Analysis on Soft-Decision-and-Forward Cooperative Networks with Multiple Relays

Kyoung-Young SONG†(a), Jaehong KIM†(b), Nonmembers, Jong-Seon NO†(c), Member, and Habong CHUNG††(d), Nonmember

SUMMARY In this paper, we analyze the best relay selection scheme for the soft-decision-and-forward (SDF) cooperative networks with multiple relays. The term ‘best relay selection’ implies that the relay having the largest end-to-end signal-to-noise ratio is selected to transmit in the second phase transmission. The approximate performances in terms of pairwise error probability (PEP) and bit error rate (BER) are analyzed and compared with the conventional multiple-relay transmission scheme where all the relays participate in the second phase transmission. Using the asymptotics of the Fox’s $H$-function, the diversity orders of the best relay selection and conventional relay scheme for the SDF cooperative networks are derived. It is shown that both have the same full diversity order. The numerical results show that the best relay selection scheme outperforms the conventional one in terms of bit error rate.

key words: cooperative diversity, Fox’s $H$-function, moment generating function (MGF), relay selection, soft-decision-and-forward (SDF)

1. Introduction

Next-generation wireless communication systems such as IMT-Advanced [1] require higher spectral efficiency and data rate, that is, 100 Mbps for high-speed mobility and 1 Gbps for low-speed mobility. These demands might be achieved by using multiple-antenna technique [2] which increases the channel capacity. However, due to the limitation on implementation, standardizations such as IEEE 802.16j [3] and Long-Term Evolution (LTE)-Advanced [4] for IMT-Advanced recommend the virtual multiple-antenna technique in a distributed sense.

Sendonaris, Erkip, and Aazhang [5], [6] proposed the cooperative diversity using the cooperation between source and relay. In [7], Zhao, Adve, and Lim analyzed outage behaviors such as outage capacity and outage probability of two different amplify-and-forward (AF) relay schemes with single antenna, namely, all-participated relay transmission and relay selection. They also proposed the power allocation strategies for those two schemes. Similarly, Ikki, and Ahmed proposed and analyzed the best relay selection scheme based on AF protocol with single antenna at each node in [8]. Bletsas et al. [9] described the forward channel estimation for the opportunistic relay selection in case of multiple relays. Yang, Song, No, and Shin [10] proposed the maximum-likelihood (ML) decoding for AF and soft-decision-and-forward (SDF) protocols with multiple antennas using Alamouti codes [11]. Since the relay nodes can separate the signals in SDF protocol while they simply amplify the received signals according to the power constraint in AF protocol, SDF becomes substantially different from AF in the case of multiple antennas. In [12], performance on SDF with single relay where each node is equipped with two antennas was analyzed in terms of bit error rate (BER). Furthermore, power allocation scheme between source and relay nodes was proposed so as to maximize the instantaneous end-to-end signal-to-noise ratio (SNR).

In this paper, the best relay selection scheme using Alamouti code for the SDF cooperative networks with multiple relays is analyzed. The indications of the performance such as the pairwise-error probability (PEP) and the diversity order of the best relay selection scheme are compared with those of the conventional multiple-relay transmission scheme where all the relays participate in the second phase transmission. For these two schemes, we express the end-to-end SNRs and derive the PEPs under the ML decoding proposed in [10]. From the derived PEPs for the multiple-relay cooperative networks with SDF protocol, the diversity orders and the approximate BERs are obtained. The best relay selection scheme has an advantage over the conventional one in terms of BER and throughput.

This paper is organized as follows. Section 2 describes the SDF protocol and reviews the results in [12]. Sections 3 and 4 address the conventional multiple-relay transmission and the ‘best relay selection’ schemes, respectively. The analytical and simulation results are shown in Sect. 5. Finally, the concluding remarks are given in Sect. 6.

Throughout this paper, the following notations are used. $\mathbb{E}[\cdot]$ denotes the expectation of a random variable. $X \sim \mathcal{CN}(0, \sigma^2)$ means that $X$ is a complex normal random variable with zero mean and variance $\sigma^2/2$ in both real and imaginary parts, respectively. $\Re\{\cdot\}$ denotes the real part of a complex number. $(\cdot)^T$, $(\cdot)^\dagger$, and $\| \cdot \|$ denote the transpose of a matrix, the conjugate transpose of a matrix, and the Frobenius norm of a matrix or a vector, respectively. Bold-face uppercase and lowercase letters denote matrices and vectors, respectively. If $X$ is a sum of $K$
independent exponential random variables, each of which has the same mean $\Omega$, then, $X$ is called a gamma random variable whose probability density function (PDF) is expressed in terms of a shape parameter $K$ and a scale parameter $\Omega$. We denote it by $X \sim \mathcal{G}(K, \Omega)$. The Alamouti code $\begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix}$ is denoted by $A(a, b)$. Also, for any $2 \times 2$ matrix $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, the $4 \times 2$ matrix $B'$ and the vector $cv(B)$ are defined as $B' = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}$ and $cv(B) = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}^T$.

2. Soft-Decision-and-Forward Protocol

The system model of SDF protocol [10] with multiple relays where each node is equipped with two antennas is depicted in Fig. 1. This cooperative communication system is composed of one source (S), one destination (D), and $M$ relays ($R_m, m = 1, \cdots, M$). In the second phase transmission, the following two transmission methods for the multiple relays are considered: conventional multiple-relay transmission and the best relay selection.

The total transmit power $P$ in the network is defined as the sum of the source power $P_1$ at S and the total relay power $P_2$ at $R_m$'s. The channel gains of each link $S \rightarrow D$, $S \rightarrow R_m$, and $R_m \rightarrow D$ are assumed to be Rayleigh-faded, i.e., $f_{ij}^0 \sim \mathcal{CN}(0, \sigma_{RSD}^2)$, $g_{ij}^0 \sim \mathcal{CN}(0, \sigma_{RSD}^2)$, and $g_{ij}^m \sim \mathcal{CN}(0, \sigma_{RSD}^2)$, where $f_{ij}^0$ and $g_{ij}^m, i, j = 1, 2, m = 1, \cdots, M$, denote the path gain from the $i$th transmit antenna at S to the $j$th receive antenna at D, from the $i$th transmit antenna at S to the $j$th receive antenna at $R_m$, and from the $i$th transmit antenna at $R_m$ to the $j$th receive antenna at D, respectively. These path gains are represented as the channel matrices $F_0 = [f_{ij}^0]$, $F_m = [g_{ij}^m]$, and $G_m = [g_{ij}^m]$.

To help understand the SDF protocol, we briefly review the SDF protocol with single relay, where each node has two antennas. The transmission is composed of two phases. In the first phase, S transmits the signal using Alamouti code to R and D. Thus, the received signals at R and D are represented, respectively, as:

$$ Y_R = \sqrt{\frac{P_1}{2}} X F_1 + N_R $$

$$ Y_D = \sqrt{\frac{P_1}{2}} X F_0 + N_D $$

where $X = A(x_1, x_2)$ is the transmit codeword at S in the first phase, $F_0$ and $F_1$ denote the channel matrices of $S \rightarrow D$ and $D \rightarrow R$, respectively, and $N_R$ and $N_D$ are the $2 \times 2$ additive white Gaussian noise (AWGN) matrices with zero-mean and unit-variance entries. During the intermediate decoding at R, R obtains the soft-decision values from the received signals using maximal ratio combining as:

$$ \mathbf{x} = [\hat{x}_1 \hat{x}_2]^T = \lambda F_1^\dagger cv(Y_R) $$

where

$$ cv(Y_R) = [y_{11}^{(R)} y_{12}^{(R)} y_{21}^{(R)} y_{22}^{(R)}]^T = \sqrt{\frac{P_1}{2}} F_1^\dagger x + cv(N_R) $$

$$ \lambda = \sqrt{\frac{2}{||F_1||^2 ||P_1||^2 + 2}} $$

where $x = [x_1 \ x_2]^T$ is the transmitted signal vector at S.

And then, R transmits the following codeword into D

$$ X_R = A(\hat{x}_1, \hat{x}_2) = [\hat{x}_1 \ \hat{x}_2]^T. $$

In the second phase, the received signal at D is expressed as

$$ Y_{D2} = \sqrt{\frac{P_2}{2}} X_R G_1 + N_{D2} $$

where $G_1$ is the channel matrix of $R \rightarrow D$ and $N_{D2}$ denotes the $2 \times 2$ AWGN matrix with zero-mean and unit-variance entries. Converting the matrix form into the vector form gives the following alternative expression:

$$ cv(Y_{D2}) = \frac{\sqrt{P_1 P_2}}{2} ||F_1||^2 G_1^\dagger x + \sqrt{\frac{P_2}{2}} ||G_1||^2 F_1^\dagger cv(N_R) + cv(N_{D2}). $$

The received signal at D during two phases can be rewritten as an equivalent vector model

$$ \begin{bmatrix} cv(Y_{D1}) \\ cv(Y_{D2}) \end{bmatrix} = \begin{bmatrix} \frac{P_1}{2} \sqrt{\frac{P_2}{2}} ||F_1||^2 G_1^\dagger x + cv(N_{D1}) \\ \frac{P_1}{2} \sqrt{\frac{P_2}{2}} ||F_1||^2 G_1^\dagger x + cv(N_{D1}) \end{bmatrix}, $$

where $cv(N_D)$ means the equivalent noise at D in the vector form, given by

$$ cv(N_D) = \sqrt{\frac{P_2}{2}} ||G_1||^2 F_1^\dagger cv(N_R) + cv(N_{D2}). $$
\[ \hat{x}_i = \arg \min_{x_i} \left( \frac{P_1}{2} ||F||^2 + \frac{P_1 P_2 ||F||^2 ||G||^2}{2(P_1 ||F||^2 + P_2 ||G||^2 + 2)} \right) |x_i|^2 - 2 \Re (y_i x_i) \]

The ML decoding rule for the SDF protocol can be written as
\[ \hat{x} = \arg \min_x [ (y - Hx)^T \mathcal{K}_n^{-1} (y - Hx) ] \]
\[ \hat{x} = \arg \min_x \left[ x^T \mathcal{K}_n^{-1} Hx - 2 \Re (y^T \mathcal{K}_n^{-1} Hx) \right] \]

where \( \mathcal{K}_n = \mathbb{E}[nn^T] = \begin{bmatrix} I_k & 0 \\ 0 & \mathcal{K}_{n}(\mathbb{C}) \end{bmatrix} \) with \( \mathcal{K}_{n}(\mathbb{C}) = I_k + P_2/(P_1 ||F||^2 + 2) \cdot G F^T \). The ML decoder for SDF protocol [10] chooses \( \hat{x} \) such as (2) at the top of this page, where \( \eta_1 = \eta_2 = y^T \mathcal{K}_n^{-1} H \). It is important to estimate \( \mathcal{K}_n \) as exact as possible. In this paper, we assume that the exact estimation on the covariance matrix at \( D \) is possible.

Using (1), the conditional PEP can be written as
\[ \Pr(x \rightarrow \hat{x}(H)) = Q \left( \frac{1}{2} \gamma \sqrt{\mathcal{K}_n^{-1} H (\hat{x} - x)^T} \right) \]

where \( Q(x) = \int_x^\infty e^{-u^2/2} \sqrt{2\pi} du, \delta_x = ||x - x|| \)

\[ \gamma_0 = \frac{P_1}{2} ||F||^2 \sim G(4, \sigma_{SD}^2, P_1/2) \]
\[ \gamma_1 = \frac{P_1}{2} ||F||^2 \sim G(4, \sigma_{SR}^2, P_1/2) \]
\[ \gamma_2 = \frac{P_2}{2} ||G||^2 \sim G(4, \sigma_{RD}^2, P_2/2) \]

It is shown in [12] that the PEP depends on the instantaneous end-to-end SNR, \( \gamma_{eq} \), which is the sum of two SNRs, i.e.,
\[ \gamma_{eq} = \gamma_0 + \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \]

And also, (4) can be upper and lower bounded as
\[ \gamma_0 + \frac{\gamma_1 \gamma_2}{c(\gamma_1 + \gamma_2)} \leq \gamma_{eq} \leq \gamma_0 + \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \]

where \( c > 1 + (\gamma_1 + \gamma_2)^{-1} \). Since \( Q(x) \) is monotonically decreasing for \( x \geq 0 \), substitution of (5) into (3) leads to the following inequalities as
\[ \Pr(x \rightarrow \hat{x}(H)) \leq Q \left( \frac{1}{2} \gamma (\gamma_0 + \frac{\gamma_1 \gamma_2}{c(\gamma_1 + \gamma_2)} \right) \delta_x^2 \]

Furthermore, the \( Q \)-function is bounded as
\[ \sum_{n=1}^{N} a_n \exp (-b_{n-1}u) \leq Q(\sqrt{u}) \leq \sum_{n=1}^{N} a_n \exp (-b_n u) \]

for \( a_n = (\theta_n - \theta_{n-1})/\pi \) and \( b_n = 1/(2 \sin^2 \theta_n) \) for \( n = 1, \cdots, N \) with \( \theta_0 = 0 \) and \( \theta_N = \pi/2 \). Then, we can bound the PEP by averaging the conditional PEP in (6) over \( H \) using the \( Q \)-function inequality in (7). It gives the upper and lower bounds of the average PEP for the SDF protocol [12] as
\[ \Pr(x \rightarrow \hat{x}) \leq \sum_{n=1}^{N} a_n M_{\gamma_0}(b_n, \delta_x^2/2) M_{H(\gamma_1, \gamma_2)}(b_n \delta_x^2/4c) \]

and
\[ \Pr(x \rightarrow \hat{x}) \geq \sum_{n=1}^{N} a_n M_{\gamma_0}(b_n, \delta_x^2/2) M_{H(\gamma_1, \gamma_2)}(b_n \delta_x^2/4c) \]

where \( M_{\gamma}(\cdot) \) is the moment generating function(MGF) for random variable \( X \) and \( H(a, b) = 2ab/(a + b) \).

3. SDF Protocol with the Conventional Multiple-Relay Transmission

In this section, the PEP and diversity order of the ‘conventional’ SDF cooperative networks with multiple relays are derived, where all relays participate in the second phase transmission.

3.1 System Model

The signal transmission in the cooperative networks is composed of two phases. In the first phase, \( S \) transmits the signals using Alamouti code to \( R_m, m = 1, \cdots, M \) and \( D \). The received signals at \( R_m \) and \( D \) are represented, respectively, as
\[ Y_{R_m} = \sqrt{\frac{P_1}{2}} X F_m + N_{R_m}, \quad m = 1, \cdots, M \]
\[ Y_{D} = \sqrt{\frac{P_1}{2}} X F_0 + N_{D} \]

where \( X = A(x_1, x_2) \) is the transmit codeword for the message vector \( x = [x_1, x_2]^T \) at \( S \) in the first phase, \( F_0 \) and \( F_m \) denote the channel matrices of \( S \rightarrow D \) and \( S \rightarrow R_m \), respectively, and \( N_{R_m} \) and \( N_D \) are the \( 2 \times 2 \) AWGN matrices with zero-mean and unit-variance entries. Transforming matrix
form into vector form, (10) can be rewritten as
\[ c^T(Y_{R_m}) = \begin{bmatrix} y_{11}^{(R_m)} & y_{21}^{(R_m)} & y_{12}^{(R_m)} & y_{22}^{(R_m)} \end{bmatrix}^T \]
\[ = \frac{P_1}{2} F_m x + c^T(N_{R_m}) \]
\[ c^T(Y_{D_1}) = \begin{bmatrix} y_{11}^{(D_1)} & y_{21}^{(D_1)} & y_{12}^{(D_1)} & y_{22}^{(D_1)} \end{bmatrix}^T \]
\[ = \frac{P_1}{2} F_m x + c^T(N_{D_1}) \]

During the intermediate decoding at \( R_m \), the soft-decision values are obtained from the received signals using the maximal ratio combining as
\[ \hat{x}_m \triangleq \begin{bmatrix} \hat{x}_{m,1} & \hat{x}_{m,2} \end{bmatrix}^T = \lambda_m F_m^\dagger c^T(Y_{R_m}) \]
where
\[ \lambda_m = \frac{2}{||F_m||^2(P_1||F_m||^2 + 2)} \]

And then, \( R_m \) transmits the following codeword to \( D \)
\[ X_{R_m} = A(\hat{x}_{m,1}, \hat{x}_{m,2}) = \begin{bmatrix} \hat{x}_{m,1} & \hat{x}_{m,2} \end{bmatrix} \]

In the second phase, the re-encoded codewords of \( M \) relays are transmitted during \( M \) time slots and the received signal at \( D \) in the \( m \)th time slot is expressed as
\[ Y_{D_2}^{(m)} = \frac{P_2}{2} X_{R_m} G_m + N_{D_2}^{(m)} \]

where \( G_m \) is the channel matrix of \( R_m \rightarrow D \) and \( N_{D_2}^{(m)} \) denotes the \( 2 \times 2 \) AWGN matrix with each element having zero-mean and unit-variance. The signals during the \( M \) time slots in the second phase are transmitted from \( M \) relays in the orthogonal transmission manner. Thus, the received signals at \( D \) during the second phase can be written as
\[ Y_{D_2}^{(m)} = \frac{P_2}{2} X_{R_m} G_m + N_{D_2}^{(m)}, \quad m = 1, \cdots, M. \]

Converting the matrix form into the vector form gives the following alternative expression
\[ c^T(Y_{D_2}) = \frac{\sqrt{P_1 P_2}}{2} \lambda_m ||F_m||^2 G_m^\dagger x + \sqrt{P_2} \lambda_m G_m^\dagger F_m^\dagger c^T(N_{R_m}) + c^T(N_{D_2}^{(m)}) + c^T(N_{D}^{(m)}) \]

where \( c^T(N_{D}^{(m)}) \) means the equivalent noise at \( D \) in the second phase transmission.

The received signals at \( D \) in the both phases can be rewritten as an equivalent vector model
\[ \begin{bmatrix} c^T(Y_{D_1}) \\ c^T(Y_{D_2}) \end{bmatrix} = \begin{bmatrix} \frac{P_1}{2} F_m x + c^T(N_{D_1}) \\ \frac{P_2}{2} \lambda_m ||F_m||^2 G_m^\dagger \end{bmatrix} \]

\[ \frac{P_1}{2} F_m x + c^T(N_{D_1}) \]
\[ \frac{P_2}{2} \lambda_m ||F_m||^2 G_m^\dagger \]

The ML decoding rule for (11) of the SDF protocol is as follows:
\[ \hat{x} = \arg \min_x [\|y - Hx\|^2 \mathcal{K}_n^{-1}(y - Hx)] \]
\[ = \arg \min_x [x^T H^T \mathcal{K}_n^{-1} H x - 2 \Re \{x^T \mathcal{K}_n^{-1} H y\}] \]

where
\[ \mathcal{K}_n = \mathcal{E}[nn^T] = \begin{bmatrix} 1_4 & 0_4 & \cdots & 0_4 \\ 0_4 & 1_4 & \cdots & 0_4 \\ \vdots & \vdots & \ddots & \vdots \\ 0_4 & 0_4 & \cdots & 1_4 \end{bmatrix} \]

with
\[ \mathcal{K}^{\mathcal{E}(N_{D_1})} = 1_4 + \frac{P_2}{P_1||F_m||^2 + 2} \lambda_m G_m^\dagger \]

The ML decoder in (12) can be restated as (13) at the top in this page, where \( y^T \mathcal{K}_n^{-1} H = \begin{bmatrix} \eta_1 & \eta_2 \end{bmatrix} \).

3.2 PEP and Diversity Order

Let \( \gamma_0, \gamma_{m,1}, \) and \( \gamma_{m,2} \) be the SNRs of \( S \rightarrow D, \ S \rightarrow R_m, \) and \( R_m \rightarrow D \) links defined by \( \gamma_0 = P_l||F_0||^2/2, \gamma_{m,1} = P_1||F_m||^2/2, \) and \( \gamma_{m,2} = P_2||G_m||^2/2, \) respectively. Then, the instantaneous end-to-end SNR for the conventional multiple-relay transmission under ML decoder can be written as
\[ \gamma_{eq} = \gamma_0 + \sum_{m=1}^{M} \frac{\gamma_{m,1} \gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1} \]

where \( \gamma_0 \sim \mathcal{G} \left( 4, \frac{\sigma^{2}_{\text{noise}} P_1}{2} \right), \gamma_{m,1} \sim \mathcal{G} \left( 4, \frac{\sigma^{2}_{\text{noise}} P_1}{2} \right), \) and \( \gamma_{m,2} \sim \mathcal{G} \left( 4, \frac{\sigma^{2}_{\text{noise}} P_2}{2} \right), \) respectively.

Applying the similar approach in Sect. 2 to the conventional scheme, the upper and lower bounds on the conditional PEP can be expressed as
In this section, the PEP and diversity order of the SDF cooperative networks with multiple relays are derived. From the above results, we conclude that the diversity or-

criterion assumes that information on \( \gamma_{m,1} \) for each relay has to be notified to the destination node. Here, information is only the channel norm between \( S \) and \( R_m \). Note that this criterion guarantees the maximum channel capacity as well as the minimum PEP.

Then, the selected \( \hat{m} \)th relay \( R_{\hat{m}} \) transmits the following codeword to \( D \)

\[ X_{R_{\hat{m}}} = A(\tilde{x}_{\hat{m},1}, \tilde{x}_{\hat{m},2}) = \begin{bmatrix} \tilde{x}_{\hat{m},1} & \tilde{x}_{\hat{m},2} \end{bmatrix} \sqrt{\frac{P_{2\hat{m}}}{2}} G_{\hat{m}} + N_{D2} \]

In the second phase, the destination \( D \) receives the signal from the \( \hat{m} \)th relay as

\[ Y_{D2} = \sqrt{\frac{P_{2\hat{m}}}{2}} X_{R_{\hat{m}}} G_{\hat{m}} + N_{D2} \]

where \( G_{\hat{m}} \) is the channel matrix of \( R_{\hat{m}} \rightarrow D \) and \( N_{D2} \) denotes the \( 2 \times 2 \) AWGN matrix with zero-mean and unit-variance entries. Converting the matrix equation into the vector form gives us the following alternative expression

\[ cv(Y_{D2}) = \sqrt{\frac{P_{1} P_{2\hat{m}}}{2}} \lambda_{0}(\|F_{\hat{m}}\|^{2} G_{\hat{m}}^{\dagger} x + \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0} G_{\hat{m}}^{\dagger} F_{\hat{m}}^{\dagger} cv(N_{R_{\hat{m}}}) + cv(N_{D2})}. \]

The received signal at \( D \) in both phases can be rewritten as an equivalent vector model

\[ \begin{bmatrix} cv(Y_{D1}) \\ cv(Y_{D2}) \end{bmatrix} = \sqrt{\frac{P_{1}}{2}} \begin{bmatrix} F_{\hat{m}}^{\dagger} \\ \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0}(\|F_{\hat{m}}\|^{2} G_{\hat{m}}^{\dagger} x + \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0} G_{\hat{m}}^{\dagger} F_{\hat{m}}^{\dagger} cv(N_{R_{\hat{m}}}) + cv(N_{D2}) \end{bmatrix} + \begin{bmatrix} cv(N_{D1}) \\ cv(N_{D2}) \end{bmatrix} \]

where \( cv(N_{D}) \) means the equivalent noise at \( D \) in the vector form given by

\[ cv(N_{D}) = \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0} G_{\hat{m}}^{\dagger} F_{\hat{m}}^{\dagger} cv(N_{R_{\hat{m}}}) + cv(N_{D2})}. \]

The ML decoder for the best relay selection is the same as the one for a single relay case.

4.2 PEP and Diversity Order

In the best relay selection scheme, the relay \( R_{\hat{m}} \) selected according to the selection criterion in (18) transmits the signals with power \( P_{2} \) in the second phase. For the easy derivation of PEP, it is assumed that the uniform power allocation is used between \( S \) and \( R_{\hat{m}} \), i.e., \( P_{1} = P_{2} = P/2 \). Let

in the first phase is the same as the conventional scheme. In contrast to the conventional multiple-relay transmission, the signal in the second phase of the best relay selection scheme is transmitted from only one relay \( R_{\hat{m}} \) according to the relay selection criterion

\[ \hat{m} = \arg \max_{m} \left\{ \frac{\gamma_{m,1} \gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1} \right\}, \quad (18) \]

This criterion assumes that information on \( \gamma_{m,1} \) for each relay has to be notified to the destination node. Here, information is only the channel norm between \( S \) and \( R_m \). Note that this criterion guarantees the maximum channel capacity as well as the minimum PEP.

Then, the selected \( \hat{m} \)th relay \( R_{\hat{m}} \) transmits the following codeword to \( D \)

\[ X_{R_{\hat{m}}} = A(\tilde{x}_{\hat{m},1}, \tilde{x}_{\hat{m},2}) = \begin{bmatrix} \tilde{x}_{\hat{m},1} & \tilde{x}_{\hat{m},2} \end{bmatrix} \sqrt{\frac{P_{2\hat{m}}}{2}} G_{\hat{m}} + N_{D2} \]

In the second phase, the destination \( D \) receives the signal from the \( \hat{m} \)th relay as

\[ Y_{D2} = \sqrt{\frac{P_{2\hat{m}}}{2}} X_{R_{\hat{m}}} G_{\hat{m}} + N_{D2} \]

where \( G_{\hat{m}} \) is the channel matrix of \( R_{\hat{m}} \rightarrow D \) and \( N_{D2} \) denotes the \( 2 \times 2 \) AWGN matrix with zero-mean and unit-variance entries. Converting the matrix equation into the vector form gives us the following alternative expression

\[ cv(Y_{D2}) = \sqrt{\frac{P_{1} P_{2\hat{m}}}{2}} \lambda_{0}(\|F_{\hat{m}}\|^{2} G_{\hat{m}}^{\dagger} x + \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0} G_{\hat{m}}^{\dagger} F_{\hat{m}}^{\dagger} cv(N_{R_{\hat{m}}}) + cv(N_{D2})}. \]

The received signal at \( D \) in both phases can be rewritten as an equivalent vector model

\[ \begin{bmatrix} cv(Y_{D1}) \\ cv(Y_{D2}) \end{bmatrix} = \sqrt{\frac{P_{1}}{2}} \begin{bmatrix} F_{\hat{m}}^{\dagger} \\ \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0}(\|F_{\hat{m}}\|^{2} G_{\hat{m}}^{\dagger} x + \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0} G_{\hat{m}}^{\dagger} F_{\hat{m}}^{\dagger} cv(N_{R_{\hat{m}}}) + cv(N_{D2}) \end{bmatrix} + \begin{bmatrix} cv(N_{D1}) \\ cv(N_{D2}) \end{bmatrix} \]

where \( cv(N_{D}) \) means the equivalent noise at \( D \) in the vector form given by

\[ cv(N_{D}) = \sqrt{\frac{P_{2\hat{m}}}{2}} \lambda_{0} G_{\hat{m}}^{\dagger} F_{\hat{m}}^{\dagger} cv(N_{R_{\hat{m}}}) + cv(N_{D2})}. \]

The ML decoder for the best relay selection is the same as the one for a single relay case.

4.2 PEP and Diversity Order

In the best relay selection scheme, the relay \( R_{\hat{m}} \) selected according to the selection criterion in (18) transmits the signals with power \( P_{2} \) in the second phase. For the easy derivation of PEP, it is assumed that the uniform power allocation is used between \( S \) and \( R_{\hat{m}} \), i.e., \( P_{1} = P_{2} = P/2 \). Let
order of $S \rightarrow D$ is four, the diversity order of the cooperative network with the best relay selection under ML decoder becomes $4(M + 1)$, which is the same as that of the conventional multiple-relay scheme.

5. Simulation Results

It is assumed that the channel is Rayleigh-faded and frequency-flat quasi-static, i.e., the channel state does not change within one phase but varies independently from phase to phase. Quadrature phase-shift keying (QPSK) is used. For the sake of simplicity, the symmetric channel is considered, i.e., $\sigma^2_{SD} = \sigma^2_{SR_m} = \sigma^2_{R_mD} = 1$. The total transmit power during two phases is set to $P$. The cooperative networks with $M$ relays equipped with two antennas both at the transmitter and receiver are considered. The analytical results for BERs of both the conventional and the best relay selection schemes under ML decoder can be obtained from the PEPs by using the relation between BER and PEP in [19].

Figures 2 and 3 depict the BER performance of the conventional multiple-relay transmission. In Fig. 2, the analytical and numerical results for the conventional multiple-relay SDF cooperative network are shown when the uniform power allocation between $S$ and $R_m$ is used. In this figure, the analytical results match well with the simulation results in the high SNR region where the discrepancy due to using the bounds of (14) becomes negligible. Furthermore, in the low SNR region, the larger the number of the relays used, the worse the BERs. This can be explained from the fact that the effect of $S \rightarrow R_m \rightarrow D$ becomes more dominant than that of $S \rightarrow D$ as the number of the relays increases. Figure 3 describes the effect of the BER performance with respect to the power allocations, where the following cases are considered:

Uniform: $P_1 = P_2 = \frac{P}{M+1}$

![Fig. 2 BERs of SDF cooperative network with the conventional multiple-relay transmission.](image-url)
SONG et al.: ANALYSIS ON SOFT-DECISION-AND-FORWARD COOPERATIVE NETWORKS WITH MULTIPLE RELAYS

515

Fig. 3 Comparison of BERs of SDF for the conventional multiple-relay transmission between uniform and non-uniform power allocation.

Fig. 4 BERs of SDF cooperative network with the best relay selection.

Non-uniform: \( P_1 = \frac{P}{2}, \ P_2 = \frac{P}{2M} \).

From the numerical results, the conventional SDF cooperative network with uniform power allocation outperforms that with non-uniform power allocation in the relatively large SNR region.

Figure 4 shows the analytical and numerical results for the SDF cooperative network with the best relay selection when the uniform power allocation between \( S \) and \( R_m \) is used, i.e., \( P_1 = P_2 = P/2 \). Unlike the conventional scheme, as the number of the relays increases, the BER performance of the best relay selection scheme is always enhanced.

In Fig. 5, two schemes with uniform power allocation are compared in terms of BER. From the numerical results, the best relay selection scheme outperforms the conventional multiple-relay transmission in the all SNR region. Furthermore, since less time slots are used for the transmission of the best relay selection scheme in the second phase, the best relay selection scheme has also an advantage of the throughput (spectral efficiency) against the conventional one.

Finally, Fig. 6 shows the analytical and numerical results for the cooperative network with the best relay selection when the uniform power allocation between \( S \) and \( R_m \) is used, i.e., \( P_1 = P_2 = P/2 \). The ‘Pade approximation’ results are obtained using Padé approximation technique [13]. And also, diversity orders from PEP are plotted. From this result, we can confirm that the proposed scheme has full diversity order.

6. Conclusion

In this paper, the performance of the SDF cooperative networks with two different relay-assisted transmission schemes has been analyzed. The PEP of the SDF cooper-
ative networks with the conventional multiple-relay transmission scheme is derived by using Gauss’ hypergeometric function. And it has been shown that the conventional scheme has full diversity. Also, the diversity order for the SDF cooperative networks with the best relay selection function. And it has been shown that the conventional scheme under ML decoder is obtained by using Fox’s $H$-function. From the numerical results, it has been shown that the best relay selection scheme outperforms the conventional multiple-relay transmission scheme in terms of BER and throughput.

Acknowledgement

This work was partly supported by the IT R&D program of MKE/ITIA [2008-F-007-02, Intelligent Wireless Communication Systems in 3 Dimensional Environment] and the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MEST) (No. 2009-0081441).

References

[4] 3GPP TS 36.211 v10.2.0, “Evolved universal terrestrial radio access (E-UTRA); Physical channels and modulation (Release 10),” June 2011.

Appendix: Proof of Theorem 1

The distribution of a general non-negative random variable, i.e., $\Pr(X < 0) = 0$, can be expressed in terms of Fox’s $H$-function [15]. Then, the random variable $X$ is said to be $H$-function distributed [14] and its probability density function (PDF) is given as

$$f_X(x) = \begin{cases} k \gamma_{p,q} \left[ \frac{1}{2} \left( (a_1, \alpha_1), \ldots, (a_p, \alpha_p), \beta_1, \ldots, \beta_q, \beta_q \right) \right], & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

This fact will be used in the derivation of the diversity order for the best relay selection scheme.

For the calculation of the asymptotic behavior of $H$-function, the following lemma is used:

**Lemma 1** ([15]). Let $\Delta \leq 0$. Suppose that some poles of the gamma functions $\Gamma(1 - a_i - \alpha_i s_i) (i = 1, \ldots, n)$ coincide. Let $N_{\Delta k}$ be the orders of these poles if some poles $a_{\Delta k}$ coincide. Then, the principal term of the asymptotic expansion of $\gamma_{p,q}^{m,n}[z]$ at the near infinity is given as

$$\gamma_{p,q}^{m,n}[z] \approx \sum_{i} h_i z^{\alpha_i - 1} + \sum_{i} H_{\Delta k} z^{\alpha_i - 1} \log z^{N_{\Delta k}}$$

where $h_i$ and $H_{\Delta k}$ are defined at the top in the next page.

Proof of Theorem: Let $Z_m = Y_m^r$, where $Y_m = \mu_H(\gamma_{m,1}, \gamma_{m,2})$ for $\gamma_{m,1}, \gamma_{m,2} \sim \mathcal{G}(K, \Omega)$. From the result of
In this case, \( K \) is calculated at the top in (A-4). According to Lemma 1, the following relation is held at the infinity for \( s \):

\[
g^{1.2}(c, s) \approx \sum_{i} h_i(c, s)^{\frac{a_i}{\Delta}} + \sum_{j} H_{i,j}(c, s)^{\frac{a_j}{\Delta}} \log(c, s)
\]

\( = h_1(c, s)^{-\frac{a_1}{\Delta}} + [H_{1,1}(c, s)^{-\frac{a_1}{\Delta}} + H_{2,0}(c, s)^{-\frac{a_1}{\Delta}}] \log(c, s).\)  

(A-4)

The constants \( h_1, H_{1,1}, \) and \( H_{2,0} \) are expressed as

\[
h_1 = \frac{1}{\alpha_1} \binom{\frac{\alpha_1}{2}}{\frac{\alpha_1}{2}} \Gamma\left(\frac{\alpha_1}{2}\right) / \Gamma\left(\frac{\alpha_1}{2}\right)
\]

\[
H_{1,1} = \frac{1}{\alpha_1} \binom{\frac{\alpha_1}{2}}{\frac{\alpha_1}{2}} \Gamma\left(\frac{\alpha_1}{2}\right) / \Gamma\left(\frac{\alpha_1}{2}\right)
\]

\[
H_{2,0} = \frac{1}{\alpha_1} \binom{\frac{\alpha_1}{2}}{\frac{\alpha_1}{2}} \Gamma\left(\frac{\alpha_1}{2}\right) / \Gamma\left(\frac{\alpha_1}{2}\right)
\]

Thus, using the relation

\[
\lim_{r \to \infty} \frac{\Gamma(\hat{s})}{\hat{s}} = 1 \quad \text{and} \quad \lim_{s \to \infty} s^{-\frac{\hat{s}}{2}} = 1,
\]

the limits of each term in (A-4) are calculated at the top in the next page.

Summing up the above results, (A-3) is approximated as

\[
\lim_{r \to \infty} M_{Z_m}(s) \approx \frac{1}{2^{1032}} \left(\frac{\Omega}{2}\right)^{-4} \left[ 3 + \frac{95}{27} \left(\frac{\Omega}{2}\right)^{-4} \log\left(\frac{\Omega}{2}\right) \right]^M.
\]
\[
\lim_{r \to \infty} \left[ h_1(cs) \right] = \lim_{r \to \infty} \left[ \frac{1}{r} \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(4)}{\Gamma\left(\frac{1}{2}\right)} \left(\frac{\Omega}{2}\right)^{-4} s^\frac{1}{2} \right] = \frac{3}{2\sqrt{\pi}} \left(\frac{\Omega}{2}\right)^{-4}
\]

\[
\lim_{r \to \infty} \left[ H_1(cs) \right] = \lim_{r \to \infty} \left[ \frac{1}{2^2} \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \left(\frac{\Omega}{2}\right)^{-8} s^\frac{1}{2} \log\left(\frac{\Omega}{2}\right) \right] = \frac{7 \cdot 5 \cdot 3}{2^2 \sqrt{\pi}} \left(\frac{\Omega}{2}\right)^{-8} \log\left(\frac{\Omega}{2}\right).
\]

\[
\lim_{r \to \infty} \left[ H_2(cs) \right] = \lim_{r \to \infty} \left[ \frac{1}{2^2} \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \left(\frac{\Omega}{2}\right)^{-8} s^\frac{1}{2} \log\left(\frac{\Omega}{2}\right) \right] = -\frac{1}{2^2 \sqrt{\pi}} \left(\frac{\Omega}{2}\right)^{-8} \log\left(\frac{\Omega}{2}\right).
\]

**Jong-Seon No** received the B.S. and M.S.E.E. degrees in Electronics Engineering from Seoul National University, Seoul, Korea, in 1981 and 1984, respectively, and the Ph.D. degree in Electrical Engineering from the University of Southern California, Los Angeles, in 1988. He was a Senior MTS with Hughes Network Systems, Germantown, MD, from February 1988 to July 1990. He was an Associate Professor with the Department of Electronic Engineering, Konkuk University, Seoul, from September 1990 to July 1999. He joined the faculty of the Department of Electrical Engineering and Computer Science, Seoul National University, in August 1999, where he is currently a Professor. His research interests include error-correcting codes, sequences, cryptography, space-time codes, LDPC codes, and wireless communication systems.

**Habong Chung** received the B.S. degree from Seoul National University, Seoul, in 1981 and the M.S. and the Ph.D. degrees from the University of Southern California, Los Angeles, in 1985 and 1988, respectively. From 1988 to 1991, he was an Assistant Professor in the Department of Electrical and Computer Engineering, the State University of New York at Buffalo. Since 1991, he has been with the School of Electronic and Electrical Engineering, Hongik University, Seoul, where he is a Professor. His research interests include coding theory, combinatorics, and sequence design.