M},$$

as $r \to \infty$, where $\Omega = \sigma^2 P/4$. ■



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$$\lim_{r\to\infty}\left[h_1(cs)^{-\frac{4}{r}}\right] = \lim_{r\to\infty}\left[\frac{1}{r}\frac{\Gamma(\frac{4}{r})\Gamma(4)}{\Gamma(\frac{1}{2})}\left(\frac{\Omega}{2}\right)^{-4} s^{-\frac{4}{r}}\right] = \frac{3}{2\sqrt{\pi}}\left(\frac{\Omega}{2}\right)^{-4}$$

$$\lim_{r\to\infty}\left[H_{1,4}(cs)^{-\frac{8}{r}}\log(cs)\right] = \lim_{r\to\infty}\left[\frac{1}{2r^2}\frac{\Gamma(\frac{8}{r})}{\Gamma(-\frac{7}{2})}\left(\frac{\Omega}{2}\right)^{-8} s^{-\frac{8}{r}}\log\left(\frac{\Omega}{2}\right)^{r}\right] = \frac{7\cdot 5\cdot 3}{2^8\sqrt{\pi}}\left(\frac{\Omega}{2}\right)^{-8}\log\left(\frac{\Omega}{2}\right)$$

$$\lim_{r\to\infty}\left[H_{2,0}(cs)^{-\frac{8}{r}}\log(cs)\right] = \lim_{r\to\infty}\left[\frac{1}{2r^2}\frac{\Gamma(\frac{8}{r})}{\Gamma(-\frac{1}{2})}\left(\frac{\Omega}{2}\right)^{-8} s^{-\frac{8}{r}}\log\left(\frac{\Omega}{2}\right)^{r}\right] = -\frac{1}{2^5\sqrt{\pi}}\left(\frac{\Omega}{2}\right)^{-8}\log\left(\frac{\Omega}{2}\right).$$

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