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New interference alignment schemes with full and half-duplex relays for the quasi-static *X* channel

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Abstract: In this study, two relay-aided interference alignment (IA) schemes are proposed for the quasi-static $M \times 2$ X channel where M is the number of transmitters and two is the number of receivers, respectively. The first scheme utilises one full-duplex relay and the second one utilises two half-duplex relays. In the proposed schemes, the transmitters transmit signals in every time slot and the relays operate in the amplify-and-forward (AF) mode. By verifying the linear independence between the desired signals and the interference, it is shown that the proposed schemes can achieve $2M/(M+1)$ degrees of freedom (DoF) which is the maximum DoF for the $M \times 2$ X channel. It is also shown by using the concept of reciprocity that the proposed IA schemes are applicable to the $2 \times M$ X channel. Finally, through the numerical analysis, it is confirmed that the proposed IA schemes provide good alternatives to the previously known relay-aided IA scheme.

1 Introduction

As the number of multimedia mobile devices such as smart phones is dramatically increasing, data communication with very high spectral efficiency must be guaranteed to support various mobile multimedia services. For that purpose, multi-user interference must be mitigated and consequently many interference management schemes have been proposed [1–4].

Recently, interference alignment (IA) has attracted much attention for interference management. Especially, the IA scheme for the general K-user interference channel is proposed in [5] and for the $M \times N$ X channel, where M is the number of transmitters and N is the number of receivers, an IA scheme is also proposed in [6]. By extending the symbols of the transmitters in the time domain, they can asymptotically achieve the maximum degrees of freedom (DoF) for each channel. Also, to achieve the maximum DoF, a relay-aided approach was investigated for the three-user interference channel [7], which can achieve 3/2 DoF. However, these schemes assume that the channel varies at each symbol time. Since all the transmitters and the receivers need global channel state information, which requires a lot of feedback information, these schemes may not be practical for the time-varying channel. Therefore research on the IA in the slow fading environment has been conducted from the practical viewpoint [7–9].

To implement the IA scheme in the slow fading environment, multiple channels such as multiple carriers or multiple antennas can be used. However, such resources are sometimes limited or hard to implement. Therefore the IA scheme with time extension may be preferred to support multiple users.

Nourani *et al.* [10, 11] proposed relay-aided IA schemes for the quasi-static X channel and interference channel, respectively. They used full-duplex relays with some memories and proved that the beamforming vectors for the IA can be designed for the quasi-static fading channel environment. These schemes imply that the IA can be practically implemented by using the time extension. However, since the transmitted signal from a relay can be fed back to the relay as a self-interference in full-duplex mode, called echo, the half-duplex relays are widely used instead of the full-duplex relays.

In this paper, a new simple IA scheme with a full-duplex relay and an IA scheme with two half-duplex relays are proposed for the quasi-static X channel. Compared with the scheme in [10], the first proposed scheme reduces the hardware complexity by using only one memory and the second one uses two half-duplex relays instead of the full-duplex relay. Therefore the proposed schemes can be thought to be more practical than the scheme in [10]. By verifying that the desired and the interference signal vectors are linearly independent with each other, the maximum DoF for the proposed IA schemes is derived. Also, it is shown that the proposed IA schemes are applicable to the $2 \times M$ X channel by using the reciprocity. In fact, the main idea of this paper is based on the result in [12], however, the achievability for the perfect IA is proved more strictly than in [12] and we extend this idea to the $2 \times M$ X channel by using reciprocity. In addition, through the numerical analysis, it is confirmed that the proposed IA schemes can achieve the maximum DoF and the IA scheme with two half-duplex relays shows slightly better performance than the other IA schemes with relay.

This paper is organised as follows. In Section 2, some preliminaries for the X channel and IA scheme are explained and in Section 3, the proposed schemes and their system models are described. In Section 4, it is proved that the proposed schemes can achieve $2M/(M+1)$ DoF, which is the maximum DoF for the $M\times 2$ X channel [6]. In Section 5, it is shown that the proposed schemes can be applied to the $2\times M$ X channel. In Section 6, some numerical results are presented and the concluding remark is given in Section 7.

2 Preliminaries: *X* channel and IA

In this section, the X channel and IA scheme in [6] are described. Assuming that each node has a single antenna, the $M\times N$ X channel can be defined as a communication channel with M transmitters and N receivers for sending MN independent messages, where each message is transmitted from each transmitter to each receiver. Fig. 1 shows the 3×2 X channel, where W_{ji} denotes the message from the transmitter i to the receiver j. In general, the $M\times N$ X channel can be thought of as a channel where each transmitter has N messages to be transmitted to all the receivers and each receiver receives M messages from all the transmitters.

In [6], an IA scheme for the X channel was proposed and it was proved that the maximum DoF of the $M\times N$ X channel is $MN/(M+N-1)$. However, in general, an infinite symbol extension is required to achieve this maximum DoF but for the $\mathrm{M}\times 2$ X channel, $2M/(M+1)$ DoF can be obtained by using the $M+1$ symbol extension.

In this paper, we focus on the $M\times 2$ X channel with relays. It is well-known that the symbol extension and the design of the beamforming vectors are essential for the IA schemes, where the beamforming vectors make the interference signals properly aligned for the receivers.

The beamforming vectors for the IA can be designed by using the channel matrices as follows. First, we should design linearly independent beamforming vectors $\boldsymbol{b}_{11}$ and $\boldsymbol{b}_{21}$ for the signals from transmitter 1 to the receivers 1 and 2, respectively. Then, the other beamforming vectors $\boldsymbol{b}_{1i}$ and $\boldsymbol{b}_{2i}$ from transmitter i to the receivers 1 and 2 are designed by using the following relations [6]

$$\boldsymbol{H}_{2i}\boldsymbol{b}_{1i} = \boldsymbol{H}_{21}\boldsymbol{b}_{11}, \quad i = 2, \ldots, M \tag{1}$$

$$\boldsymbol{H}_{1i}\boldsymbol{b}_{2i} = \boldsymbol{H}_{11}\boldsymbol{b}_{21}, \quad i = 2, \ldots, M \tag{2}$$

where $\boldsymbol{b}_{ji}$ and $\boldsymbol{H}_{ji}$ denote the $(M+1)\times 1$ beamforming vector and the $(M+1)\times(M+1)$ channel matrix between the transmitter i and the receiver j, respectively. The equality in (1) and (2) means that the two vectors on the left- and right-hand sides of the equality are placed in the same direction in the vector space.

The received signal spaces at each receiver can be given as

$$\Gamma_1 = \left[\boldsymbol{H}_{11}\boldsymbol{b}_{11} \quad \boldsymbol{H}_{12}\boldsymbol{b}_{12} \quad \cdots \quad \boldsymbol{H}_{1M}\boldsymbol{b}_{1M} \quad \boldsymbol{H}_{11}\boldsymbol{b}_{21}\right] \tag{3}$$

$$\Gamma_2 = \left[\boldsymbol{H}_{21}\boldsymbol{b}_{21} \quad \boldsymbol{H}_{22}\boldsymbol{b}_{22} \quad \cdots \quad \boldsymbol{H}_{2M}\boldsymbol{b}_{2M} \quad \boldsymbol{H}_{21}\boldsymbol{b}_{11}\right] \tag{4}$$

If these matrices have the full rank, then each receiver can detect all the desired signal vectors from the M transmitters by the zero-forcing filter, which is called perfect IA in this paper. In [6], it was proved that these matrices have the full rank, which guarantees $2M/(M+1)$ DoF. Fig. 2 describes the IA scheme for the 2×2 X channel, where d_{ji} is the datastream from the transmitter i to the receiver j. By using the beamforming vectors b_{ji}, each transmitter can align its signals along the directions different from those of the interference signals so that the desired signals are linearly independent from the interference signals at each receiver. However, in [6], it is assumed that every channel is time-varying and the equivalent channel matrix has the diagonal form. Since the quasi-static X channel with relays is assumed in this paper, the analysis in [6] cannot be applied. Therefore it should be proved that Γ_1 and Γ_2 have the full rank for the proposed schemes.

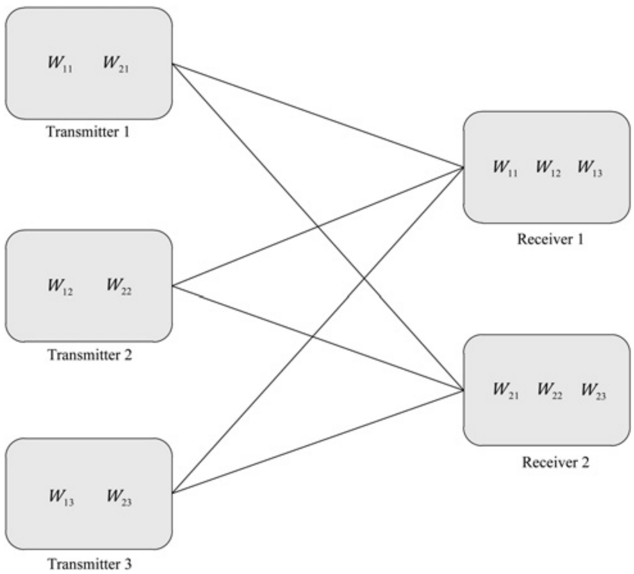

Fig. 1 *3 × 2 X channel*

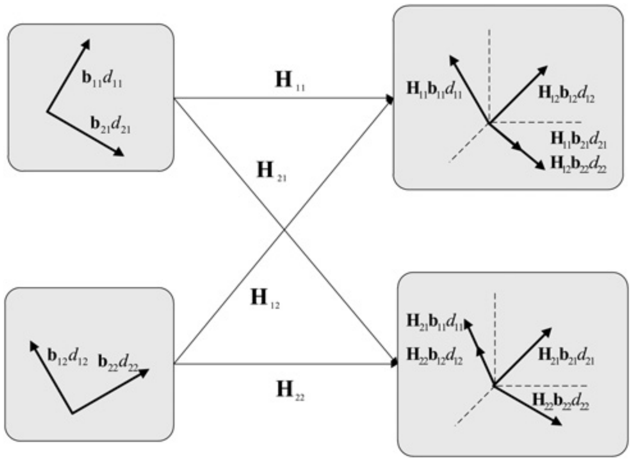

Fig. 2 *Interference alignment for the 2 × 2 X channel*

3 Proposed schemes and system models

In this section, two IA schemes with relays are proposed and their corresponding system models are given.

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3.1 Two proposed schemes

- *A simple IA scheme with a full-duplex relay:* Actually, this scheme described in Fig. 3*a* is the low-complexity version of the IA scheme with a full-duplex relay in [10], that is, the relay in the proposed scheme receives signals in every time slot and amplifies and forwards them to the receivers in the next time slot. In the IA scheme in [10], the relay stores all the received signals which are needed for symbol extension, and amplifies and adds all the received signals with different amplification gains and forwards them to the receivers. Thus, the IA scheme in [10] can have some performance gain by optimising the amplification gain for each received signal but this optimisation is complicated and requires lots of memory to store all the received signals. In the proposed IA scheme, the maximum DoF can be obtained without storing all the received signals, which will be proved in Section 4. Therefore the proposed scheme can be a good simple alternative method for the IA scheme in [10].
- *An IA scheme with two half-duplex relays:* It is hard to implement a full-duplex relay because the power of the transmit signal is much stronger than that of the received signal at the relay and thus the transmit signal can cause a strong self-interference at the relay. Therefore an IA scheme with half-duplex relays is preferred from the practical viewpoint and thus, we propose an IA scheme with two half-duplex relays.

By using two half-duplex relays as in Fig. 3*b*, we can obtain the same effect as that of the IA scheme in [10] which has to store all the received signals and forward them. If two half-duplex relays transmit and receive by turns in the amplify-and-forward (AF) mode, the received signal at each relay in each time slot contains all the previous and current received signals. If this scheme achieves the maximum DoF for the $M \times 2$ X channel, it can be considered more practical than the other schemes with a full-duplex relay. In the next section, it will be proved that the proposed IA scheme with two half-duplex relays can obtain the maximum DoF for the $M \times 2$ X channel.

3.2 System models for the proposed schemes

It is assumed that the number of transmitters is M and the number of receivers is 2. It is also assumed that each node has one antenna and each channel is quasi-static Rayleigh fading channel. All the transmitters and the receivers know all the channel state information. Fig. 3 shows the system models for the two proposed relay-aided IA schemes.

- *The system model using a full-duplex relay in Fig. 3a:* In this model, a full-duplex relay operates in the AF mode. Let $y_r(k)$ and $y_j(k)$ be the received signals at the relay and the receiver j in the time slot k, $k = 0, 1, \ldots, M$. Then, the system model is given as

$$y_r(k) = \sum_{i=1}^{M} h_{ri} x_i(k) + n_r(k) \tag{5}$$

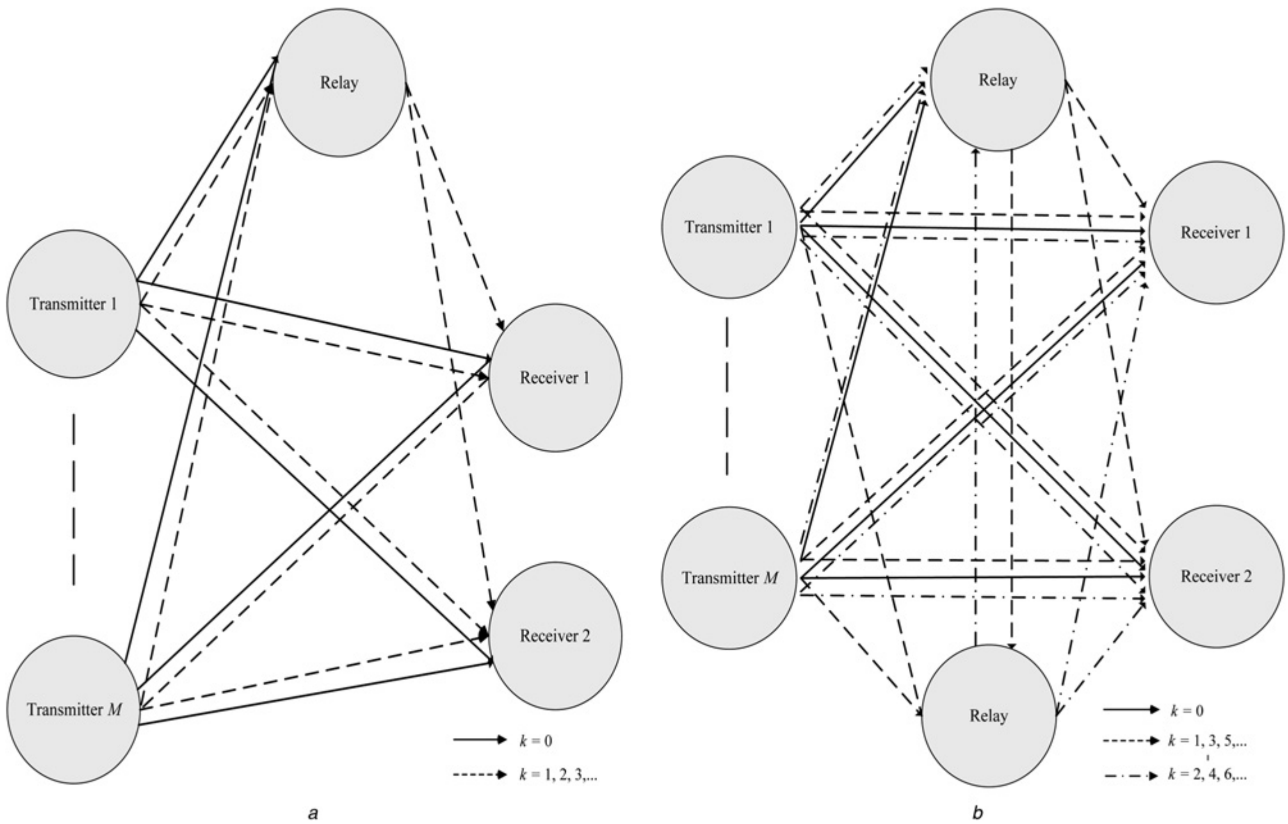

Fig. 3 *System models for the proposed schemes over the M × 2 X channel: aided by*

a Full-duplex relay and
b Two half-duplex relays

$$y_j(k) = \sum_{i=1}^{M} h_{ji}x_i(k) + u(k)h_{jr}y_r(k-1) + n_j(k),\quad j = 1, 2 \tag{6}$$

where $u(k)$ denotes the relay gain in the AF mode and $y_r(k) = 0$, $k < 0$, and h_{ri} and h_{jr} denote the channel coefficients between the transmitter i and the relay and between the relay and the receiver j, respectively. $n_r(k)$ and $n_j(k)$ are the additive complex white Gaussian noises at the relay and receiver j, respectively. $x_i(k)$ denotes the transmitted signal from the transmitter i, which is expressed as $x_i(k) = b_{1i,\ k}d_{1i} + b_{2i,\ k}d_{2i}$, where $b_{ji,k}$ is the kth element of the beamforming vector b_{ji} and d_{ji} is the desired data symbol from the transmitter i to the receiver j. For simplicity, it is assumed that h_{ri} and h_{jr} are independent continuous random variables and $n_r(k)$, and $n_j(k)$ are independent complex white Gaussian random variables with the distribution $C(0, 1)$. $C(a, b)$ denotes the complex Gaussian distribution with mean a and variance b.

This model is simpler than that in [10] because the receiver j receives only $x_i(k)$ and $y_r(k-1)$ at time k. In this model, each transmitter uses the $M+1$ symbol extension to transmit two datastreams because we have M transmitters, which implies that the maximum $2M/(M+1)$ DoF can be achieved. The achievability of the DoF will be proved in Section 4.

- *The system model using two half-duplex relays in Fig. 3b:* In this model, two half-duplex relays are used between the M transmitters and the two receivers. Since two half-duplex relays transmit and receive signals by turns, the received signals at the relays are given as

$$\begin{aligned} y_{r_1}(k) &= \sum_{i=1}^{M} h_{r_1 i}x_i(k) + g_2u(k)y_{r_2}(k-1) + n_{r_1}(k),\\ &\quad k = 0, 2, 4, \ldots,\quad \text{at relay 1}\\ y_{r_2}(k) &= \sum_{i=1}^{M} h_{r_2 i}x_i(k) + g_1u(k)y_{r_1}(k-1) + n_{r_2}(k),\\ &\quad k = 1, 3, 5, \ldots,\quad \text{at relay 2} \end{aligned} \tag{7}$$

where the received signal at each receiver is also given as (see (8))

where $y_{r1}(k) = 0$, $y_{r2}(k) = 0$ for $k < 0$, $j = 1$, 2 and $h_{r_1 i}$, h_{jr_1}, $h_{r_2 i}$ and h_{jr_2} denote the channel coefficients from the transmitter i to the relay 1, from the relay 1 to the receiver j, from the transmitter i to the relay 2 and from the relay 2 to the receiver j, respectively and g_1 and g_2 are the channel coefficients from relays 1 to 2 and from relays 2 and 1, respectively. All the channel coefficients are statistically independent with some continuous distributions. n_{r1} and n_{r2} are the additive complex white Gaussian noises at relays 1 and 2, respectively, with the distribution $C(0, 1)$.

In this model, the transmitters transmit new signals at each time slot and each relay operates in the AF mode (non-orthogonal AF). However, if one relay transmits the signal, the other should receive. In other words, they transmit and receive the signals by turns. Since a relay receives signals from the transmitters and the other relay, this signal contains all the previous and current received signals. Note that each transmitter also uses the $M+1$ symbol extension to transmit two datastreams.

4 Achievability of the proposed IA schemes

In this section, we investigate the equivalent channel matrix of each system model and show that the perfect IA can be achieved by the proposed relay-aided IA schemes.

4.1 IA scheme with a full-duplex relay

The equivalent channel matrix between the transmitter i and the receiver j in Fig. 3a is given as (see (9))

In this $(M+1)\times(M+1)$ matrix, the main diagonal elements and subdiagonal elements are not zero but the other elements are all zero.

Its inverse matrix $\boldsymbol{H}_{ji}^{-1}$ is given as (see at the bottom of next page (10))

The received signal space Γ_1 for receiver 1 can be given as (see at the bottom of next page (11))

where $\tilde{\boldsymbol{b}}_{11} = \boldsymbol{H}_{11}\boldsymbol{b}_{11}$, $\boldsymbol{b}'_{11} = \boldsymbol{H}_{21}\boldsymbol{b}_{11}$ and $\tilde{\boldsymbol{b}}_{21} = \boldsymbol{H}_{11}\boldsymbol{b}_{21}$.

For simplicity, let $u(k) = 1$ and $\tilde{h}_{ji} = h_{ri}h_{jr}/h_{ji}$. Even though we set $u(k) = 1$, by normalising the relay power with total power, the power constraint can be easily satisfied. Therefore this assumption does not affect the proof of the

$$y_j(k) = \begin{cases} \sum_{i=1}^{M} h_{ji}x_i(k) + u(k)h_{jr_2}y_{r_2}(k-1) + n_j(k), & \text{for}\quad k = 0, 2, 4, \ldots \\ \sum_{i=1}^{M} h_{ji}x_i(k) + u(k)h_{jr_1}y_{r_1}(k-1) + n_j(k), & \text{for}\quad k = 1, 3, 5, \ldots \end{cases} \tag{8}$$

$$\boldsymbol{H}_{ji} = h_{ji}\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \dfrac{u(1)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & 0 & \cdots & 0 \\ 0 & \dfrac{u(2)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \dfrac{u(3)h_{ri}h_{jr}}{h_{ji}} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \dfrac{u(M)h_{ri}h_{jr}}{h_{ji}} & 1 \end{bmatrix} \tag{9}$$

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perfect IA. Then, $\boldsymbol{H}_{1i}\boldsymbol{H}_{2i}^{-1}$ is given as (see (12))

The ith column Γ_{1i} of Γ_1 in (11), $i \neq 1, M+1$, is given as (see (13))

where $b'_{11,k}$ is the kth element of b'_{11}.

To confirm that Γ_1 has full rank, it is sufficient to show that $|\Gamma_1|$ is not zero with Probability 1. Now, by keeping the highest order term in $\tilde{h}_{2i}$ and removing all the other terms in each element of Γ_1, we obtain $\tilde{\Gamma}_1$ as (see (14))

where $\tilde{b}_{11,k}$ and $\tilde{b}_{21,l}$ denote the kth element of $\tilde{b}_{11}$ and the lth element of $\tilde{b}_{12}$, respectively. Then, the determinant of $\tilde{\Gamma}_1$ is given as

$$\boldsymbol{H}_{ji}^{-1} = h_{ji}^{-1}\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -\dfrac{u(1)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & 0 & \cdots & 0 \\ \dfrac{u(1)u(2)(h_{ri}h_{jr})^2}{h_{ji}^2} & -\dfrac{u(2)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & \cdots & 0 \\ -\dfrac{u(1)u(2)u(3)(h_{ri}h_{jr})^3}{h_{ji}^3} & \dfrac{u(2)u(3)(h_{ri}h_{jr})^2}{h_{ji}^2} & -\dfrac{u(3)(h_{ri}h_{jr})}{h_{ji}} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ (-1)^M\dfrac{\prod_{k=1}^{M}u(k)(h_{ri}h_{jr})^M}{h_{ji}^M} & (-1)^{M-1}\dfrac{\prod_{k=2}^{M}u(k)(h_{ri}h_{jr})^{M-1}}{h_{ji}^{M-1}} & \cdots & \cdots & -\dfrac{u(M)h_{ri}h_{jr}}{h_{ji}} & 1 \end{bmatrix} \quad (10)$$

$$\begin{aligned}\Gamma_1 = {} & \left[\boldsymbol{H}_{11}\boldsymbol{b}_{11} \quad \boldsymbol{H}_{12}\boldsymbol{H}_{22}^{-1}\boldsymbol{H}_{21}\boldsymbol{b}_{11} \quad \cdots \quad \boldsymbol{H}_{1M}\boldsymbol{H}_{2M}^{-1}\boldsymbol{H}_{21}\boldsymbol{b}_{11} \quad \boldsymbol{H}_{11}\boldsymbol{b}_{21}\right] \\ & \times\left[\tilde{\boldsymbol{b}}_{11} \quad \boldsymbol{H}_{12}\boldsymbol{H}_{22}^{-1}\boldsymbol{b}'_{11} \quad \cdots \quad \boldsymbol{H}_{1M}\boldsymbol{H}_{2M}^{-1}\boldsymbol{b}'_{11} \quad \tilde{\boldsymbol{b}}_{11}\right]\end{aligned} \quad (11)$$

$$\boldsymbol{H}_{1i}\boldsymbol{H}_{2i}^{-1} = \frac{h_{1i}}{h_{2i}}\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \tilde{h}_{1i}-\tilde{h}_{2i} & 1 & 0 & 0 & \cdots & 0 \\ -\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2 & \tilde{h}_{1i}-\tilde{h}_{2i} & 1 & 0 & \cdots & 0 \\ \tilde{h}_{1i}\tilde{h}_{2i}^2-\tilde{h}_{2i}^3 & -\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2 & \tilde{h}_{1i}-\tilde{h}_{2i} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ (-1)^{M-1}\left(\tilde{h}_{1i}\tilde{h}_{2i}^{M-1}-\tilde{h}_{2i}^M\right) & (-1)^{M-2}\left(\tilde{h}_{1i}\tilde{h}_{2i}^{M-2}-\tilde{h}_{2i}^{M-1}\right) & \cdots & \cdots & \tilde{h}_{1i}-\tilde{h}_{2i} & 1 \end{bmatrix} \quad (12)$$

$$\Gamma_{1i} = \boldsymbol{H}_{1i}\boldsymbol{H}_{2i}^{-1}\boldsymbol{b}'_{11} = \frac{h_{1i}}{h_{2i}}\begin{bmatrix} b'_{11,1} \\ (\tilde{h}_{1i}-\tilde{h}_{2i})b'_{11,1}+b'_{11,2} \\ \left(-\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2\right)b'_{11,1}+(\tilde{h}_{1i}-\tilde{h}_{2i})b'_{11,2}+b'_{11,3} \\ \left(\tilde{h}_{1i}\tilde{h}_{2i}^2-\tilde{h}_{2i}^3\right)b'_{11,1}+\left(-\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2\right)b'_{11,2}+(\tilde{h}_{1i}-\tilde{h}_{2i})b'_{11,3}+b'_{11,4} \\ \vdots \\ \sum_{k=1}^{M}(-1)^{k-1}\left(\tilde{h}_{1i}\tilde{h}_{2i}^{k-1}-\tilde{h}_{2i}^k\right)b'_{11,M+1-k}+b'_{11,M+1} \end{bmatrix} \quad (13)$$

$$\tilde{\Gamma}_1 = \begin{bmatrix} \tilde{b}_{11,1} & \dfrac{h_{12}}{h_{22}}b'_{11,1} & \dfrac{h_{13}}{h_{23}}b'_{11,1} & \cdots & \dfrac{h_{1M}}{h_{2M}}b'_{11,1} & \tilde{b}_{21,1} \\ \tilde{b}_{11,2} & -\dfrac{h_{12}}{h_{22}}\tilde{h}_{22}b'_{11,1} & -\dfrac{h_{13}}{h_{23}}\tilde{h}_{23}b'_{11,1} & \cdots & -\dfrac{h_{1M}}{h_{2M}}\tilde{h}_{2M}b'_{11,1} & \tilde{b}_{21,2} \\ \tilde{b}_{11,3} & \dfrac{h_{12}}{h_{22}}\tilde{h}_{22}^2b'_{11,1} & \dfrac{h_{13}}{h_{23}}\tilde{h}_{23}^2b'_{11,1} & \cdots & \dfrac{h_{1M}}{h_{2M}}\tilde{h}_{2M}^2b'_{11,1} & \tilde{b}_{21,3} \\ \tilde{b}_{11,4} & -\dfrac{h_{12}}{h_{22}}\tilde{h}_{22}^3b'_{11,1} & -\dfrac{h_{13}}{h_{23}}\tilde{h}_{23}^3b'_{11,1} & \cdots & -\dfrac{h_{1M}}{h_{2M}}\tilde{h}_{2M}^3b'_{11,1} & \tilde{b}_{21,4} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{b}_{11,M+1} & (-1)^{M-1}\dfrac{h_{12}}{h_{22}}\tilde{h}_{22}^Mb'_{11,1} & (-1)^{M-1}\dfrac{h_{13}}{h_{23}}\tilde{h}_{23}^Mb'_{11,1} & \cdots & (-1)^{M-1}\dfrac{h_{1M}}{h_{2M}}\tilde{h}_{2M}^Mb'_{11,1} & \tilde{b}_{21,M+1} \end{bmatrix} \quad (14)$$

$$|\tilde{\Gamma}_1| = \tilde{b}_{11,1}C_{11} - \frac{h_{12}}{h_{22}}b'_{11,1}C_{12} + \cdots + (-1)^{M+1}\tilde{b}_{21,1}C_{1(M+1)} \tag{15}$$

where C_{ij} is the cofactor, which is the determinant of the square matrix obtained after deleting the ith row and the jth column of $\tilde{\Gamma}_1$. Note that $\tilde{b}_{11,i}$, $i \neq 1$ and $\tilde{b}_{12,j}$ appear only in the first column and the $(M+1)$th column, respectively and $\tilde{h}_{2i}$ appears only in the ith column, $2 \leq i \leq M$. Therefore the cofactor C_{1j}'s cannot be canceled out in (15) and $|\tilde{\Gamma}_1|$ cannot be a zero-polynomial.

Since $|\tilde{\Gamma}_1|$ is not a zero-polynomial, it can be easily seen that $|\Gamma_1|$ cannot be a zero-polynomial. In fact, the set V of the root of $|\Gamma_1| = 0$ is an affine variety. Since $|\Gamma_1|$ is a non-zero polynomial, the dimension of this variety is lower than that of the continuous space U with $\tilde{b}_{11}$, $\tilde{b}_{12}$, b'_{11}, h'_{ji}, h'_{ri} and h'_{jr} [13]. Therefore V does not have volume in U.

Finally, since the volume of V is zero and all the random variables are continuous, we have

$$\Pr(|\Gamma_1| = 0) = \int_V f\left(\tilde{b}_{11}, \tilde{b}_{12}, b'_{11}, h_{ji}', h_{ri}', h_{jr}'\right) \mathrm{d}U = 0 \tag{16}$$

where $f(\cdot)$ is the joint probability density function. Since $|\Gamma_1|$ is not zero with probability 1, Γ_1 has full rank with probability 1 and thus the perfect IA can be achieved.

For receiver 2, it can be similarly proved that $|\Gamma_2|$ is not zero with probability 1 and therefore, the proposed IA scheme with a full-duplex relay can achieve the $2M/(M+1)$ DoF.

4.2 IA scheme with two half-duplex relays

By considering arbitrary time extension, the equivalent channel matrix between the transmitter i and the receiver j for this scheme is given as (see (17))

When M is odd, the last low in (17) is given as

$$\frac{\prod_{k=1}^{M} u(k) g_1^{(M-1/(2))} g_2^{(M-1/(2))} h_{r_1 i} h_{jr_1}}{h_{ji}} \; \frac{\prod_{k=2}^{M} u(k) g_1^{(M-3/(2))} g_2^{(M-1/(2))} h_{r_2 i} h_{jr_1}}{h_{ji}} \cdots \; \cdots \frac{u(M)u(M-1) g_2 h_{r_2 i} h_{jr_1}}{h_{ji}} \; \frac{u(M) h_{r_1 i} h_{jr_1}}{h_{ji}} \quad 1 \tag{18}$$

When M is even, the last low in (17) is given as

$$\frac{\prod_{k=1}^{M} u(k) g_1^{(M/2)} g_2^{(M-2)} h_{r_2 i} h_{jr_2}}{h_{ji}} \; \frac{\prod_{k=2}^{M} u(k) g_1^{(M-2)/2)} g_2^{(M-2/(2))} h_{r_1 i} h_{jr_2}}{h_{ji}} \cdots \; \cdots \frac{u(M)u(M-1) g_1 h_{r_1 i} h_{jr_2}}{h_{ji}} \; \frac{u(M) h_{r_2 i} h_{jr_2}}{h_{ji}} \quad 1 \tag{19}$$

For simplicity, we assume that $u(k) = 1$ for all k. Let $\boldsymbol{H}_{ji} = h_{ji}(\boldsymbol{I} - 1/h_{ji}\boldsymbol{G}_{ji})$, where $\boldsymbol{G}_{ji}$ is the matrix consisting of zero diagonal elements and the negative values of the off-diagonal entries of $\boldsymbol{H}_{ji}$. Clearly, $\boldsymbol{G}_{ji}$ is a nilpotent matrix and thus

$$\boldsymbol{H}_{ji}^{-1} = \frac{1}{h_{ji}} \sum_{k=0}^{M} h_{ji}^{-k} \boldsymbol{G}_{ji}^{k} \tag{20}$$

and

$$\boldsymbol{H}_{1i}\boldsymbol{H}_{2i}^{-1} = \frac{h_{1i}}{h_{2i}}\left(\boldsymbol{I} - \frac{1}{h_{1i}}\boldsymbol{G}_{1i}\right) \sum_{k=0}^{M} h_{2i}^{-k} \boldsymbol{G}_{2i}^{k} \tag{21}$$

The first k rows of $\boldsymbol{G}_{2i}^{k}$ are all zero and $\sum_{k=0}^{M} h_{ji}^{-k} \boldsymbol{G}_{ji}^{k}$ contains higher order terms as the row index increases. The received signal space Γ_1 for receiver 1 is defined similarly to (11). Then, the ith column $\Gamma_{1i} = \boldsymbol{H}_{1i}\boldsymbol{H}_{2i}^{-1}\boldsymbol{b}'_{11}$ of Γ_1 contains the polynomials of higher degrees as the row index increases. Consider the highest order terms of each entry of Γ_1 and then, $\tilde{\Gamma}_1$ can be obtained similarly to (14). Similar to the full-duplex relay case, $|\Gamma_1|$ can be shown to be a non-zero polynomial and thus the variety that $|\Gamma_1| = 0$ has lower dimensions than the union space consisting of all the variables in $|\Gamma_1| = 0$. Therefore the probability that $|\Gamma_1| = 0$ is zero and Γ_1 has full rank with probability one.

This proof can also be applied to receiver 2 and therefore this scheme also achieves the maximum DoF $2M/(M+1)$ for the $M \times 2$ X channel. It is clear that the proposed IA scheme with two half-duplex relays only needs to store the current received signal instead of all the received signals as in the conventional full-duplex relay-aided IA scheme [10].

In the AF relay mode, the transmission power of the relay at time slot k is given as

$$P_{\hat{r}_k} = |u(k)|^2 \left(\sum_{i=1}^{M} \left| h_{\hat{r}_k i} \right|^2 |x_i(k)|^2 + N \right) \tag{22}$$

$$\boldsymbol{H}_{ji} = h_{ji} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \dfrac{u(1)h_{r_1 i}h_{jr_1}}{h_{ji}} & 1 & 0 & 0 & 0 & \cdots & 0 \\ \dfrac{u(2)u(1)g_1 h_{r_1 i}h_{jr_2}}{h_{ji}} & \dfrac{u(2)h_{r_2 i}h_{jr_2}}{h_{ji}} & 1 & 0 & 0 & \cdots & \vdots \\ \dfrac{u(3)u(2)u(1)g_1 g_2 h_{r_1 i}h_{jr_1}}{h_{ji}} & \dfrac{u(3)u(2)g_2 h_{r_2 i}h_{jr_1}}{h_{ji}} & \dfrac{u(3)h_{r_1 i}h_{jr_1}}{h_{ji}} & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & & \ddots & \ddots & 0 \\ \text{Last low} & & & & & & \end{bmatrix} \tag{17}$$

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where N is the power of noise at the relay and it is assumed that $N = 1$. To keep the relay power stable on average for the proposed IA scheme with two half-duplex relays, $u(k)$ can be determined as

$$u(k) = 1 \Bigg/ \sqrt{\sum_{i=1}^{M} \left|h_{r_1 i}\right|^2 + 1/\rho} \quad \text{for } k = 1$$

$$\text{and} \quad u(k) = 1 \Bigg/ \sqrt{\sum_{i=1}^{M} \left|h_{\hat{r}_k i}\right|^2 + |g_{\tilde{r}_k}|^2 + 1/\rho}$$

for $k > 1$ where ρ denotes the transmit signal-to-noise power ratio (SNR) at the transmitters and relays. $\hat{r}_k$ denotes r_1 and r_2 when k is odd and even, respectively and $\hat{r}_k$ is 2 when $\hat{r}_k$ is r_1 and 1 when $\hat{r}_k$ is r_2. Then, for $E(|x|^2) = \rho$, it can be seen that $E(P_{\hat{r}_k}) = \rho$. It is clear that $u(k)$ increases as $|g_{\tilde{r}_k}|$ goes to 0 for $k > 1$, and the matrix $\boldsymbol{H}_{ji}$ in (17) has the similar form as that of the IA scheme with a full-duplex relay. Therefore the perfect IA can be achieved in this case.

However, if $|g_{\tilde{r}_k}|$ goes to infinity, only the elements of the first column and diagonal elements $\boldsymbol{H}_{ji}$ in (17) are non-zero. In this case, the perfect IA is not feasible for even the 2×2 X channel. To overcome this problem, the proposed IA scheme with two half-duplex relays should be modified. Actually, the channel capacity between the two half-duplex relays goes to infinity in this case and thus each relay can send its information by using a very small portion of the bandwidth or time. This implies that the two relays can share their information with almost no time delay and the two half-duplex relays can operate like a full-duplex relay. Therefore the proposed IA scheme with two half-duplex relays can behave like the proposed IA scheme with a full-duplex relay which can achieve the perfect IA.

5 Achievability of the proposed IA schemes for the 2 × *M X* channel

In this section, by using the reciprocity of the $2 \times M$ channel, it is shown that the perfect IA for the $2 \times M$ X channel is also possible for the proposed IA schemes and $2M/(M+1)$ DoF can be achieved [6]. The relation of the original $2 \times M$ X channel and its reciprocal $M \times 2$ X channel are explained in Fig. 4 [6].

Let $\boldsymbol{H}_{ji}$ and $\tilde{\boldsymbol{H}}_{ij}$ denote the $(M+1) \times (M+1)$ channel matrices between the transmitter i and the receiver j for the original $2 \times M$ X channel and between the transmitter j and the receiver i for its reciprocal $M \times 2$ X channel in Fig. 4, respectively. Then, we have the relation $\boldsymbol{H}_{ji} = \tilde{\boldsymbol{H}}_{ij}^{*}$ [6], where $(\cdot)^*$ denotes the complex conjugate and transpose of a matrix. $\boldsymbol{z}_{ji}$ denotes the zero-forcing vector for the beamforming vector $\boldsymbol{b}_{ji}$ at the receiver j in the original $2 \times M$ X channel and $\boldsymbol{z}_{ji}^{r}$ denotes the zero-forcing vector for the beamforming vector $\boldsymbol{b}_{ji}^{r}$ in the reciprocal $M \times 2$ X channel.

From Fig. 4, it is easy to check that the zero-forcing vectors for the original channel can be used as the beamforming vectors for the reciprocal channel and the beamforming vectors for the original channel can be used as the zero-forcing vectors for the reciprocal channel [6]. Therefore if the perfect IA is possible for the reciprocal channel, it is also possible for the original channel.

In this section, it is assumed that the original channel is the $2 \times M$ X channel with relay and the reciprocal channel is the $M \times 2$ X channel with relay and the $M+1$ symbol extension is used. The system models are given as follows by considering a full-duplex relay and two half-duplex relays:

- *The case of one full-duplex relay:* The received signals at the relay and the receiver for the original channel are given as

$$y_r(k) = \sum_{i=1}^{2} h_{ri} x_i(k) + n_r(k) \tag{23}$$

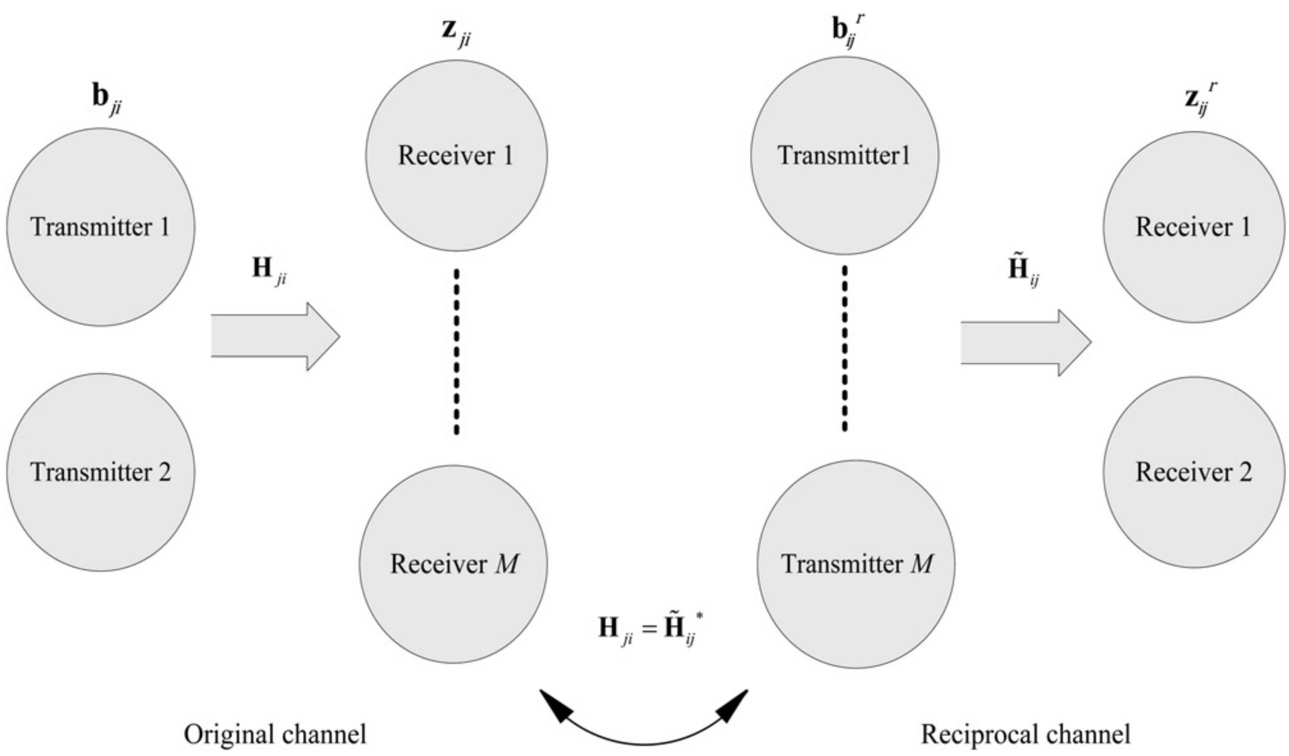

Fig. 4 *Reciprocal relation between 2 × M and M × 2 X channels*

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$$y_j(k) = \sum_{i=1}^{2} h_{ji}x_i(k) + u(k)h_{jr}y_r(k-1) + n_j(k), \quad j = 1, \ldots, M \tag{24}$$

where $k = 0, \ldots, M$, $y_r(k) = 0$ for $k < 0$ and $u(k)$ is the relay gain.

- *The case of two half-duplex relays:* For the original channel, the received signals at the relays are given as

$$\begin{aligned} y_{r_1}(k) &= \sum_{i=1}^{2} h_{r_1 i}x_i(k) + g_2 u(k) y_{r_2}(k-1) + n_{r_1}(k), \\ &\qquad k = 0, 2, 4, \ldots \quad \text{at relay 1} \\ y_{r_2}(k) &= \sum_{i=1}^{2} h_{r_2 i}x_i(k) + g_1 u(k) y_{r_1}(k-1) + n_{r_2}(k), \\ &\qquad k = 1, 3, 5, \ldots \quad \text{at relay 2} \end{aligned} \tag{25}$$

and the received signals at the receivers are also given as (see (26))

where $y_{r_1}(k) = 0, y_{r_2}(k) = 0 \quad$ for $k < 0$.

For the original channel, the channel matrices with a full-duplex relay and two half-duplex relays are identical to $\boldsymbol{H}_{ji}$ in (9) and (17), respectively. Therefore the channel matrices for the reciprocal channels $\tilde{\boldsymbol{H}}_{ij} = \boldsymbol{H}_{ji}^*$ become upper triangular matrices.

To verify the achievability of the proposed IA schemes for the $2 \times M$ X channel, we can use the results in Section 3 for the $M \times 2$ X channel by replacing the channel matrix with the upper triangular matrix $\tilde{\boldsymbol{H}}_{ji}$. Similar to the proof in Section 4, it is easy to show that Γ_1 for the reciprocal $M \times 2$ X channel in Fig. 4 has full rank, that is, a perfect IA is achieved for the reciprocal $M \times 2$ X channel. Therefore it is clear that by setting $\boldsymbol{b}_{ji} = \boldsymbol{z}_{ij}^{r*}$, a perfect IA for the original $2 \times M$ channel is achieved by the IA schemes in [6].

6 Numerical analysis

In this section, some numerical results are given to compare the proposed schemes with the relay-aided IA scheme in [10]. All the channel coefficients and noises at the relays and the receivers are assumed as complex Gaussian random variables distributed as $\mathcal{C}(0, 1)$. To satisfy the relay power constraint, the relay gain $u(k)$ is given as (see (27))

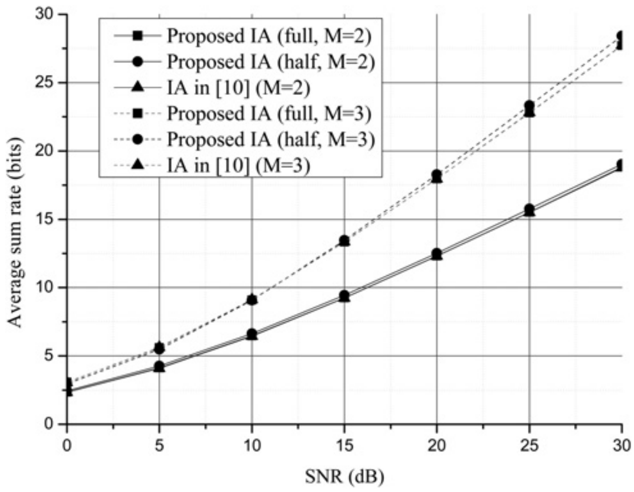

Fig. 5 *Average sum rate at receiver 1 for various relay-aided IA schemes*

where $k = 0, 1, \ldots, M$, with $u(0) = 0$. By setting the amplification gain of the relay as (27), the relay can keep its power stable on average. Actually, in [10], the amplification gain was set in a different way from (27). Since the relay stores all the received signals in the $M+1$ time slots, the relay can amplify for each received signal by using different gain. Thus, optimisation of the IA schemes in [10] is possible. However, it is beyond the scope of this paper and thus, the relay gain for the IA schemes in [10] is set as (27) for simplicity. Also, the zero-forcing vectors are obtained by using the method in [14].

Fig. 5 shows the average sum rate of the receiver 1 for various IA schemes with $M = 2, 3$, where the 'Proposed IA (full)' denotes the proposed IA scheme with a full duplex relay and 'Proposed IA (half)' denotes the proposed IA scheme with two half-duplex relays. From Fig. 5, it can be seen that the sum rates of the proposed schemes are close to that in [10] and their slopes are also similar. Theoretically, for $M = 2$, each scheme can achieve DoF $2M/(M+1) = 4/3$ and thus, the DoF at receiver 1 is 2/3, where DoF d is defined as

$$\text{sum rate} \simeq d\log_2(\text{SNR}) \tag{28}$$

$$y_j(k) = \begin{cases} \sum_{i=1}^{2} h_{ji}x_i(k) + u(k)h_{jr_2}y_{r_2}(k-1) + n_j(k), & k = 0, 2, 4, \ldots \\ \sum_{i=1}^{2} h_{ji}x_i(k) + u(k)h_{jr_2}y_{r_2}(k-1) + n_j(k), & k = 0, 2, 4, \ldots \end{cases} \tag{26}$$

$$u(k) = \begin{cases} \dfrac{1}{\sqrt{\sum_{i=1}^{M} |h_{ri}|^2 + 1/\rho}}, & \text{for the IA scheme with a full–duplex relay} \\ \dfrac{1}{\sqrt{\sum_{i=1}^{M} |h_{r_1 i}|^2 + 1/\rho}}, \ k = 1, & \text{for the IA scheme with two half–duplex relays} \\ \dfrac{1}{\sqrt{\sum_{i=1}^{M} |h_{\hat{r}_k i}|^2 + |g_{\tilde{r}_k}|^2 + 1/\rho}}, \ k \neq 1, & \text{for the IA scheme with two half–duplex relays} \\ \dfrac{1}{\sqrt{n\left(\sum_{i=1}^{M} |h_{ri}|^2 + 1/\rho\right)}}, & \text{for the IA scheme in [10]} \end{cases} \tag{27}$$

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when the SNR goes to infinity. In Fig. 5, we do not normalise the DoF by the time extension and if DoF 2 and DoF 3 are achieved at receiver 1, it implies that the theoretical maximum DoF is achieved. In Fig. 5, the increase rate of the sum rate per one interval goes close to 3.3 after about 25 dB SNR for $M = 2$, that is

$$2\left(\log_2 \mathrm{SNR}_2 - \log_2 \mathrm{SNR}_1\right) = 2\log_2 10^{\Delta} \simeq 3.32 \qquad (29)$$

where $\Delta = \log_{10} \mathrm{SNR}_2/\mathrm{SNR}_1 = 0.5$ corresponds to the interval of the horizontal axis, 5 dB SNR. Therefore it is confirmed that the proposed schemes achieve the maximum DoF of the $M \times 2$ X channel. For $M = 3$, it can also be seen that the theoretical DoF is achieved by the proposed schemes.

In fact, the proposed IA scheme with two half-duplex relays shows a little better performance than the others. In the proposed IA scheme with two half-duplex relays, the receivers combine the signals from two independent channels because of two relays. It can be thought that the diversity effect is obtained a little and it provides a little better performance than when only one channel is used. However, there is a disadvantage that more CSI is needed at the transmitters and the receivers.

7 Conclusion

In this paper, two relay-aided IA schemes are proposed for the quasi-static $M \times 2$ X channel by using a full-duplex relay and two half-duplex relays. It is proved that the proposed schemes can achieve the perfect IA and therefore, the maximum DoF $2M/(M + 1)$ is also achieved. Through the numerical analysis, it is also shown that the proposed schemes can achieve the sum rate close to that of the previous relay-aided IA scheme. Therefore the proposed IA schemes can be good practical alternative schemes for the conventional relay-aided IA scheme. Especially, the second proposed scheme replaces the full-duplex relay with the two half-duplex relays for the IA which is more practical from the viewpoint of implementation. As a further work, we can consider the IA scheme with one half-duplex relay and extension for the general X channel. In addition, the research on the combination of IA and the diversity technique can also be a good topic in order to obtain high throughput and reliability simultaneously.

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