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New stopping criteria for iterative decoding of LDPC codes in H-ARQ systems[‡]

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SUMMARY

Under severely unreliable channel, decoding of error-correcting codes frequently fails, which requires a lot of computational complexity, especially, in the iterative decoding algorithm. In hybrid automatic repeat request systems, most of computation power is wasted on failed decoding if a codeword is retransmitted many times. Therefore, early stopping of iterative decoding needs to be adopted. In this paper, we propose a new stopping algorithm of iterative belief propagation decoding for low-density parity-check codes, which is effective on both high and low signal-to-noise ratio ranges and scalable to variable code rate and length. The proposed stopping algorithm combines several good stopping criteria. Each criterion is extremely simple and will not be a burden to the overall system. With the proposed stopping algorithm, it is shown via numerical analysis that the decoding complexity of hybrid automatic repeat request system with adaptive modulation and coding scheme can be fairly reduced. Copyright © 2012 John Wiley & Sons, Ltd.

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KEY WORDS: hybrid automatic repeat request (H-ARQ); iterative decoding; low-density parity-check (LDPC) codes; stopping criteria

1. INTRODUCTION

The invention of turbo codes [1] ignited the explosive growth of the research on finding capacity-achieving error-correcting codes with decoding algorithm of moderate complexity [2–4]. Accordingly, low-density parity-check (LDPC) codes were rediscovered and shown to have the performance close to the Shannon limit with the iterative belief propagation (BP) decoding. As in the case of turbo codes, the essence of these new error-correcting codes is their low-complexity decoding, that is, a finite number of repetitions of a linear-complexity decoding. One of the main advantages of LDPC codes over turbo codes is the implementation feasibility of extremely fast decoder demanded in current and future high speed wireless communication systems. Even though the decoding complexity of these powerful error-correcting codes came down to the practical level [5, 6], the iterative decoder still requires massive computation so that it is one of the principal power consumers in the communication receivers.

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In order to alleviate the computational burden of iterative decoding, several stopping criteria have been proposed for turbo codes [7]. It turned out that those stopping criteria work very effectively. With regard to this, LDPC codes have an advantage of having an inherent method to stop iterative decoding, which stops the decoding if the syndrome of the tentative hard-decision codeword bits at some iteration is zero. This conventional syndrome stopping is optimal in the sense of detecting successful decoding.

At high signal-to-noise ratio (SNR) region, the average number of iterations in LDPC decoding with the syndrome stopping is substantially small because the iterative decoding is seldom performed up to the predetermined maximum number of iterations. Therefore, the conventional syndrome stopping is good enough for high SNR region.

Typically, low SNR region where the decoder may frequently hit the maximum number of iterations has not been of the concern. However, in this paper, we will mostly concern about the low SNR case that hybrid automatic repeat request (H-ARQ), which is an advanced error control scheme guaranteeing the reliable communication by adapting the channel condition, is generally adopted. [8]. In H-ARQ systems, when codewords are not successfully decoded at the receiver, NAK message is fed back to the transmitter for requesting the retransmission. Frequent decoding failures imply that the decoder reaches the maximum number of iterations many times, which causes high decoding complexity. It is a bit ironical to consume most of power on decoding failures, but it is hard to detect bad codewords prior to decoding because of the imperfect knowledge on the received signal characterization of decoding failure.

Instead, we want to know if the received codeword is good (enough to be successfully decoded) or not in the early stage of the decoding procedure. An early stopping algorithm, which is effective even at low SNR under additive white Gaussian noise (AWGN) channel was proposed for the iterative decoding algorithm assuming perfect SNR estimation [9]. However, this stopping algorithm can cause the performance degradation in fast-varying fading channel, where the assumptions may not hold. A stopping criterion based on the convergence of mean magnitude [10] may be effective for the H-ARQ system under fast fading environment. Also, a stopping algorithm with simpler criteria [11] was proposed for H-ARQ systems. However, the explicit explanation on the benefit of the complexity reduction of H-ARQ systems at low SNR has not been addressed so far.

In this paper, we propose a new stopping algorithm of iterative (BP) decoding for LDPC codes, which are effective on both high and low SNR ranges and scalable to variable code rate and length. By combining several stopping criteria including the conventional syndrome stopping, a new stopping algorithm is proposed. Note that each criterion is extremely simple and shall not be a burden to the overall system. It is shown that the proposed algorithm dramatically reduces the decoding complexity in the H-ARQ system with adaptive modulation and coding (AMC) scheme [13]. It is also shown that the proposed algorithm has lower computational complexity than other stopping algorithms in [10] and [11] under the same throughput. Needless to say, the proposed algorithm is well compatible with other types of message passing decoding [14] of LDPC codes.

The rest of the paper is organized as follows: Section 2 is devoted to some important observations on the behavior of iterative BP decoder, especially, at low SNR region and describes the proposed algorithm. Some numerical results are presented in Section 3 to confirm the validity of the proposed algorithm, and the conclusions are given in Section 4.

2. NEW STOPPING CRITERIA

2.1. Stagnancy check

From the observation that the number of unsatisfied check nodes in almost all erroneously decoded frames after iterative decoding converges to a fixed value after some iterations as in Figure 1, a stopping criterion to detect a stagnant state of iterative BP decoder can be obtained as follows. Let $\mathbf{s}_i$ denote the syndrome of the tentative hard-decision codeword bits after the i-th iteration of decoding. Then, the weight of $\mathbf{s}_i$, $w(\mathbf{s}_i)$, equals the number of unsatisfied check nodes after the i-th iteration. Let $N_i^{\max}$ and $N_i^{\min}$ be the maximum and the minimum values of $w(\mathbf{s}_i)$ during the past

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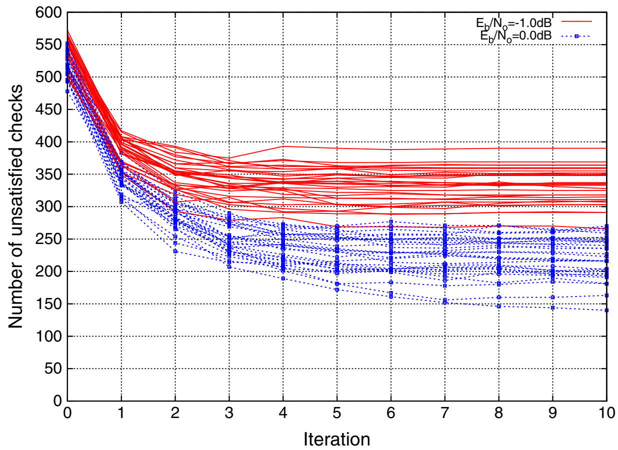

Figure 1. Iteration versus $w(\mathbf{s}_i)$ of 30 erroneously decoded frames at $E_b/N_o = -1.0$ (solid red) and 0.0 dB (dashed blue with square), respectively (IEEE802.16e LDPC code [12] with $n = 2304$, $R = 1/2$).

I_{stag} iterations from the i-th iteration, respectively. Then the decoding of a frame is considered to fall into *stagnancy* at the i-th iteration if

$$N_i^{\max} - N_i^{\min} \leqslant \delta_{\text{stag}}$$

is satisfied, where I_{stag} and δ_{stag} are thresholds on the number of iterations and the number of unsatisfied check nodes, respectively, to check the stagnancy of the decoding. As I_{stag} increases and δ_{stag} decreases, the number of frames, whose decoding has stagnancy, becomes small for a fixed SNR. Accordingly, the first proposed stopping criterion is the stagnancy check, that is, the iterative BP decoding stops as soon as the decoding of a frame turns out to have stagnancy.

Restricted to relatively low SNR region, $w(\mathbf{s}_i)$'s of erroneously decoded frames converge to some non-zero value rather than oscillate in most cases. Thus, even with a tighter condition on δ_{stag} (say, $\delta_{\text{stag}} = 0$), the proposed stopping criterion enables the accurate prediction of decoding failures. From now on, we consider only the case of $\delta_{\text{stag}} = 0$ in this paper. This implies that we just check whether $N_i^{\max} = N_i^{\min}$ holds or not throughout the I_{stag} iterations. For other value of δ_{stag}, similar approach can be applied.

2.2. *Early syndrome weight check*

In low SNR region, it is observed that the erroneously decoded frames tend to suffer from stronger noise than the successfully decoded frames. Also, we can see in Figure 1 that the lower the SNR, that is, the stronger the noise, the higher the average of $w(\mathbf{s}_i)$. Because this phenomenon occurs at all iterations, it is possible to distinguish erroneously decoded frames from successfully decoded frames even at very early iterations. These observations bring a stopping criterion, which can make dramatic reduction on the decoding complexity especially at low SNR region where most of the frames are erroneously decoded.

Suppose that the syndrome check is performed at the I_{chk}-th iteration for some nonnegative integer I_{chk}. If $w(\mathbf{s}_i)$ is smaller than δ_{th}, the frame will be regarded as healthy enough to continue the decoding, where δ_{th} is a threshold on the number of unsatisfied check nodes. Otherwise, the decoding will stop. This criterion is called "early syndrome weight check" and can be used to sort out the frames at early stage, whose decoding seems to eventually fail.

For the stopping algorithm with "early syndrome weight check," it is important to determine the proper δ_{th} in advance. However, theoretical derivation of δ_{th} for each SNR is difficult because of the nonlinearity of the check node process. Instead, for a given channel and SNR range, the optimal δ_{th} can be obtained from massive simulation. Note that we observed a small change of δ_{th} that does not make a noticeable influence on the overall performance and hence this insensitivity on δ_{th} is an advantage of "early syndrome weight check."

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2.3. A new stopping algorithm

The "stagnancy check" stopping can be applied only after the first $I_{\text{stag}} - 1$ iterations while we want to distinguish very bad frames at an early stage of decoding to reduce the decoding complexity. On the other hand, the "early syndrome weight check" stopping can be applied in the first few iterations and greatly decreases the unnecessary computations, which cannot be effectively removed by "stagnancy check" algorithm (or other existing stopping algorithms).

Because both of the preceding proposed stopping criteria exploit $w(\mathbf{s}_i)$, the following stopping algorithm combining these criteria, which barely introduces additional complexity, is proposed.

- Step 1: $i \leftarrow 0$
- Step 2: If $w(\mathbf{s}_i) = 0$, stop decoding
- Step 3: $i \leftarrow i + 1$
- Step 4: Run the i-th iteration of iterative BP decoding
- Step 5: If $w(\mathbf{s}_i) = 0$, stop decoding
- Step 6: If $i \geqslant$ the maximum number of iterations $I_{\max}$, stop decoding
- Step 7: If $i = I_{\text{chk}}$ and $w(\mathbf{s}_i) \geqslant \delta_{\text{th}}$, stop decoding
- Step 8: If $i \geqslant I_{\text{stag}} - 1$, update $N_i^{\max}$ and $N_i^{\min}$ in the duration $[i - I_{\text{stag}} + 1, i]$
- Step 9: If $i \geqslant I_{\text{stag}} - 1$ and $N_i^{\max} - N_i^{\min} \leqslant \delta_{\text{stag}}$, stop decoding
- Step 10: Go to Step 3

Basically, the conventional syndrome stopping is used in Steps 2 and 5. Also, "early syndrome weight check" and "stagnancy check" are used in Steps 7 and 9, respectively. Note that the proposed algorithm is well compatible with other types of message passing decodings because it does not use any decoder-dependent parameter but only uses the number of unsatisfied check nodes. The flowchart of the proposed stopping algorithm is illustrated in Figure 2.

In the proposed stopping algorithm, "stagnancy check" and "early syndrome weight check" are independently used whereas the stopping algorithm in [11] checks the number of satisfied check nodes of only the frames, which do not pass "stagnancy check"-like criterion. This dependency makes the syndrome weight check not adopted at early stage, which results in less reduction of computational complexity.

3. NUMERICAL RESULTS

In this section, numerical results are shown through computer simulations to assess the effectiveness of the new stopping algorithm. First, the system with no H-ARQ protocol is simulated and then the simplified version of AMC scheme in IEEE802.16e [12] is applied to the simulation of H-ARQ systems. For the entire simulations, word error rate (WER) and the average number of iterations are provided to illustrate the error-correcting performance and the computational complexity of decoding, respectively. Because the proposed stopping criteria exploit the syndrome weight like the conventional syndrome stopping, extra computation and memory space are negligible so that it is reasonable to consider only the average number of iterations as a measure of the computational complexity. For LDPC decoding, the floating-point sum-product algorithm is used, and the maximum number of iterations is set to 50, which is generally good enough to show the convergence of decoding.

3.1. Additive white Gaussian noise channel

Although our main interest is H-ARQ systems over the fading channel, verification of the proposed stopping algorithm over the AWGN channel is also necessary. The rate-1/2 LDPC code with code length $n = 2304$ in IEEE802.16e standard is used for the simulation.

As shown in Figures 3 and 4, via massive simulations, we can find adequate parameters for the proposed stopping algorithm to avoid unnecessary iterations of decoding with negligible performance degradation. We can see that the stopping algorithm using only the parameter I_{stag} reduces the average number of iterations without significant loss of performance. However, because the stop

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of decoding using only "stagnancy check" is determined after at least I_{stag} iterations, the resulting average number of iterations is still large as shown in Figure 4. Also, "early syndrome weight check" makes a big drop of the average number of iterations at very low SNR region whereas "stagnancy

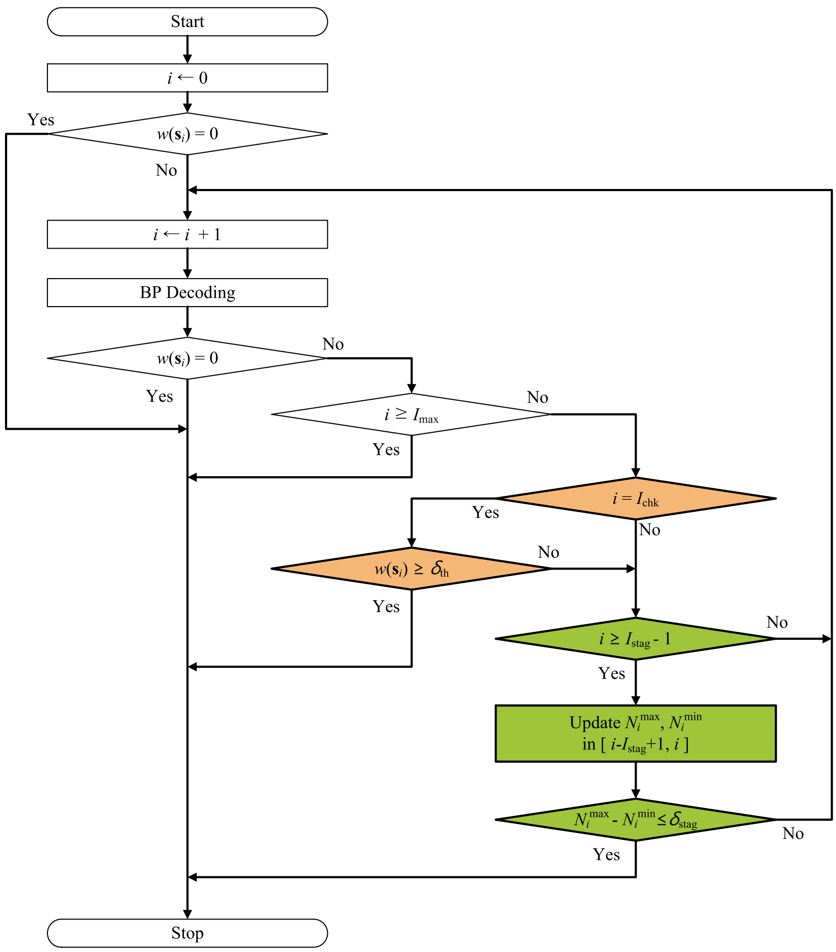

Figure 2. Flowchart of the proposed stopping algorithm.

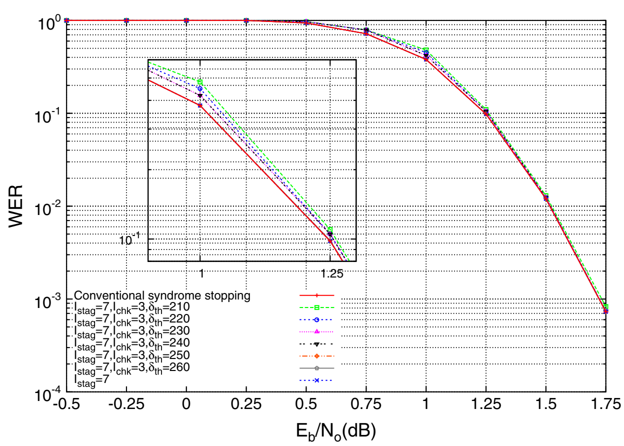

Figure 3. WER performance of an LDPC code (IEEE802.16e, $n = 2304$, $R = 1/2$) with the proposed stopping algorithm in AWGN channel.

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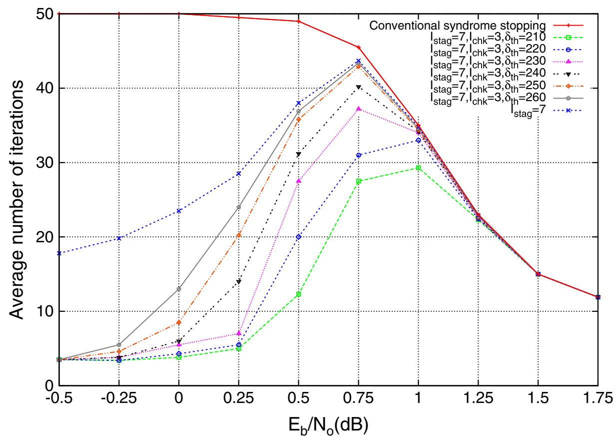

Figure 4. Average number of iterations of iterative BP decoding for an LDPC code (IEEE802.16e, $n = 2304$, $R = 1/2$) with the proposed stopping algorithm in AWGN channel.

check" lowers the average number of iterations over low-to-mid range of SNR. Apparently, as δ_{th} increases, the average number of iterations increases, but WER is improved.

3.2. *Fading channel*

Before verifying the performance of the proposed stopping algorithm in H-ARQ systems with AMC scheme, simulations are performed to show the effectiveness of the proposed algorithm for the single transmission on fading channel. The three-path fading channel [15] with equi-power paths is used and 288 subcarriers in orthogonal frequency division multiplexing system are allocated as resource for transmission. LDPC codes of variable length in IEEE802.16e standard shown in Table I are used.

As an example, WER performance and the average number of iterations of a rate-$1/2$ LDPC code with $n = 1728$ are shown all together in Figure 5. We can see that the shape of the plot is similar to the AWGN case, and the proposed stopping algorithm also works well in the fading channel. For $I_{stag} = 5$ and $\delta_{th} = 389$, small I_{chk} gives rise to the degraded WER performance and the low computational complexity. Note that every 6 bits of a codeword are allocated to one subcarrier and experience the same channel.

3.3. *H-ARQ system with AMC scheme*

To verify the effectiveness of the proposed stopping algorithm in H-ARQ systems, Type-I H-ARQ system [8, 16] with maximum four transmissions per codeword and the AMC scheme in Table I are used. Applying this setting to the system only using the conventional syndrome stopping criterion,

Table I. AMC scheme in IEEE802.16e standard and all δ_{th}'s used in simulation.

	Code			δ_{th}				
Modulation	Length	Rate	# of check nodes	10%	15%	20%	25%	30%
QPSK	576	1/2	288	28	43	57	72	86
		3/4	144	14	21	28	36	43
16QAM	1152	1/2	576	57	86	115	144	172
		3/4	288	28	43	57	72	86
64QAM	1728	1/2	864	86	129	172	216	259
		2/3	576	57	86	115	144	172
		3/4	432	43	64	86	108	129
		5/6	288	28	43	57	72	86

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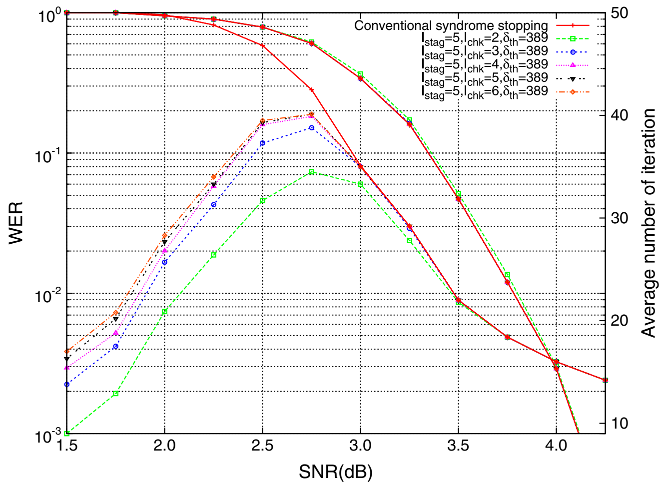

Figure 5. WER performance and the average number of iterations of an LDPC code (IEEE802.16e, $n = 1728$, $R = 1/2$) with the proposed stopping algorithm in fading channel.

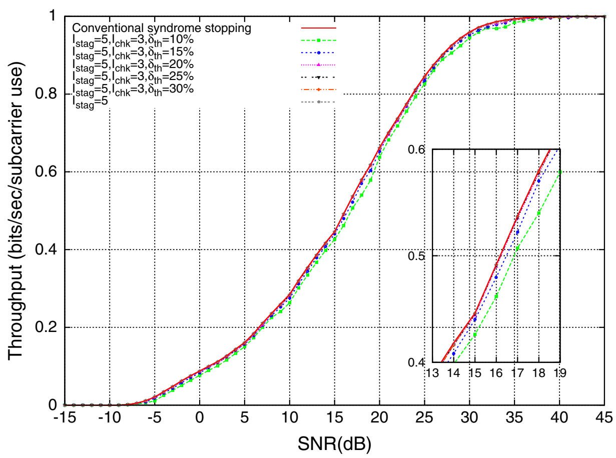

Figure 6. Throughput performance of the H-ARQ system with the proposed stopping algorithm.

solid red lines in Figures 6 and 7 are obtained as the overall results. In Figure 6, the throughput is used as the performance measure, which means the transmission rate per channel use. Note that in Figure 7, as SNR increases, some spikes appear at the SNR points where the modulation and coding scheme changes abruptly according to the AMC scheme.

For each LDPC code in the H-ARQ system with the proposed stopping algorithm, δ_{th} is set according to various ratio over the number of check nodes in the LDPC codes as shown in Table I. For simplicity and scalability, eight LDPC codes in Table I use the same ratio for δ_{th}, not the same δ_{th} itself. Based on massive numerical experiments, I_{stag} and I_{chk} are set to 5 and 3, respectively. From Figures 6 and 7, it is observed that large reduction of the average number of iterations is achieved without noticeable performance degradation. Moreover, if small and restricted amount of performance degradation is allowed, just $3 \sim 4$ ($\simeq I_{chk}$) iterations are enough over the extremely wide range of SNR. Note that all these results are attained by using a common ratio to determine δ_{th} for different LDPC codes, which verifies the scalability of the proposed algorithms contrary to other stopping algorithms.

Compared with the stopping algorithms in [10] and [11], the average number of iterations for the proposed stopping algorithm with $I_{stag} = 5$, $I_{chk} = 3$, and $\delta_{th} = 20\%$ is smaller than those

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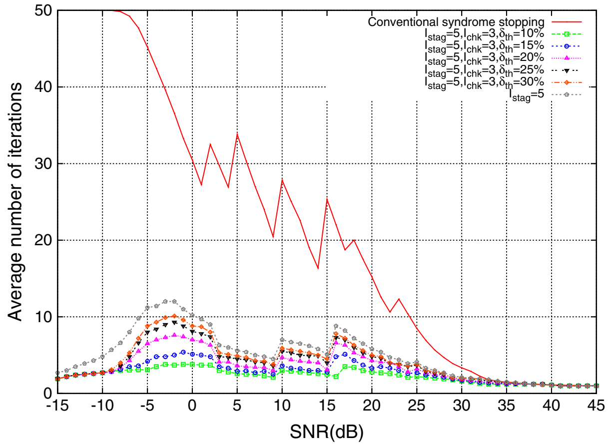

Figure 7. Average number of iterations per transmission of the H-ARQ system with the proposed stopping algorithm.

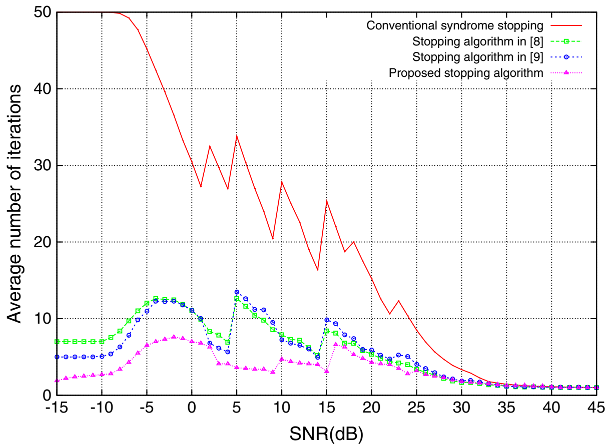

Figure 8. Average number of iterations per transmission of the H-ARQ systems for the proposed and other stopping algorithms.

of the others as shown in Figure 8. The parameters for the stopping algorithms in [10] and [11] are chosen to have almost the same throughput as our algorithm. Clearly, the proposed stopping algorithm outperforms the others over the entire SNR region, especially in low SNR region.

4. CONCLUSIONS

In this paper, we propose a new stopping algorithm for the iterative BP decoding of LDPC codes by combining "stagnancy check" and "early syndrome weight check." Its effectiveness and scalability are proved by numerical analysis. It is also shown that the decoding complexity of the H-ARQ system with AMC scheme is dramatically reduced by using the proposed stopping algorithm. We also verify that our stopping algorithm outperforms other stopping algorithms such as those in [10] and [11] with lower computational complexity. Clearly, the proposed stopping algorithm is well compatible with other types of message passing decoding algorithms such as those in [14].

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REFERENCES

1. Berrou C, Glavieux A, Thitimajshima P. Near Shannon limit error-correcting coding and decoding: turbo codes. *Proceedings of IEEE International Conference on Communications*, Geneva, Switzerland, May 1993; 1064–1070.
2. MacKay D, Neal R. Near Shannon limit performance of low density parity check codes. *Electronics Letters* 1996; **32**:1645–1646.
3. Jin H, Khandekar A, McEliece R, Irregular repeat–accumulate codes. *Proceedings of the 2nd International Symposium on Turbo Codes and Related Topics*, Brest, France, September 2000; 1–8.
4. Ping L, Huang X, Phamdo N. Zigzag codes and concatenated zigzag codes. *IEEE Transactions on Information Theory* 2001; **47**:800–807.
5. Lin C, Chou S, Ku M. Operation reduced low-density parity-check decoding algorithms for low power communication systems. *International Journal of Communication Systems* 2011. DOI: 10.1002/dac.1331.
6. Cam H, Ucan O, Odabasioglu N, Sonmez C. Performance of joint multilevel/AES-LDPCC-CPFSK schemes over wireless sensor networks. *International Journal of Communication Systems* 2009; **23**:77–90.
7. Shao R, Lin S, Fossorier M. Two simple stopping criteria for turbo decoding. *IEEE Transactions on Communications* 1999; **47**:1117–1120.
8. Lin S, Costello Jr D. *Error Control Coding: Fundamentals and Applications*. Prentice-Hall: Upper Saddle River, NJ, 2004.
9. Kienle F, Wehnm N. Low complexity stopping criterion for LDPC code decoders. *Proceedings of IEEE Vehicular Technology Conference Spring*, May 2005; 606–609.
10. Li J, You X, Li J. Early stopping for LDPC decoding: convergence of mean magnitude (CMM). *IEEE Communications Letters* 2006; **10**:667–669.
11. Shin D, Ha J, Heo K, Lee H. A stopping criterion for low-density parity-check codes. *IEICE Transactions on Communications* 2008; **E92-B**:1145–1148.
12. IEEE standard for local and metropolitan area network part 16: air interface for fixed and mobile broadband access systems. *IEEE Std.802.16-2004*, 2004.
13. Bhunia S, Misra I, Sanyal S, Kundu A. Performance study of mobile WiMAX network with changing scenarios under different modulation and coding. *International Journal of Communication Systems* 2011; **24**:1087–1104.
14. Chen J, Dholakia A, Eleftheriou E, Fossorier M, Hu X. Reduced-complexity decoding of LDPC codes. *IEEE Transactions on Communications* 2005; **53**:1288–1299.
15. Lin S, Lee T, Gardina M. Diversity protections for digital radio - Summary of ten-year experiments and studies. *IEEE Communications Magazine* 1988; **26**:51–64.
16. Zhang L, Li V, Cao Z. Non-binary type-II HARQ and its application to an adaptive system. *International Journal of Communication Systems* 2005; **18**:97–117.

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