On the Properties of Cubic Metric for OFDM Signals

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***Abstract*—As a metric for amplitude fluctuation of orthogonal frequency division multiplexing (OFDM) signal, cubic metric (CM) has received an increasing attention because it is more closely related to the distortion induced by nonlinear devices than the well-known peak-to-average power ratio (PAPR). In this letter, the properties of CM of OFDM signal is investigated. First, asymptotic distribution of CM is derived. Second, it is verified that 1.7 times oversampling rate is good enough to capture the CM of continuous OFDM signals in terms of mean square error, which is also practically meaningful because the fast Fourier transform size is typically 1.7 times larger than the nominal bandwidth in the long-term evolution (LTE) cellular communication systems.**

***Index Terms*—Cubic metric (CM), fast Fourier transform (FFT), orthogonal frequency division multiplexing (OFDM), oversampling, peak-to-average power ratio (PAPR).**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) signals suffer from high amplitude fluctuation which causes performance degradation due to nonlinear devices. A well-known metric for amplitude fluctuation of OFDM signal is peak-to-average power ratio (PAPR). Many research efforts have been carried out to find efficient PAPR reduction techniques [1]. Also, the distribution of the PAPR of continuous OFDM signals was derived [2][3] and it is widely accepted that four times oversampling is good enough to capture the PAPR of continuous OFDM signals [4]. However, there are research results showing that PAPR may not be the best metric to measure the magnitude of the envelope fluctuations. For example, Bento *et al.* [5] pointed out the dependency of PAPR and the number of subcarriers.

Another metric for amplitude fluctuation of OFDM signals has been considered [6][7], which is known as cubic metric (CM) [8]. Studies on PAPR and CM suggest that, except for large power backoff, CM is more closely related to the amount of distortion induced by a nonlinear power amplifier than PAPR [7]. Moreover, after analyzing certain OFDM-type signals that are considered to meet the goal of the long-term evolution (LTE), it was shown in the 3GPP that CM predicts amplifier power de-rating more accurately than PAPR [9]. Thus, research to reduce the CM for the LTE systems has recently been carried out [10]–[12].

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A great deal of literature has been devoted to derive the properties of the PAPR metric as in [2][4]. On the contrary, such analysis on the CM has not been done yet. Thus, it is worth revealing more about the behavior of this metric. In this letter, an asymptotic probability distribution of CM for continuous OFDM signals is derived. Also, sufficient oversampling rate for capturing the CM of continuous OFDM signals is obtained.

Cubic Metric: The CM of OFDM signals is defined as [9]

$$\mathrm{CM}|_{\mathrm{dB}} \triangleq \frac{\mathrm{RCM}|_{\mathrm{dB}} - \mathrm{RCM}_{\mathrm{ref}}|_{\mathrm{dB}}}{K} \tag{1}$$

where RCM is the raw CM defined by

$$\mathrm{RCM}[s(t)]|_{\mathrm{dB}} \triangleq 20\log\left[\mathrm{rms}\left[\left(\frac{|s(t)|}{\mathrm{rms}[s(t)]}\right)^3\right]\right] \tag{2}$$

for a continuous OFDM signal $s(t)$ and both $\mathrm{RCM}_{\mathrm{ref}}|_{\mathrm{dB}}$ and K are determined according to the considered OFDM systems [9]. As an example, in the downlink of LTE, $\mathrm{RCM}_{\mathrm{ref}}|_{\mathrm{dB}} = 1.52$ dB and $K = 1.56$ are used [9]. Thus, from (2), we are only interested in

$$\mathrm{RCM}[s(t)] = \sqrt{\mathbb{E}\left[\left(\frac{|s(t)|}{\sqrt{P_{\mathrm{av}}}}\right)^6\right]} \tag{3}$$

where P_{av} is the average power of the continuous OFDM signal $s(t)$. To simplify analysis, we will use the square of it as

$$\xi \triangleq (\mathrm{RCM}[s(t)])^2 = \mathbb{E}\left[\left(\frac{|s(t)|}{\sqrt{P_{\mathrm{av}}}}\right)^6\right]. \tag{4}$$

In practice, instead of calculating the CM of continuous OFDM signals, we calculate the CM of discrete OFDM signals. Let $s(t)$ be a continuous OFDM signal and its L times oversampled OFDM signal sequence be represented as

$$s_{n,L} \triangleq s(nT_s/LN), \quad 0 \le n \le LN-1 \tag{5}$$

where T_s is the OFDM signal period, N is the number of subcarriers, and L is a real number larger than or equal to one.

Without loss of generality, the input symbols in frequency domain are assumed to be statistically independent, identically distributed (i.i.d.) random variables with zero mean, where the input symbol is the complex data symbol assigned to each subcarrier. Then the OFDM signal components in time domain are given by the sum of i.i.d. random variables. Thus, from the central limit theorem (CLT), the magnitude of $s_{n,L}$ is Rayleigh distributed [2]. Therefore, if it is normalized as

$$r(t) \triangleq |s(t)|/\sqrt{P_{\mathrm{av}}} \tag{6}$$

$$r_{n,L} \triangleq |s_{n,L}|/\sqrt{P_{\mathrm{av}}}, \tag{7}$$

the probability density functions (PDFs) of $r(t)$ and $r_{n,L}$ are given as

$$f_{r(t)}(r) = f_{r_{n,L}}(r) = 2re^{-r^2}. \quad (8)$$

Finally, for the discrete OFDM signal sequence obtained by L times oversampling, the corresponding ξ and RCM are expressed as

$$\xi_L = \frac{1}{LN}\sum_{n=0}^{LN-1} r_{n,L}^6 \quad (9)$$

and

$$\mathrm{RCM}_L = \sqrt{\xi_L}. \quad (10)$$

II. Properties of $r_{n,L}^6$

In (9), ξ_L is the sample mean of $r_{n,L}^6$'s and thus in this section we investigate the properties of the random variable $r_{n,L}^6$. By the definition of Weibull distribution, the power transformation $w_{n,L} \triangleq r_{n,L}^6$ of the Rayleigh distributed random variable $r_{n,L}$ follows Weibull distribution [13]. The PDF and cumulative distribution function (CDF) of $w_{n,L}$ are given as

$$f_{w_{n,L}}(w) = \frac{1}{3}w^{-\frac{2}{3}}\exp(-w^{\frac{1}{3}}) \quad (11)$$

and

$$F_{w_{n,L}}(w) = 1 - \exp(-w^{\frac{1}{3}}), \quad (12)$$

respectively. The kth-order moment of $w_{n,L}$ is known as $\mathbb{E}[w_{n,L}^k] = \Gamma(1+3k)$, where $\Gamma(a) = (a-1)!$ is the Gamma function for an integer a and $\mathbb{E}[\cdot]$ denotes the expectation. Then we have

$$\mathbb{E}[w_{n,L}] = 3! = 6 \quad (13)$$
$$\mathrm{var}(w_{n,L}) = 684 \quad (14)$$

where $\mathrm{var}(\cdot)$ denotes the variance.

Now, we calculate the covariance of two random variables $w_{n,L}$ and $w_{n',L'}$, which will be denoted as $\mathrm{cov}(w_{n,L}, w_{n',L'})$. First, we calculate $\mathrm{cov}(w(t), w(t+\tau))$ for the continuous time lag τ, where $w(t) = r^6(t)$. The joint moment of $w(t)$ and $w(t+\tau)$ is expressed as [13]

$$\mathbb{E}[w^p(t)w^q(t+\tau)] = (1-\rho_\tau^2)^{1+3p+3q}\Gamma(1+3p)\Gamma(1+3q)_2 \times F_1(1+3p, 1+3q; 1; \rho_\tau^2) \quad (15)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function and $\rho_\tau = \rho_{s(t),s(t+\tau)}$ is the Pearson's correlation coefficient between $s(t)$ and $s(t+\tau)$ [13]. From (13) and (15), we have

$$\mathrm{cov}(w(t), w(t+\tau)) = \mathbb{E}[w(t)w(t+\tau)] - \mathbb{E}[w(t)]\mathbb{E}[w(t+\tau)] = 36(9\rho_\tau^2 + 9\rho_\tau^4 + \rho_\tau^6). \quad (16)$$

It can be assumed as in [2] that the power spectrum of baseband OFDM signal has conjugate symmetry, which is valid because the power spectrum of baseband OFDM signal can be designed to have symmetry at the center of the bandwidth by giving proper frequency offset. This assumption guarantees the autocorrelation function of $s(t)$, $\mathbb{E}[s(t)s(t+\tau)^*]$, to be a real function. In any case, the frequency offset is immaterial to our analysis because it does not change the magnitude of the envelope of OFDM signal. Thus, without loss of generality, this assumption can be used to simplify the derivation as follows. In this case, the normalized autocorrelation function ρ_τ is known as [2]

$$\rho_\tau = \frac{\mathbb{E}[s(t)s(t+\tau)^*]}{\mathbb{E}[s(t)s(t)^*]} = \frac{\sin(\pi N\tau/T_s)}{N\sin(\pi\tau/T_s)} \quad (17)$$

for $\tau \neq 0$ and clearly $\rho_\tau = 1$ for $\tau = 0$. Note that $\rho_\tau = 0$ when

$$\tau = \pm\frac{T_s}{N}, \pm\frac{2T_s}{N}, \pm\frac{3T_s}{N}, \cdots \quad (18)$$

which implies the well-known fact that the elements of the Nyquist sampled OFDM signal sequence are statistically independent.

The time lag τ between two discrete samples $w_{n,L}$ and $w_{n',L'}$ is

$$\tau = \frac{nT_s}{LN} - \frac{n'T_s}{L'N}. \quad (19)$$

Finally, the covariance of $w_{n,L}$ and $w_{n',L'}$ is given as

$$\mathrm{cov}(w_{n,L}, w_{n',L'}) = 36(9\rho_\tau^2 + 9\rho_\tau^4 + \rho_\tau^6)\Big|_{\tau=\frac{nT_s}{LN}-\frac{n'T_s}{L'N}}. \quad (20)$$

III. Distribution of RCM

A. Mean and Variance of ξ_L

The mean of ξ_L is clearly 6 from (13) because ξ_L is the sample mean of $w_{n,L}$. To find the variance of ξ_L, suppose that LN is an odd integer. Even though LN can be any real number larger than or equal to N, it is not difficult to show that the discrepancy by the assumption is negligible. Since $s_{n,L}$ is a complex stationary Gaussian process, both $r_{n,L}$ and $w_{n,L}$ are also stationary random process. Therefore, the variance of ξ_L becomes

$$\mathrm{var}(\xi_L) = \sigma_L^2 = \frac{\mathrm{var}(w_{n,L})}{LN} + \frac{2}{(LN)^2}\sum_{k=1}^{LN-1}(LN-k)\mathrm{cov}(w_{0,L}, w_{k,L}). \quad (21)$$

We can separate the summation in (21) into two parts and change the index of variable as

$$\sigma_L^2 = \frac{\mathrm{var}(w_{n,L})}{LN} + \frac{2}{(LN)^2}\left(\sum_{k=1}^{\frac{LN-1}{2}}(LN-k)\mathrm{cov}(w_{0,L}, w_{k,L}) + \sum_{k=1}^{\frac{LN-1}{2}} k\,\mathrm{cov}(w_{0,L}, w_{LN-k,L})\right). \quad (22)$$

Using (14) and the fact that the covariance $\mathrm{cov}(w_{0,L}, w_{k,L})$ in (20) is symmetric and periodic with the period LN, (22) is rewritten as

$$\sigma_L^2 = \frac{684}{LN} + \frac{2}{LN}\sum_{k=1}^{\frac{LN-1}{2}}\mathrm{cov}(w_{0,L}, w_{k,L}). \quad (23)$$

Consider the following two extreme cases using (20).

1) For the Nyquist Sampling Rate: We have

$$\sigma_1^2 = \frac{684}{N}. \quad (24)$$

2) For the Continuous OFDM Signal: We have

$$\begin{aligned}\sigma_\infty^2 &= \lim_{L\to\infty}\sigma_L^2 = \lim_{L\to\infty}\frac{2}{LN}\sum_{k=1}^{\frac{LN-1}{2}}\mathrm{cov}(w_{0,L},w_{k,L})\\ &= \frac{72}{N\pi}\lim_{L\to\infty}\sum_{k=1}^{\frac{LN-1}{2}}\left(9\left(\frac{\sin\left(\frac{k\pi}{L}\right)}{N\sin\left(\frac{k\pi}{LN}\right)}\right)^2\right.\\ &\quad \left.+9\left(\frac{\sin\left(\frac{k\pi}{L}\right)}{N\sin\left(\frac{k\pi}{LN}\right)}\right)^4+\left(\frac{\sin\left(\frac{k\pi}{L}\right)}{N\sin\left(\frac{k\pi}{LN}\right)}\right)^6\right)\frac{\pi}{L}\\ &= \frac{72}{N\pi}\int_0^{\frac{N\pi}{2}} 9\left(\frac{\sin(x)}{N\sin\left(\frac{x}{N}\right)}\right)^2\\ &\quad +9\left(\frac{\sin(x)}{N\sin\left(\frac{x}{N}\right)}\right)^4+\left(\frac{\sin(x)}{N\sin\left(\frac{x}{N}\right)}\right)^6 dx\\ &= \frac{36}{5N^5}+\frac{117}{N^3}+\frac{2799}{5N}. \qquad (25)\end{aligned}$$

Unless N is too small, it becomes approximately $\sigma_\infty^2 \approx 2799/5N$. The detailed derivation of the integration in (25) is easy to show by using trigonometric identities and omitted due to the lack of space.

B. Distribution of ξ_L

Since the exact distribution of the sum of Weibull random variables has not been derived yet, there have been many attempts to approximate the distribution of the sum of Weibull random variables [15][16]. However, as the number of Weibull random variables increases, the result in [15] becomes inaccurate and the result in [16] becomes too complicated. In this letter, we apply CLT to express the distribution of the sum of Weibull random variables even if CLT gives an accurate distribution when the number of Weibull random variables is large.

It is known that CLT can be applied to m-dependent random process [14], where m-dependent means that two samples from a random process with the interval larger than m have no statistical dependency. $w_{n,L}$ is an approximately m-dependent random process for $m \gg L$ from the fact that the correlation of the stationary random process $w_{n,L}$ may rapidly diminish as the interval exceeds T_s/N [2], for any real number L larger than one. Thus, ξ_L can be assumed as asymptotically Gaussian distributed as

$$\xi_L \overset{a.s.}{\sim} \mathcal{N}(6,\sigma_L^2) \qquad (26)$$

where $a.s.$ means that the random variable is asymptotically distributed.

C. Distribution of $\mathrm{RCM}_L|_{\mathrm{dB}}$

Clearly, the complementary CDF (CCDF) of $\mathrm{RCM}_L|_{\mathrm{dB}}$ is

$$P\left(\mathrm{RCM}_L|_{\mathrm{dB}} > a\right) = \frac{1}{2}\left[1-\mathrm{erf}\left(\frac{10^{\frac{a}{10}}-6}{\sigma_L\sqrt{2}}\right)\right] \qquad (27)$$

where $\mathrm{erf}(a) = \frac{2}{\sqrt{\pi}}\int_0^a e^{-t^2}dt$ is the error function. Fig. 1 compares the simulated and analytical CCDFs of $\mathrm{RCM}_\infty|_{\mathrm{dB}}$. Though we have obtained the distribution of $\mathrm{RCM}_L|_{\mathrm{dB}}$ for various values of L, we only provide the distribution when $L\to\infty$, which is of practical importance. In the simulation, L is set to 32, which is good enough to represent the continuous OFDM signal and 16-quadrature amplitude modulation (QAM) is used. It is widely known that the metrics describing the envelope behavior of OFDM signals such as CM and PAPR do not depend on the modulation order. Note that analysis is performed based on Gaussian approximation in Section IV-B. Unfortunately, Weibull distribution is a heavy-tailed distribution and thus the sum of Weibull random variables slowly converges to Gaussian distribution as N increases but it shows good agreement when N is larger than 2048 as shown in Fig. 1.

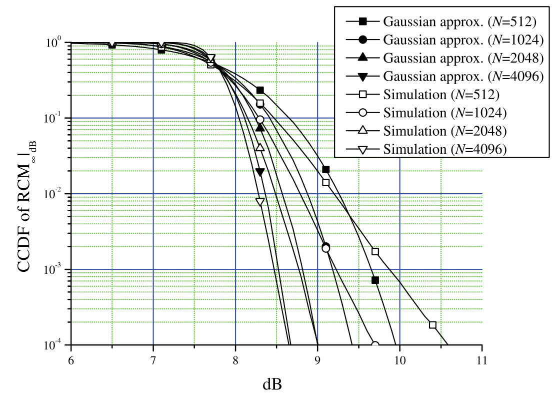

Fig. 1. Comparison of simulated and analytical CCDFs of $\mathrm{RCM}_\infty|_{\mathrm{dB}}$.

IV. Oversampling Rate to Capture the CM of Continuous OFDM Signal

To be precise, the metrics such as PAPR and CM have to be calculated from continuous OFDM signals. However, it is impossible to handle the continuous OFDM signal and instead one calculates the metrics from the sufficiently oversampled discrete OFDM signal. In this case, the metrics calculated from the oversampled OFDM signal can be viewed as an estimator for the metrics of the continuous OFDM signal. For instance, in the case of CM calculation, RCM_L is the estimator for true parameter RCM_∞. It is natural that a high sampling rate guarantees low estimation error but it also entails high complexity. Thus, finding a sufficient sampling rate is of great importance.

In [4], upper bounds on the estimation error of PAPR according to L is derived and it is proposed that four times oversampling rate is good enough to capture the PAPR of continuous OFDM signals. However, in the case of CM, this approach is not valid. Unlike the PAPR, the estimator RCM_L rarely shows extremely large estimation error. Therefore, the total inspection approach is not suitable for the case of CM. This is due to the fact that the Weibull distribution is a heavy-tailed distribution, which means that its variance is quite large, and thus the sample mean of Weibull distributions also has a large fluctuation. Then the square error $|\mathrm{RCM}_L-\mathrm{RCM}_\infty|^2$ also has a large fluctuation, too. For instance, when the OFDM signal sequence is an impulse signal with $N = 1024$, it has $\mathrm{RCM}_1 = 1024$ and $\mathrm{RCM}_\infty \simeq 759$. In this case, the square error $|\mathrm{RCM}_1 - \mathrm{RCM}_\infty|^2 \simeq 7\times10^4$ is much larger than the mean square error (MSE) $\mathbb{E}[|\mathrm{RCM}_1-\mathrm{RCM}_\infty|^2] \simeq 5\times10^{-3}$. Thus, in this section we statistically approach this problem by deriving the MSE $\mathbb{E}[|\mathrm{RCM}_L-\mathrm{RCM}_\infty|^2]$ according to L.

A. Joint PDF of ξ_L and ξ_∞

To obtain the MSE $\mathbb{E}[|\mathrm{RCM}_L - \mathrm{RCM}_\infty|^2] = \mathbb{E}[|\sqrt{\xi_L} - \sqrt{\xi_\infty}|^2]$, first we find the joint PDF of ξ_L and ξ_∞, which is clearly bivariate Gaussian distribution from our investigation in Section IV-B. We already checked their mean values $\mathbb{E}[\xi_L] = \mathbb{E}[\xi_\infty] = 6$. Next, the correlation coefficient between ξ_L and ξ_∞ is given as

$$\begin{aligned}
\rho_{\xi_L,\xi_\infty} &= \frac{\mathrm{cov}(\xi_L, \xi_\infty)}{\sigma_L \sigma_\infty} \\
&= \lim_{L' \to \infty} \frac{\sum_{n=0}^{LN-1} \sum_{n'=0}^{L'N-1} \mathrm{cov}(w_{n,L}, w_{n',L'})}{\sigma_L \sigma_\infty L L' N^2} \\
&= \lim_{L' \to \infty} \frac{\sum_{n=0}^{LN-1} \sum_{n'=0}^{L'N-1} 36(9\rho_\tau^2 + 9\rho_\tau^4 + \rho_\tau^6)|_{\tau = \frac{nT_s}{LN} - \frac{n'T_s}{L'N}}}{\sigma_L \sigma_\infty L L' N^2} \\
&= \frac{36}{\sigma_L \sigma_\infty \pi L N^2} \cdot \sum_{n=0}^{LN-1} \int_{-\frac{n\pi}{L}}^{-\frac{n\pi}{L}+N\pi} 9 \left(\frac{\sin(x)}{N \sin\left(\frac{x}{N}\right)} \right)^2 \\
&\quad + 9 \left(\frac{\sin(x)}{N \sin(\frac{x}{N})} \right)^4 + \left(\frac{\sin(x)}{N \sin\left(\frac{x}{N}\right)} \right)^6 dx
\end{aligned} \tag{28}$$

where the integrand is periodic with the period $N\pi$. Thus, we have

$$\begin{aligned}
\rho_{\xi_L,\xi_\infty} &= \frac{36}{\sigma_L \sigma_\infty \pi N} \int_0^{N\pi} 9 \left(\frac{\sin(x)}{N \sin\left(\frac{x}{N}\right)} \right)^2 \\
&\quad + 9 \left(\frac{\sin(x)}{N \sin\left(\frac{x}{N}\right)} \right)^4 + \left(\frac{\sin(x)}{N \sin\left(\frac{x}{N}\right)} \right)^6 dx \\
&= \frac{\sigma_\infty}{\sigma_L}.
\end{aligned} \tag{29}$$

In terms of estimation theory, $\rho_{\xi_L,\xi_\infty} = \sigma_\infty / \sigma_L$ implies that ξ_L can be considered as an unbiased minimum MSE estimator of ξ_∞. That is, orthogonality principle $\mathbb{E}[(\xi_L - \xi_\infty)\xi_\infty] = 0$ and unbiased property $\mathbb{E}[\xi_L] = \mathbb{E}[\xi_\infty]$ are satisfied.

B. MSE Between RCM_L and RCM_∞

The MSE between RCM_L and RCM_∞ can be obtained as

$$\begin{aligned}
&\mathbb{E}[|\mathrm{RCM}_L - \mathrm{RCM}_\infty|^2] = \mathbb{E}[|\sqrt{\xi_L} - \sqrt{\xi_\infty}|^2] \\
&= 12 - 2 \cdot \mathbb{E}[\sqrt{\xi_L \xi_\infty}] \\
&\overset{a.s.}{\sim} 12 - 2 \int_0^\infty \int_0^\infty \sqrt{\xi_L \xi_\infty} f_{\xi_L,\xi_\infty}(\xi_1, \xi_2) d\xi_1 d\xi_2
\end{aligned} \tag{30}$$

where $f_{\xi_L,\xi_\infty}(\xi_1, \xi_2)$ is the joint Gaussian PDF of ξ_1 and ξ_2 with the correlation coefficient ρ_{ξ_L,ξ_∞} in (29).

Fig. 2 compares $\mathbb{E}[|\mathrm{RCM}_L - \mathrm{RCM}_\infty|^2]$ obtained by simulation and analysis using Gaussian approximation, where simulation results are obtained by testing randomly generated 10^5 OFDM signal sequences. From a practical viewpoint, MSE is usually normalized as $\mathbb{E}[|\mathrm{RCM}_L - \mathrm{RCM}_\infty|^2]/\mathbb{E}[|\mathrm{RCM}_\infty|^2] = \mathbb{E}[|\mathrm{RCM}_L - \mathrm{RCM}_\infty|^2]/6$. Thus, one can conclude that 1.7 times oversampling gives the normalized MSE smaller than about 10^{-4} for practical value of $N \geq 256$, which is remarkable because the FFT size is typically 1.7 times larger than the nominal bandwidth for the LTE cellular communication systems [17].

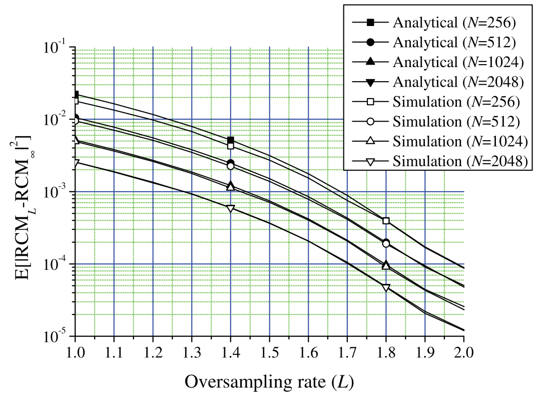

Fig. 2. Comparison of simulated and analytical $\mathbb{E}[|\mathrm{RCM}_L - \mathrm{RCM}_\infty|^2]$ for various L using Gaussian approximation.

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