A New Low-Complexity PTS Scheme Based on Successive Local Search Using Sequences

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***Abstract*—A new partial transmit sequence (PTS) scheme with low computational complexity is proposed, where two search steps are taken to find a subset of phase rotating vectors showing good peak-to-average power ratio (PAPR) reduction performance. In the first step, sequences with low correlation are used as phase rotating vectors for PTS scheme, which are called the initial phase vectors. In the second step, local search is performed based on the initial phase vectors to find additional phase rotating vectors which show good PAPR reduction performance. Numerical analysis shows that the proposed PTS scheme achieves better PAPR reduction performance with significantly reduced computational complexity than the existing low–complexity PTS schemes.**

***Index Terms*—Kasami sequences, OFDM, PAPR, PTS, quaternary sequences.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) has received considerable attention for its bandwidth efficiency and robustness against frequency-selective fading channels. An inverse fast Fourier transform (IFFT) used for the baseband modulation simplifies the transceiver design and provides an efficient hardware implementation. Thus, it has been employed in various broadband communication systems such as IEEE 802.11 wireless local area networks. However, one major drawback of the OFDM system is its high peak-to-average power ratio (PAPR) of the transmitted signal in time domain, which brings on the OFDM signal distortion in the nonlinear region of high power amplifier (HPA).

In order to alleviate the PAPR problem of OFDM signals, many PAPR reduction techniques have been proposed [1]. Among them, the partial transmit sequence (PTS) scheme can effectively reduce the PAPR of OFDM signals without distorting them, but it requires an exhaustive search over all the phase rotating vectors, which results in large computational complexity [2].

In this paper, a new low-complexity PTS scheme taking two search steps is proposed, which finds a good subset of phase rotating vectors. Through numerical analysis, the PAPR reduction performance and the computational complexity of the proposed PTS scheme are compared with those of the existing low-complexity PTS schemes.

Manuscript received May 15, 2012. The associate editor coordinating the review of this letter and approving it for publication was G. Ginis.

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2012-0000186).

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Digital Object Identifier 10.1109/LCOMM.2012.071612.121055

II. System Model and Conventional PTS Scheme

Let $\mathbf{X}$ be an input symbol sequence of length N in OFDM system, that is, $\mathbf{X} = [X_0, X_1, \cdots, X_{N-1}]^T$. An OFDM signal for $\mathbf{X}$ as a sampled complex baseband signal is obtained by applying IFFT to $\mathbf{X}$ as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N},\ 0 \le n \le N-1 \quad (1)$$

where $X_k, k = 0, 1, \cdots, N-1$, are input symbols modulated by MPSK or QAM. The PAPR of an OFDM signal $\mathbf{x} = [x_0, x_1, \cdots, x_{N-1}]^T$ is defined as

$$PAPR = \frac{\max_{0\le n\le N-1} |x_n|^2}{E[|\mathbf{x}|^2]} \quad (2)$$

where $E[\cdot]$ denotes the expectation.

In the conventional PTS scheme, an input symbol sequence $\mathbf{X}$ is partitioned into M disjoint symbol subblocks $\mathbf{X}_m = [X_{m,0}, X_{m,1}, \cdots, X_{m,N-1}]^T, 0 \le m \le M-1$, such that $\mathbf{X} = \sum_{m=0}^{M-1} \mathbf{X}_m$. The time domain signal vector $\mathbf{x}_m = [x_{m,0}, x_{m,1}, \cdots, x_{m,N-1}]^T$ is generated by applying IFFT to the symbol subblock $\mathbf{X}_m$. Next, each $\mathbf{x}_m$ is independently rotated by multiplying phase rotating factor $b_m^i \in \{e^{j2\pi l/W} | l = 0, 1, \cdots, W-1\}$, where W is the number of phase rotating factors. The phase rotating factors b_m^i constitute a phase rotating vector $\mathbf{b}^i = [b_0^i, b_1^i, \cdots, b_{M-1}^i]$. Then the i-th alternative OFDM signal sequence $\mathbf{x}^{(i)}$ is generated by

$$\mathbf{x}^{(i)} = \sum_{m=0}^{M-1} b_m^i \mathbf{x}_m, i = 0, 1, \cdots, U-1 \quad (3)$$

where U is the number of alternative OFDM signal sequences.

The objective of the conventional PTS scheme is to search for the optimal phase rotating vector that yields the alternative OFDM signal sequence with the minimum PAPR. Finally, the alternative OFDM signal sequence with the minimum PAPR is transmitted with the side information on the index of the optimal phase rotating vector. Note that the total number of alternative OFDM signal sequences in the conventional PTS scheme is $U = W^{M-1}$, where the first phase rotating factor can be fixed without any performance loss.

III. A New PTS Scheme Based on Sequences with Good Correlation

In this section, a new PTS scheme with two search steps is proposed, where the initial phase vectors are generated by using sequences with good correlation property in the first step and then additional phase rotating vectors are generated by changing one symbol of the initial phase vectors in the second step.

A. First Step : Initial Phase Vectors

Kasami sequences [3] and quaternary sequences of Family A [4] are considered as initial phase vectors for the proposed PTS scheme. Kasami sequences are important binary sequences having very low cross-correlation values, which are generated by taking the modulo-2 sum of binary m-sequence and its decimated sequences [3]. There are two different sets of Kasami sequences, a large set and a small set. In this paper, we use a small set of Kasami sequences having an optimal correlation property.

To generate a small set of Kasami sequences, we begin with an m-sequence μ of period $2^r - 1$ for even r and a shorter-period sequence $\mu^{'}$ obtained by decimating μ by $2^{r/2} + 1$. Note that the resulting sequence $\mu^{'}$ is an m-sequence of period $2^{r/2}-1$. Then a small set of Kasami sequences is generated by taking the modulo-2 sum of μ and all the cyclically shifted sequences of $\mu^{'}$, which results in $2^{r/2}$ binary sequences of period $2^r - 1$. The total number of Kasami sequences and their all cyclically shifted sequences is $2^{r/2}(2^r - 1)$. Thus, the total number N_K of phase rotating vectors selected from them should satisfy

$$N_K \leq 2^{r/2}(2^r - 1). \tag{4}$$

These N_K binary sequences are used as initial phase vectors for the proposed PTS scheme, where the alphabet size for the phase rotating factors is $W = 2$.

A family of quaternary sequences over Z_4 with optimal correlation property, called Family A, has been proposed in [4], which consists of $2^r + 1$ quaternary sequences of period $2^r - 1$. Clearly, the total number of quaternary sequences of Family A and their all cyclically shifted sequences is $(2^r + 1)(2^r - 1)$. Then, the total number N_Q of initial phase vectors selected from them should satisfy

$$N_Q \leq (2^r + 1)(2^r - 1). \tag{5}$$

For example, the Kasami and quaternary sequences of period 15 can be used as the initial phase vectors for the proposed PTS scheme with $M = 16$, where the phase rotating factor b_0^i for the first subblock is fixed to one. In this case, the maximum number of binary initial phase vectors of length 15 is $N_K = 60$ from (4), and the maximum number of quaternary initial phase vectors of length 15 is $N_Q = 225$ from (5). However, it is not guaranteed to find a good solution for the proposed PTS scheme only by using these initial phase vectors. Therefore, in the second step, one symbol of each initial phase vector is further changed to generate additional phase rotating vectors. Note that, by using sequences with low correlation as initial phase vectors, the search in the second step becomes more efficient.

B. Second Step : Local Search

Suppose that P_0 initial phase vectors are generated in the first step, where $P_0 = N_K$ or N_Q. Then P_1 vectors giving the smallest PAPRs are selected from these P_0 initial phase vectors, $0 < P_1 \leq P_0$, which are used to generate additional phase rotating vectors by changing one symbol from each of them, called *local search*.

TABLE I
P_m VALUES FOR 900 PHASE ROTATING VECTORS IN THE PROPOSED PTS SCHEME USING KASAMI SEQUENCES WHEN $M = 16$

P_0	$P_1(W-1)$	$P_2(W-1)$	$P_3(W-1)$
60	60×1	60×1	60×1
$P_4(W-1)$	$P_5(W-1)$	$P_6(W-1)$	$P_7(W-1)$
60×1	60×1	60×1	60×1
$P_8(W-1)$	$P_9(W-1)$	$P_{10}(W-1)$	$P_{11}(W-1)$
60×1	60×1	60×1	60×1
$P_{12}(W-1)$	$P_{13}(W-1)$	$P_{14}(W-1)$	$P_{15}(W-1)$
60×1	60×1	30×1	30×1

This local search will be explained by using an example. Assume that the phase rotating factor for the second subblock of the alternative OFDM signal sequence $\mathbf{x}^{(i)}$ in (3) is changed from b_1^i to $b_1^{i'}$, where $b_1^{i'}$ can take any phase rotating factor except b_1^i. Then the additional alternative OFDM signal sequence $\mathbf{x}^{(i')}$ can be easily obtained as

$$\mathbf{x}^{(i')} = \mathbf{x}^{(i)} + (b_1^{i'} - b_1^i)\mathbf{x}_1 \tag{6}$$

without summing all $\mathbf{x}_m$'s weighted by a new phase rotating vector $\mathbf{b}^{i'}$ as in (3). Compared with (3), the computational complexity to obtain additional alternative OFDM signal sequences in (6) can be substantially reduced. The phase rotating factor for the second subblock of other alternative OFDM signal sequence can be changed in the same fashion to generate additional alternative OFDM signal sequences. Then, $P_1(W-1)$ additional phase rotating vectors are generated by changing the phase rotating factor for the second subblock. After calculating PAPRs of these $P_1(W-1)$ alternative OFDM signal sequences and comparing PAPRs of total $P_1 W$ alternative OFDM signal sequences, we can select P_2 phase rotating vectors giving the smallest PAPRs.

Similar to the second block case, $P_2(W-1)$ additional phase rotating vectors are generated from these P_2 phase rotating vectors by changing the phase rotating factor for the third subblock, and the same comparison and selection are performed. This procedure continues up to the last subblock and the total number T of phase rotating vectors in the proposed PTS scheme becomes

$$T = P_0 + (W-1)\sum_{m=1}^{M-1} P_m. \tag{7}$$

Extensive simulation can be performed to find P_m for good PAPR reduction performance. As an example, for the case of $M = 16$, the number $P_m(W-1)$ of additional phase rotating vectors generated by local search for each subblock is listed in Table I, where a small set of Kasami sequences of length 15 and local search are used to select $T = 900$ phase rotating vectors out of $2^{15} = 32768$ binary vectors.

IV. COMPUTATIONAL COMPLEXITY AND SIMULATION RESULTS

A. Comparison of Computational Complexity

The computational complexity of PTS scheme is determined by the following three parts: a) M IFFTs for M subblocks; b) generation of U alternative OFDM signal sequences; c) computation and comparison of PAPRs of U alternative OFDM signal sequences. In general, when the number M

TABLE II
COMPARISON OF COMPUTATIONAL COMPLEXITY OF THE PROPOSED PTS SCHEME AND OTHER PTS SCHEMES (PERCENTAGE IS COMPARED FOR $M = 16$, $L = 4$, $W = 2$, $N = 256$, $A = 16253$ AND $T = 900$)

PTS Schemes	No. of Complex Multiplications	Percentage
Conventional PTS	$(M-1)LNW^{M-1}$	100%
Optimal Search	$(M-1)LNA$	49.6%
Parallel TS–PTS	$(M-1)LNT$	2.75%
ABC–PTS	$(M-1)LNT$	2.75%
Proposed PTS	$(M-1)LNP_0$ $+LN(W-1)\sum_{m=1}^{M-1} P_m$	0.42%

of subblocks is fixed, the computational complexity for the part a) is also fixed, and the part b) is mainly considered for reducing the computational complexity of the PTS scheme, while the complexity of part c) is negligible.

In order to reduce the computational complexity, an optimal search has been proposed in [5], where the computational complexity is reduced by restricting searching among the alternative OFDM signal sequences inside a sphere by using sphere decoding algorithm. Recently, combinatorial optimization algorithms including artificial bee colony algorithm (ABC–PTS) [6] and parallel tabu search algorithm (parallel TS–PTS) [7] have been used to efficiently search a good subset of phase rotating vectors for the PTS scheme to further reduce the complexity of the part b).

Table II compares the computational complexity of the conventional PTS, optimal search, parallel TS–PTS, ABC–PTS, and the proposed PTS scheme for $M = 16$, $L = 4$, $W = 2$, $N = 256$, and $T = 900$, where L is the oversampling factor. Since the complexity due to the complex additions shows the same tendency, only the complex multiplications for generating alternative OFDM signal sequences in the part b) are considered in Table II.

The optimal search algorithm in [8] searchs the alternative OFDM signal sequences inside a sphere corresponding to $\gamma^2 = 6.8$, which results in generating $A = 16253$ alternative OFDM signal sequences on average. Note that while the conventional PTS and optimal search generate $W^{M-1} = 2^{15} = 32768$ and $A = 16253$ alternative OFDM signal sequences, respectively, the other low–complexity PTS schemes generate $T = 900$ alternative OFDM signal sequences. Table II shows that, compared with the number of complex multiplications required by the conventional PTS, the optimal search requires 49.6% of complexity, and each of parallel TS–PTS and ABC–PTS requires 2.75% of complexity, whereas the proposed PTS scheme requires only 0.42% of complexity by using P_m in Table I. Clearly, the proposed PTS scheme shows the lowest computational complexity among other low–complexity PTS schemes.

In the next subsection, it will be shown that the proposed PTS scheme can give almost the same PAPR reduction performance as the conventional PTS scheme.

B. Simulation Results

Fig. 1 compares the PAPR reduction performance of the conventional PTS scheme, the proposed PTS scheme with

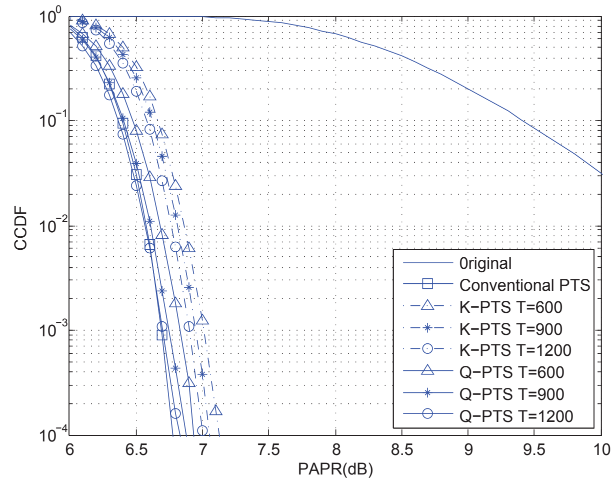

Fig. 1. Comparison of PAPR reduction performance of the conventional PTS and the proposed PTS scheme using various sequences with $M = 16$, $L = 4$, $N = 256$, and 16QAM.

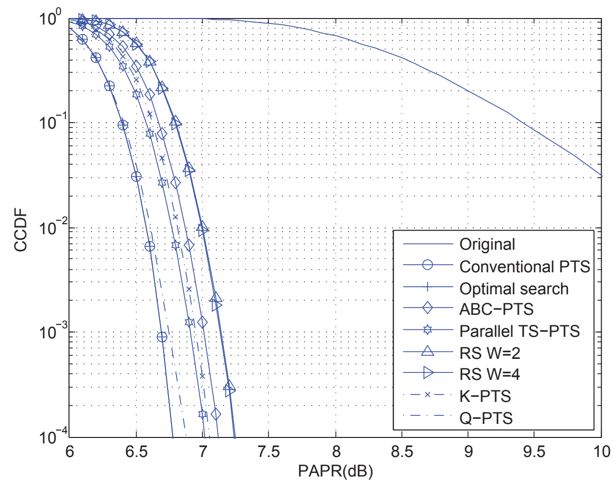

Fig. 2. Comparison of PAPR reduction performance of various PTS schemes with $M = 16$, $L = 4$, $T = 900$, $N = 256$, and 16QAM.

Kasami sequences (K–PTS), and the proposed PTS scheme with quaternary sequences of Family A (Q–PTS) for $M = 16$, $L = 4$, $N = 256$, and 16QAM. Fig. 1 shows that Q–PTS can outperform K–PTS. Note that Q–PTS with $T = 1200$ shows almost the same PAPR reduction performance as the conventional PTS with $W = 2$ and $U = 32768$.

Fig. 2 compares the PAPR reduction performance of the conventional PTS, optimal search, ABC-PTS, and parallel TS-PTS for $W = 2$, random search (RS) for $W = 2$ and 4, and the proposed K–PTS and Q–PTS. An OFDM system with $M = 16$, $L = 4$, $N = 256$, and 16QAM is considered. For optimal search, $A = 16253$ alternative OFDM signal sequences are generated and for other low–complexity PTS schemes, $T = 900$ alternative OFDM signal sequences are generated. It can be seen that the PAPRs of the random search at $CCDF = 10^{-3}$ for $W = 2$ and $W = 4$ are the same 7.15dB. Meanwhile, the PAPRs of ABC–PTS, parallel TS–PTS, K–PTS, and Q–PTS are 7.02dB, 6.85dB, 6.9dB, and 6.72dB at $CCDF = 10^{-3}$, respectively. As expected, the optimal search shows identical PAPR reduction performance with the conventional PTS scheme. Compared with other PTS schemes, the proposed PTS scheme shows similar or better

PAPR reduction performance with much lower computational complexity as given in Table II.

V. Conclusions

In this paper, a new two-step search algorithm for PTS scheme is proposed to reduce the computational complexity. In the first step, sequences with good correlation property such as Kasami and quaternary sequences are used as the initial phase vectors. In the second step, by using the initial phase vectors, local search is performed for further searching the phase rotating vectors with very low computational complexity. Numerical analysis shows that the proposed PTS scheme can achieve almost the same PAPR reduction performance as the conventional PTS scheme with much lower computational complexity than other low–complexity PTS schemes.

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