Clipping Noise Cancelation for OFDM Systems Using Reliable Observations Based on Compressed Sensing

Kee-Hoon Kim, Hosung Park, Jong-Seon No, *Fellow, IEEE*, Habong Chung, *Member, IEEE*, and Dong-Joon Shin, *Senior Member, IEEE*

***Abstract*—In this paper, a clipping noise cancelation scheme using compressed sensing (CS) technique is proposed for orthogonal frequency division multiplexing systems. The proposed scheme does not need reserved tones or pilot tones, which is different from the previous works using CS technique. Instead, observations of the clipping noise in data tones are exploited, which leads to no loss of data rate. Also, in contrast with the previous works, the proposed scheme selectively exploits the reliable observations of the clipping noise instead of using whole observations, which results in minimizing the bad influence of channel noise. From the selected reliable observations, the clipping noise in time domain is reconstructed and canceled by using CS technique. Simulation results show that the proposed scheme performs well compared to other conventional clipping noise cancelation schemes and shows the best performance in some cases.**

***Index Terms*—Clipping noise, compressed sensing (CS), fast Fourier transform (FFT), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is known as one of the best modulation schemes for high-rate data transmission in wireless communications due to its robustness against multipath fading, bandwidth efficiency, and simple implementation. However, due to high peak-to-average power ratio (PAPR) of an OFDM signal, OFDM systems require expensive high power amplifiers having a large dynamic range.

Therefore, many PAPR reduction schemes for OFDM systems have been proposed [1]–[15] and among them, clipping [8]–[15] is the simplest one. Clipping at the Nyquist sampling rate has been used for low-complexity applications but suffers from peak regrowth after digital-to-analog (D/A) conversion. It is known that clipping an oversampled OFDM signal reduces the peak regrowth after D/A conversion, but it causes out-of-band radiation which has to be filtered [10]. The distortion of the OFDM signal caused by clipping is called clipping noise which has sparsity in time domain. There are several schemes to mitigate clipping noise [16]–[19], among which the scheme in [17] performs iterative maximum likelihood (ML) estimation for all tones and recreates clipping procedure in order to reconstruct clipping noise.

According to recent results in sparse signal processing, also known as compressed sensing (CS) theory [20]–[23], a sparse signal can be reconstructed from its compressed observations. In this context, clipping noise can be effectively reconstructed at the receiver by CS reconstruction algorithms. As the first work for this, a tone reservation scheme using CS is proposed in [24] and [25], where several tones are reserved at the transmitter before clipping, and the receiver reconstructs the clipping noise by exploiting the compressed observations of the reserved tones. However, in this scheme, the reserved tones induce data rate loss, and due to the vulnerability of CS reconstruction algorithms to the channel noise the bit error rate (BER) performance is poor. Another clipping noise cancelation scheme using CS is proposed in [26], motivated by the results in [25]. The scheme in [26] does not induce data rate loss because the compressed observations of the pilot tones are exploited. However, it still shows poor BER performance due to its vulnerability to the channel noise.

In this paper, we propose a new clipping noise cancelation scheme using CS, which selectively uses observations of data tones. That is, reliable observations contaminated by less channel noise are selected, and then the clipping noise is reconstructed from these compressed observations by using a CS reconstruction algorithm. The proposed scheme has the following three major advantages compared to the schemes in [25] and [26].

- In contrast with the scheme in [25], the proposed scheme does not reserve tones and instead exploits compressed observations of the underlying clipping noise in data tones, which leads to no data rate loss.
- In practice, some OFDM systems do not insert pilot tones into every OFDM signal. Even in this case, the proposed scheme works well in contrast with the scheme in [26], which exploits pilot tones.
- The biggest difference is that the schemes in [25] and [26] use all the compressed observations without considering the reliability of observations, which may result in including the observations severely contaminated by the channel noise, and thus it leads to inaccurate reconstruction of the clipping noise. However, the proposed scheme selects the observations less contaminated by the channel noise in order to utilize reliable compressed observations. By doing this, we successfully overcome the vulnerability of CS reconstruction to the channel noise. Note that the simulation results in Section IV show that the proposed scheme

Manuscript received March 14, 2014; revised October 28, 2014; accepted November 6, 2014. Date of publication December 18, 2014; date of current version February 28, 2015.

K.-H. Kim and J.-S. No are with the Department of Electrical and Computer Engineering, Institute of New Media and Communications, Seoul National University, Seoul 151-744, Korea (e-mail: kkh@ccl.snu.ac.kr).

H. Park was with the Department of Electrical and Computer Engineering, Institute of New Media and Communications, Seoul National University, Seoul 151-744, Korea. He is now with California Institute for Telecommunications and Information Technology, University of California, San Diego, CA 92093 USA.

H. Chung is with the School of Electronics and Electrical Engineering, Hong-Ik University, Seoul 121-791, Korea.

D.-J. Shin is with the Department of Electronic Engineering, Hanyang University, Seoul 133-791, Korea.

This paper is the contemporary work with the works in [35] and [36]. We made our first paper public at 19 Oct. 2011 (http://arxiv.org/abs/1110.4174v1) in [34], which is earlier than the papers by Al-Safadi and Al-Naffouri [35] and [36]. Of course, this paper has been significantly improved compared to the first paper in [34] for two years. The basic idea of the data aided CS is common to both this paper and the work by Al-Safadi and Al-Naffouri [35] and [36]. We think that it is a just coincidence. There are many differences between the work by Al-Safadi and Al-Naffouri [35] and [36] and this paper. We explained the detailed differences in Section I and showed comparison through simulation in Section IV, which shows that the proposed scheme is more intelligent and shows superior performance than the scheme in [35].

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TBC.2014.2374222

mitigates the clipping noise well over both an additive white Gaussian noise (AWGN) channel and a Rayleigh fading channel.

Also, in [35], the authors in [25] proposed a clipping noise cancelation scheme exploiting reliable observations of data tones, which can be viewed as a contemporary work with our work. The basic idea of the data aided CS is common. However, the approach in [35] is entirely different from our approach.

Firstly, to improve the performance, we exploit a statistical model for a clipped signal derived by using the Bussgang's theorem. But, the scheme in [35] is based on a naive assumption on a clipped signal. In [35], the clipping noise in frequency domain is modeled as complex Gaussian. Secondly, we consider not only the clipping at the Nyquist sampling rate but also the clipping and filtering at an oversampling rate. The scheme in [35] only considers the former case. Note that the latter has been widely studied because it mitigates the peak regrowth after D/A conversion. Thirdly, the scheme in [35] needs optimization of the number of compressed observations for a given signal-to-noise ratio (SNR) point, while our scheme needs no optimization process. Finally, the scheme in [35] only considers the decision error probability of received symbols to measure the reliability of observations, while we also consider a level of channel noise. Due to these differences, our scheme shows superior BER performance than the scheme in [35] as shown in simulation results in Section IV.

This paper is organized as follows. OFDM, clipping, and CS are reviewed in Section II, and a new clipping noise cancelation scheme is proposed in Section III. Section IV presents simulation results, and conclusion is given in Section V.

II. Preliminaries

A. Notation

Upper and lower case letters denote signals in frequency domain and signals in time domain, respectively. The n-th component of a column vector x is denoted as $x(n)$, and bold face letters denote matrices. $E\{\cdot\}$ is the ensemble average operator. $||\cdot||_0$, $||\cdot||_1$, and $||\cdot||_2$ indicate l_0-norm (the number of nonzero elements), l_1-norm, and l_2-norm, respectively. $\text{FFT}_N(\cdot)$ and $\text{IFFT}_N(\cdot)$ denote N-point fast Fourier transform (FFT) and N-point inverse FFT (IFFT), respectively.

B. OFDM and PAPR

Let $X = (X(0), X(1), \ldots, X(N-1))^T$ be an input symbol sequence, where N is the number of subcarriers. The continuous-time baseband OFDM signal is represented as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi kt}{T}\right), \qquad 0 \le t \le T \tag{1}$$

where T is the OFDM signal duration. Let $\Delta t_L = T/LN$ be a sampling interval, where L is an oversampling rate. Then the discrete-time OFDM signal component sampled at time $n\Delta t_L$ can be expressed as

$$x_L(n) = x(n\Delta t_L), \qquad n = 0, 1, \ldots, LN - 1. \tag{2}$$

An L-times oversampled OFDM signal sequence x_L can also be obtained by IFFT after padding X with $(L-1)N$ zeros.

The PAPR of the oversampled OFDM signal sequence x_L is defined as

$$\text{PAPR} = \frac{\max_{0 \le n \le LN-1} |x_L(n)|^2}{E\left\{|x_L(n)|^2\right\}}. \tag{3}$$

C. Clipping

In our proposed scheme, ordinary clipping operation followed by filtering is performed at the transmitter [10]. Due to the filtering, there is no out-of-band radiation.

1) Clipping at the Transmitter: Clipping is performed on the oversampled OFDM signal sequence because it mitigates peak regrowth after D/A conversion. It is known that four-times ($L = 4$) oversampling is sufficient for that purpose [28]. The clipped signal $\bar{x}_L(n)$ is given as

$$\bar{x}_L(n) = \begin{cases} x_L(n), & |x_L(n)| \le A \\ A \cdot e^{j\angle x_L(n)}, & |x_L(n)| > A \end{cases} \tag{4}$$

where A is the clipping threshold. Then the clipping ratio γ is defined as

$$\gamma = \frac{A}{E\{|x_L(n)|\}}. \tag{5}$$

Clearly, the clipping ratio γ can take a value larger than one. In this paper, we consider the range $\gamma \ge 1.3 = 2.278$dB because the proposed scheme can be effectively adopted for that range. Using the clipping ratio in that range still can sufficiently suppress the PAPR of OFDM signals, compared to the other PAPR reduction schemes [1].

The clipped signal $\bar{x}_L(n)$ can be considered as the sum of $x_L(n)$ and the clipping noise $c_L(n)$ as

$$\bar{x}_L(n) = x_L(n) + c_L(n), \qquad 0 \le n \le LN - 1. \tag{6}$$

Since the envelope of $x_L(n)$ is Rayleigh distributed when N is sufficiently large, it is easily shown that the average clipped output energy is [10]

$$E\left\{||\bar{x}_L||_2^2\right\} = \left(1 - e^{-\gamma^2}\right) E\left\{||x_L||_2^2\right\}. \tag{7}$$

In order to remove the out-of-band radiation due to the clipping operation, the clipped signal $\bar{x}_L(n)$ in time domain is transformed to the one in frequency domain by taking LN-point FFT. (Filtering is not needed when $L = 1$, which is referred to as "clipping at the Nyquist sampling rate".) That is, we have $\bar{X}_L = \text{FFT}_{LN}(\bar{x}_L)$. After filtering out the out-of-band components of $\bar{X}_L$, we have clipped input symbol sequence $\bar{X}(k)$ as

$$\bar{X}(k) = X(k) + C(k), \qquad 0 \le k \le N - 1 \tag{8}$$

where $C(k)$ is the clipping noise in frequency domain, and we call $C(k)$ observations of the clipping noise c in this paper.

Finally, $x(n) + c(n)$, $n = 0, 1, \ldots, N-1$, is transmitted, where $x = \text{IFFT}_N(X)$ is the OFDM signal sequence and $c = \text{IFFT}_N(C)$ is the clipping noise which has to be recovered and canceled at the receiver. Fig. 1 summarizes the clipping procedure. With much higher computational complexity, we can also iteratively perform the clipping and filtering procedure. However, we do not consider such iterative case in this paper because it makes the problem very difficult. We leave it as a future work.

2) Statistical Model of Clipped Signals: Using the Bussgang's theorem, it was shown that the clipped signal $\bar{x}_L(n)$ can be statistically decomposed into two uncorrelated parts in [10] as

$$\bar{x}_L(n) = \alpha x_L(n) + d_L(n) \tag{9}$$

where α (≤ 1) is an attenuation factor and $d_L(n)$ is the oversampled clipping noise uncorrelated to $x_L(n)$. The attenuation factor α is given in [10] as

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}\gamma}{2}\text{erfc}(\gamma). \tag{10}$$

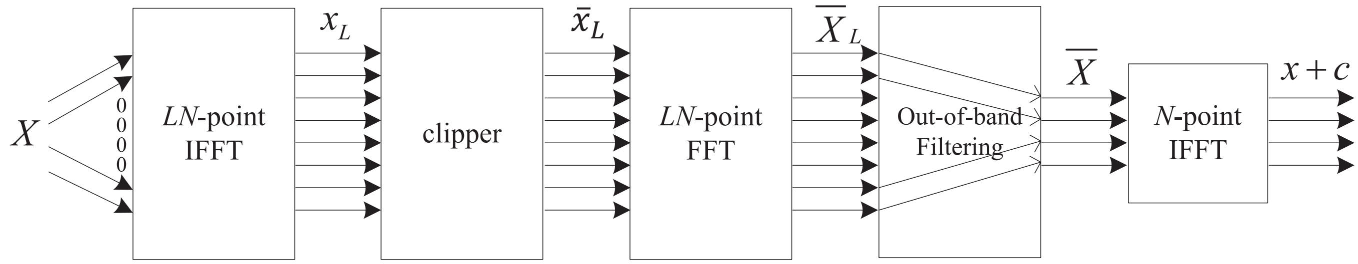

Fig. 1. Example of clipping and filtering when $L = 2$ and $N = 4$.

Note that α is only dependent on γ, and thus α is known at the receiver when γ is fixed. From the equation in (9), the clipped input symbol sequence $\bar{X}(k)$ in (8) can be statistically viewed as

$$\bar{X}(k) = \alpha X(k) + D(k), \qquad 0 \le k \le N-1 \tag{11}$$

where D is the FFTed and out-of-band filtered version of d_L, and clearly $C = (\alpha - 1)X + D$.

$D(k)$ can be assumed to be a complex Gaussian random variable with zero mean and variance $2\sigma^2_{D(k)}$ [10]. For the Nyquist sampling rate ($L = 1$), its variance is easily obtained as

$$\begin{aligned} 2\sigma^2_{D(k)} &= E\left\{|\bar{X}(k)|^2\right\} - \alpha^2 E\left\{|X(k)|^2\right\} \\ &= \left(1 - e^{-\gamma^2} - \alpha^2\right) E\left\{|X(k)|^2\right\} \end{aligned} \tag{12}$$

and then

$$E\left\{|C(k)|^2\right\} = (\alpha - 1)^2 E\left\{|X(k)|^2\right\} + 2\sigma^2_{D(k)} \tag{13}$$
$$= \left(2 - 2\alpha - e^{-\gamma^2}\right) E\left\{|X(k)|^2\right\}. \tag{14}$$

Even if $L > 1$, we can still obtain the values of $2\sigma^2_{D(k)}$ and $E\{|C(k)|^2\}$ for all k's. In many literatures [9]–[11], a power spectral density (PSD) of the oversampled clipping noise d_L is calculated from its autocorrelation function in various forms. For example, the PSD of d_L is given as [9]

$$S_{d_L d_L}(v) = \sum_{n=1}^{\infty} \frac{\beta_n}{\left(E\left\{|X(k)|^2\right\}\right)^{2n+1}} \left[\overbrace{S_{x_L x_L}(v) * \cdots * S_{x_L x_L}(v)}^{2n+1 \text{ convolutions}} \right] \tag{15}$$

where v is the frequency variable, β_n is a coefficient depending on the clipper, and $S_{x_L x_L}(v)$ is the PSD of a non-clipped signal. The exact expression of β_n can be found in [9]. Likewise, the values of $2\sigma^2_{D(k)}$ and $E\{|C(k)|^2\}$ can be calculated and stored in advance for any L. Note that, for all k's within $0 \le k \le N-1$, the values of $2\sigma^2_{D(k)}$ and $E\{|C(k)|^2\}$ when $L > 1$ are smaller than the values of those when $L = 1$ because, when $L > 1$, the clipping noise spreads out over not only in-band but also out-of-band.

3) Conventional Receiver Without Clipping Noise Cancelation Scheme: At the receiver, the received symbol $Y(k)$ in frequency domain can be expressed as

$$Y(k) = H(k)\bar{X}(k) + Z(k), \qquad 0 \le k \le N-1 \tag{16}$$

where $H(k)$ denotes the frequency domain channel response and $Z(k)$ denotes the AWGN with variance $2\sigma^2$. We assume the perfectly known channel response and the perfect synchronization which are widely adopted in many OFDM literatures such as [17] and [18]. After zero-forcing channel equalization, we obtain

$$H^{-1}(k)Y(k) = \bar{X}(k) + H^{-1}(k)Z(k). \tag{17}$$

Then, by plugging (11) into (17), we derive a maximum likelihood (ML) estimator for $X(k)$ as follows.

$$\hat{X}(k) = \arg\min_{s \in \mathcal{X}} |\alpha^{-1}H^{-1}(k)Y(k) - s| \tag{18}$$

where $\mathcal{X}$ is a signal constellation.

D. Compressed Sensing

In a typical CS problem, the goal is to reconstruct an $N \times 1$ *K-sparse signal vector c* from an $M \times 1$ *compressed observation vector* $\tilde{Y}$ under the condition $K \ll M < N$ [20]–[23]. A signal vector c is called K-sparse when it has at most K nonzeros, i.e., $||c||_0 \le K$. Then, c and $\tilde{Y}$ are linearly related to each other as

$$\tilde{Y} = \mathbf{\Phi} c + \eta \tag{19}$$

where $\mathbf{\Phi}$ is an $M \times N$ *sensing matrix* and η is the $M \times 1$ *observation noise vector* with a bounded noise level $||\eta||_2 \le \epsilon$.

To reconstruct c, the following l_1-norm minimization problem, also known as basis pursuit (BP), to obtain $\hat{c}$ is considered in [21].

$$\begin{aligned} &\arg\min_{||c||_0 \le K} \quad ||c||_1 \\ &\text{subject to} \quad ||\mathbf{\Phi} c - \tilde{Y}||_2 \le \epsilon. \end{aligned} \tag{20}$$

It is shown that if the vector c is sufficiently sparse, then the solution $\hat{c}$ in (20) is close to the true solution c within the noise level such as

$$||c - \hat{c}||_2 \le O(1) \cdot \epsilon \tag{21}$$

when the sensing matrix $\mathbf{\Phi}$ satisfies a good restricted isometry property (RIP). In [21], a good RIP says that the matrix $\mathbf{\Phi}$ acts like an almost isometry on all K-sparse vectors c.

Including a BP algorithm given in (20), a number of CS reconstruction algorithms have been proposed [20]–[22]. In this paper, for comparison purpose, we adopt a sparse approximation algorithm called orthogonal matching pursuit (OMP) [27] because of its ease of implementation and speed. Note that OMP is a greedy algorithm which iteratively finds an index whose coefficient is thought to be nonzero based on correlation calculation, and then those coefficients are estimated by least squares.

III. Clipping Noise Cancelation for OFDM Systems Based on CS

A. Sparsity of c

Due to the clipping (and filtering), the clipping noise c is added to the OFDM signal sequence x at the transmitter end. To recover and mitigate the clipping noise c by CS technique at the receiver, c needs to be sparse as much as possible.

1) Sparsity of c When $L = 1$ (Clipping at the Nyquist Sampling Rate): The probability that $|x_1(n)| > A$ is $e^{-\gamma^2}$, due to the fact that the envelope of an OFDM signal sequence is Rayleigh distributed when N is sufficiently large. Also, $x_1(n)$ can be assumed to be i.i.d. random variables. Thus, the average number of nonzero elements in c is $N \cdot e^{-\gamma^2}$. Unless the clipping ratio γ is too small, the clipping noise c can be viewed as a sparse signal. For example, $E\{||c||_0\} = N \cdot e^{-\gamma^2} \leq 0.184 \cdot N$ when $\gamma \geq 1.3 = 2.278$dB.

2) Sparsity of c When L > 1 (Clipping and Filtering at Oversampling Rate): When $L > 1$, c is the $1/L$ downsampled version of the "filtered c_L" of (6) and (8). Thus if the "filtered c_L" has sparsity, c also inherits the sparsity. On the average, the number of nonzero elements in c is $1/L$ of that in c_L. Thus we will investigate the sparsity of the filtered c_L denoted by c'_L.

It is possible that the clipping noise is characterized as a series of parabolic pulses unless γ is too small [12]. The analysis in [12] is based on continuous-time signals, which can be easily extended to the discrete-time case because the oversampling factor L takes a value to make the discrete-time signals similar to the continuous-time signals. That is, the oversampled clipping noise c_L can be represented as

$$c_L(n) = \sum_{i=1}^{N_p} f_i(n), \qquad 0 \leq n \leq LN - 1 \tag{22}$$

where $f_i(n)$ is the i-th clipping parabolic pulse having its maximum amplitude at n_i, and N_p is the number of the parabolic pulses. The average value of N_p is given as [12]

$$E\{N_p\} = N\sqrt{\frac{\pi}{3}}\gamma e^{-\gamma^2} \tag{23}$$

which is usually small one compared to N. For example, $E\{N_p\} \leq 0.245 \cdot N$ when $\gamma \geq 1.3 = 2.278$dB.

Also, the filtered clipping noise vector c'_L can be represented as a sum of N_p sinc functions as

$$c'_L(n) = \sum_{i=1}^{N_p} \alpha_i \cdot \text{sinc}\left(\frac{\pi}{L}(n - n_i)\right), \qquad 0 \leq n \leq LN - 1 \tag{24}$$

where α_i is a coefficient depending on the shape of $f_i(n)$ and $\text{sinc}(x) = \sin(x)/x$.

Unfortunately, c'_L is not an exactly sparse signal, but most of its elements may be close to zero because the sinc function is a sine wave that decays in amplitude. The peak of the first sidelobe is only 21.22% of the peak of the mainlobe and the duration of the mainlobe is only $2L$. After downsampling c'_L to obtain c that we are interested in, the duration of the mainlobe of c is reduced to $2L/L = 2$. Such signals having mostly very small nonzero elements are called compressible, approximately sparse, or relatively sparse in various contexts [21], [31]. For approximately sparse case, it is known that CS techniques can be used to recover c, which will be shown from the simulation results in Section IV.

B. Reconstruction of the Clipping Noise c by CS

In a matrix form, (17) can be rewritten as

$$\mathbf{H}^{-1}Y = \bar{X} + \mathbf{H}^{-1}Z \tag{25}$$

where $\mathbf{H} = \text{diag}(H)$, and Y, $\bar{X}$, and Z are $N \times 1$ column vectors. If we subtract $\hat{X}$ in (18) from (25), we have

$$\mathbf{H}^{-1}Y - \hat{X} = \underbrace{C}_{\text{noiseless observation vector}} + \underbrace{X - \hat{X} + \mathbf{H}^{-1}Z}_{\text{observation noise vector}} \tag{26}$$

which is the sum of the noiseless observation vector and the observation noise vector.

It is obvious that if the whole observations (or whole tones) are used, reconstruction of the clipping noise c may be inaccurate due to the fact that some observations are severely contaminated by the observation noise. Therefore, our suggestion here is to select a reliable subset of the whole observations $\mathbf{H}^{-1}Y - \hat{X}$, namely M out of N components, and then we can obtain an $M \times 1$ compressed observation vector $\tilde{Y}$. This process can be done by multiplying an $M \times N$ selection matrix $\mathbf{S}$ consisting of some M rows of $N \times N$ identity matrix $\mathbf{I}_N$. Such selection strategy will be described in the next subsection. Let $C = \mathbf{F}c$, where $\mathbf{F}$ is an $N \times N$ unitary discrete Fourier transform (DFT) matrix. Then we have

$$\begin{aligned}\tilde{Y} = \mathbf{S}\mathbf{H}^{-1}Y - \mathbf{S}\hat{X} &= \mathbf{SF}c + \mathbf{S}(X - \hat{X}) + \mathbf{S}\mathbf{H}^{-1}Z \\ &= \mathbf{\Phi}c + \underbrace{\mathbf{S}(X - \hat{X}) + \mathbf{S}\mathbf{H}^{-1}Z}_{\text{observation noise vector}} \\ &= \mathbf{\Phi}c + \eta\end{aligned} \tag{27}$$

where the matrix $\mathbf{\Phi} = \mathbf{SF}$ can be considered as the $M \times N$ *sensing matrix* from the view of CS. As one can see from [21], a sensing matrix for CS can be constructed by using a subset of rows in a DFT matrix, which shows a good RIP. Then (27) can be considered as a CS problem given in (19), where $\tilde{Y}$ is the $M \times 1$ *compressed observation vector*, the clipping noise c is the $N \times 1$ *sparse signal vector*, and $\eta = \mathbf{S}(X - \hat{X}) + \mathbf{S}\mathbf{H}^{-1}Z$ is the $M \times 1$ *observation noise vector*.

By using a CS recovery algorithm such as OMP, we can recover c denoted as $\hat{c}$ from the compressed observation vector $\tilde{Y}$ in (27). Then $\hat{C} = \text{FFT}_N(\hat{c})$ is subtracted from the equalized received symbol sequence $\mathbf{H}^{-1}Y$, and then the final decision $\hat{X}_{\text{final}}$ is made as

$$\hat{X}_{\text{final}}(k) = \arg\min_{s \in \mathcal{X}} |H^{-1}(k)Y(k) - \hat{C}(k) - s|. \tag{28}$$

Fig. 2 pictorially summarizes the proposed scheme, where y is the received OFDM signal sequence.

C. Construction of the Compressed Observation Vector $\tilde{Y}$

As we already mentioned, the compressed observation vector can be obtained by selecting some reliable components of the whole observations $\mathbf{H}^{-1}Y - \hat{X}$ whose k-th component is given as

$$\begin{aligned}H^{-1}(k)Y(k) - \hat{X}(k) = &\underbrace{C(k)}_{\text{noiseless observation}} \\ &+ \underbrace{H^{-1}(k)Z(k) + X(k) - \hat{X}(k)}_{\text{observation noise}}.\end{aligned} \tag{29}$$

1) Which Observations Should be Selected?: The observations less contaminated by observation noise should be selected. For convenience, we will use $\theta(k)$ to denote the observation noise in (29) as

$$\theta(k) = H^{-1}(k)Z(k) + X(k) - \hat{X}(k). \tag{30}$$

That is, we have to select reliable observations which contain small $|\theta(k)|$.

2) Estimation of $\theta(k)$ Based on $H^{-1}(k)Y(k)$: In this subsubsection, we will derive the minimum mean square error (MMSE) estimator of $\theta(k)$, $\hat{\theta}(k)$. For convenience, we separate $\theta(k)$ into two parts, $\theta_0(k)$ and $\theta_1(k)$, as

$$\theta(k) = \underbrace{H^{-1}(k)Z(k)}_{\theta_0(k)} + \underbrace{X(k) - \hat{X}(k)}_{\theta_1(k)}. \tag{31}$$

Also, we treat the equalized received symbol $H^{-1}(k)Y(k)$ as observation $o(k)$ as

$$o(k) = H^{-1}(k)Y(k). \tag{32}$$

Then $\hat{\theta}(k)$ can be separately obtained by

$$\hat{\theta}(k) = E\{\theta(k) \mid o(k)\} = E\{\theta_0(k) \mid o(k)\} + E\{\theta_1(k) \mid o(k)\}. \tag{33}$$

For simplicity, we drop the subcarrier index k in the following derivation.

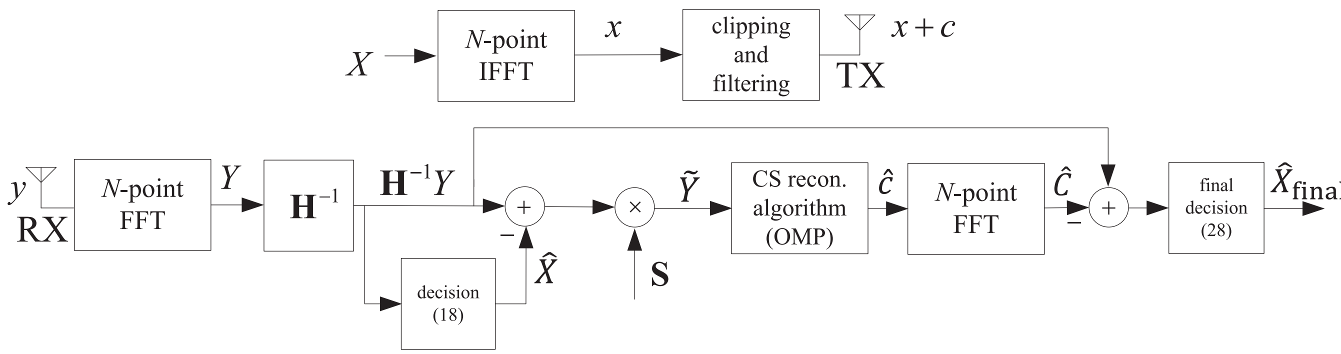

Fig. 2. Block diagram of the proposed clipping noise cancelation scheme.

First, from (11) and (17), θ_0 is linearly related to observation o as

$$o = \theta_0 + \alpha X + D \tag{34}$$

and

$$\begin{aligned}\theta_0 &\sim \mathcal{CN}(0, 2|H^{-1}|^2\sigma^2)\\ D &\sim \mathcal{CN}(0, 2\sigma_D^2)\\ X &= \mathcal{X}_i \text{ with probability } \frac{1}{|\mathcal{X}|}\end{aligned} \tag{35}$$

where $\mathcal{X}_i$ denotes the i-th constellation point of signal constellation $\mathcal{X}$, and $|\mathcal{X}|$ denotes signal constellation size. We assume that each constellation point can be transmitted with equal probability.

Then, $E\{\theta_0 \mid o\}$ can be expressed as

$$E\{\theta_0 \mid o\} = \sum_{X\in\mathcal{X}} E\{\theta_0 \mid X, o\} \cdot p(X \mid o) \tag{36}$$

where $p(\cdot)$ and $p(\cdot|\cdot)$ denote probability density function (PDF) and conditional PDF, respectively.

It is widely known that $E\{\theta_0 \mid X, o\}$ in (36) is [33]

$$E\{\theta_0 \mid X, o\} = \frac{|H^{-1}|^2\sigma^2}{|H^{-1}|^2\sigma^2 + \sigma_D^2}(o - \alpha X) \tag{37}$$

and the conditional PDF $p(X \mid o)$ in (36) is

$$\begin{aligned}p(X \mid o) &= \frac{p(o \mid X)p(X)}{p(o)}\\ &= \text{const} \cdot p(o \mid X)\\ &= \frac{p(o \mid X)}{\sum_{X'\in\mathcal{X}} p(o \mid X')}\end{aligned} \tag{38}$$

where

$$p(o \mid X) = \frac{1}{2\pi\left(|H^{-1}|^2\sigma^2 + \sigma_D^2\right)} \cdot \exp\left[-\frac{1}{2\left(|H^{-1}|^2\sigma^2 + \sigma_D^2\right)} \mid o - \alpha X \mid^2\right]. \tag{39}$$

After plugging (37) and (38) into (36), we have

$$E\{\theta_0 \mid o\} = \sum_{X\in\mathcal{X}} \frac{|H^{-1}|^2\sigma^2}{|H^{-1}|^2\sigma^2 + \sigma_D^2}(o - \alpha X) \cdot \frac{p(o \mid X)}{\sum_{X'\in\mathcal{X}} p(o \mid X')}. \tag{40}$$

Second, $E\{\theta_1 \mid o\}$ is expressed as

$$\begin{aligned}E\{\theta_1 \mid o\} &= E\{\theta_1 \mid o, \hat{X}\}\\ &= \sum_{X\in\mathcal{X}} (X - \hat{X}) \cdot p(X \mid o)\\ &= \sum_{X\in\mathcal{X}} (X - \hat{X}) \cdot \frac{p(o \mid X)}{\sum_{X'\in\mathcal{X}} p(o \mid X')}.\end{aligned} \tag{41}$$

Finally, combining (40) and (41), the MMSE estimator of θ is

$$\hat{\theta} = \sum_{X\in\mathcal{X}} \left(\frac{|H^{-1}|^2\sigma^2}{|H^{-1}|^2\sigma^2 + \sigma_D^2}(o - \alpha X) + (X - \hat{X})\right) \cdot \frac{p(o \mid X)}{\sum_{X'\in\mathcal{X}} p(o \mid X')}. \tag{42}$$

For systems in which high computational complexity is not allowed, it is too complicated to use the estimator in (42). Thus, we propose the low-complexity version of the MMSE estimator of θ by using the following approximation;

$$\sum_{X\in\mathcal{X}} X \cdot \frac{p(o \mid X)}{\sum_{X'\in\mathcal{X}} p(o \mid X')} \simeq \hat{X}. \tag{43}$$

By plugging (43) into (42), the MMSE estimator $\hat{\theta}$ is approximated as

$$\hat{\theta} \simeq \frac{|H^{-1}|^2\sigma^2}{|H^{-1}|^2\sigma^2 + \sigma_D^2}(o - \alpha\hat{X}). \tag{44}$$

In simulations, the above estimator is used for ease of implementation.

3) Selection Criterion of Observations: As we mentioned, based on the estimate of $\theta(k)$, we can select reliable observations which will be used to recover the clipping noise c. Furthermore, we use a selection criterion which selects the observations whose observation noise level is lower than the average noiseless observation power. That is, the following selection criterion is used in the proposed scheme;

$$\mathcal{K} = \left\{k : |\hat{\theta}(k)|^2 < E\left\{|C(k)|^2\right\}\right\}. \tag{45}$$

If the cardinality of $\mathcal{K}$ is M, an $M \times N$ selection matrix $\mathbf{S}$ in (27) is constructed by selecting the corresponding M rows from $\mathbf{I}_N$.

In some cases, the cardinality of the set $\mathcal{K}$ is too small to recover the clipping noise stably. In other words, there are insufficient reliable observations. In this case, it is better to use the conventional receiver without any CS recovery. Thus, we introduce some threshold M_{min}. For stability, the proposed algorithm only works when $|\mathcal{K}| = M > M_{min}$.

Kunis and Rauhut have studied the performance of OMP for signal recovery from random frequency measurements [37]. Their work suggests that $O(K \cdot \ln(N))$ measurements are sufficient for OMP to recover an K-sparse signal. Unfortunately, their work may be not suitable for very large K case, and thus we adopted $M_{min} = 0.8N$ in this case, which is empirically obtained. To sum it up, we set the threshold

$$M_{min} = \min\left(C \cdot E\{K\}\ln(N),\ 0.8N\right) \tag{46}$$

where our massive simulation gives that the constant $C = 0.8$ is suitable for our situation. $E\{K\} = E\{||c||_0\}$ for $L = 1$ and $E\{K\} = E\{N_p\}$ for $L = 4$.

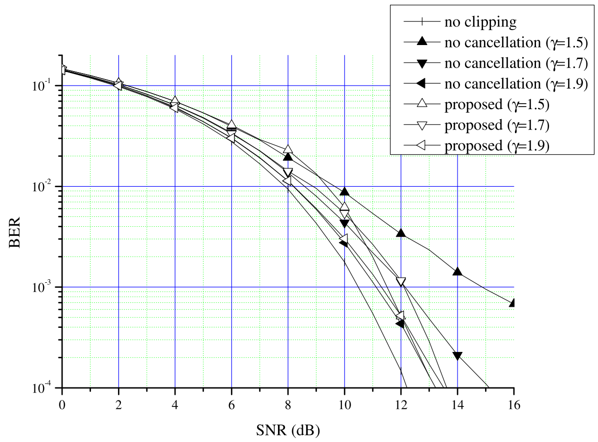

Fig. 3. BER performance of the proposed scheme for various clipping ratios γ over an AWGN channel when $L = 1$, $N = 128$, and 16-QAM are used.

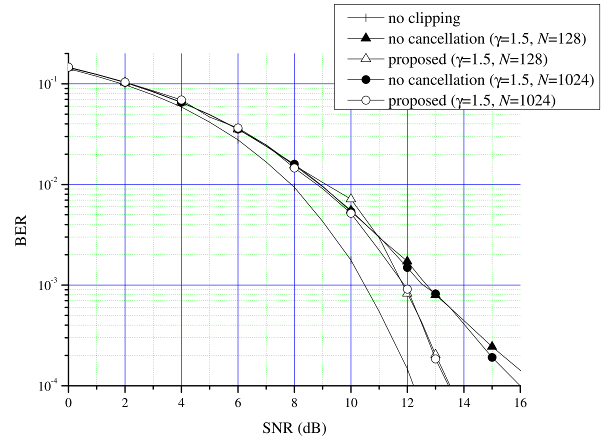

Fig. 4. BER performance of the proposed scheme for various N over an AWGN channel when $L = 4$ and 16-QAM are used.

D. Computational Complexity

In terms of computational complexity, the CS reconstruction algorithm part of the proposed scheme is a dominant factor, and each iteration of the OMP algorithm approximately requires the complexity of one N-point FFT because the sensing matrix in (27) is an $M \times N$ partial Fourier matrix [27].

For ease of implementation, in the proposed scheme, the number of iterations of the OMP algorithm is a half of the number of average dominant pulses of the clipping noise c ($0.5 \cdot E\{||c||_0\}$ for $L = 1$ and $0.5 \cdot E\{N_p\}$ for $L = 4$), based on the discussion in Section III-A. That is, we run the OMP algorithm $0.5N \cdot e^{-\gamma^2}$ times and $0.5N \cdot \sqrt{\frac{\pi}{3}}\gamma e^{-\gamma^2}$ times for $L = 1$ and $L = 4$, respectively. Although this number of iterations seems to be insufficient to recover the whole pulses of the clipping noise, it still shows good performance as shown in Section IV. This is because, in practice, the number of dominant pulses in the clipping noise are usually less than the number that we inferred in Section III-A, owing to the fact that an OFDM signal sequence is under the total energy constraint by the Parseval's theorem.

IV. Simulation Results

In this section, we evaluate the BER performance of the proposed clipping noise cancelation scheme for uncoded OFDM systems similarly as other works [18], [25]. Here, the SNR means the ratio of bit energy to the variance of AWGN. We simulate both the case of clipping at the Nyquist sampling rate ($L = 1$) and the case of clipping and filtering at an oversampling rate ($L = 4$) over an AWGN channel and a Rayleigh fading channel. To confirm the validity of the proposed scheme, we compare the proposed scheme with existing clipping noise cancelation schemes over a Rayleigh fading channel.

A. AWGN Channel

Fig. 3 shows the BER performance of the proposed scheme over an AWGN channel when the clipping at the Nyquist sampling rate is used. Also, 16-quadrature amplitude modulation (16-QAM) is used. As we mentioned, the proposed scheme can be effectively adopted for not too small clipping ratio, and thus various clipping ratios are used for simulations. Although the proposed scheme shows performance degradation in low SNR region, it clearly shows that the proposed scheme performs well for various clipping ratios in moderate SNR region that we are interested in.

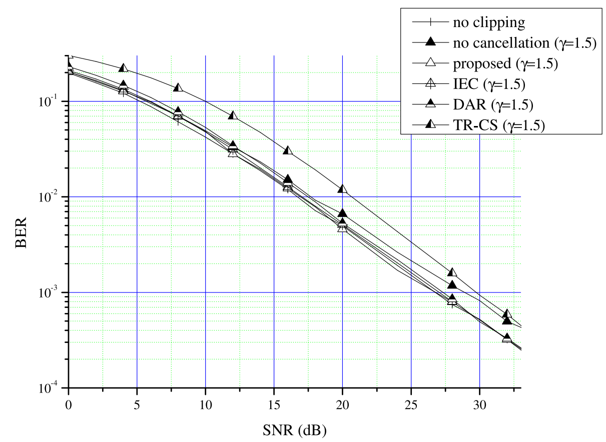

Fig. 5. BER comparison of the proposed scheme and the existing clipping noise cancelation schemes (IEC [17], DAR [18], and TR-CS [25]) over a frequency-selective fading channel when $L = 1$, $N = 128$, and 16-QAM are used.

Fig. 4 shows the BER performance of the proposed scheme over an AWGN channel when the clipping and filtering at four-times oversampling rate is used. Clipping ratio γ is fixed to $1.5 = 3.52$dB. In this case, the proposed scheme also performs well in moderate SNR region, even though the clipping noise c is not an exactly sparse signal when oversampling is used. Also, the proposed schemes performs well regardless of the number of subcarriers N.

B. Rayleigh Fading Channel

In this subsection, we compare the proposed scheme with other clipping noise cancelation schemes over a fading channel. The length of the channel is assumed to be four and the channel taps are assumed to be complex Gaussian distributed with zero-mean and variance $1/4$, i.e., frequency-selective fading channel where coefficients of taps are Rayleigh distributed. More precisely, the $N \times 1$ channel impulse response h is modeled as

$$h = (h(0), h(1), h(2), h(3), 0, \cdots, 0)^T \tag{47}$$

where $h(0), h(1), h(2), h(3) \sim \mathcal{CN}(0, 1/4)$ and $H = \mathrm{FFT}_N(h)$ shows a frequency-selective fading channel. We assume the perfect knowledge of the channel frequency response at the receiver as other works [17], [18].

Figs. 5–7 compare the BER performance of the proposed scheme, IEC [17], DAR [18], and TR-CS [25] for two sampling rates and

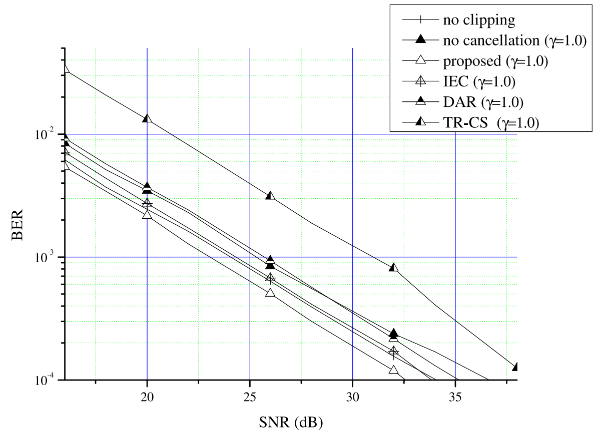

Fig. 6. BER comparison of the proposed scheme and the existing clipping noise cancelation schemes (IEC [17], DAR [18], and TR-CS [25]) over a frequency-selective fading channel when $L = 1$, $N = 128$, and QPSK are used.

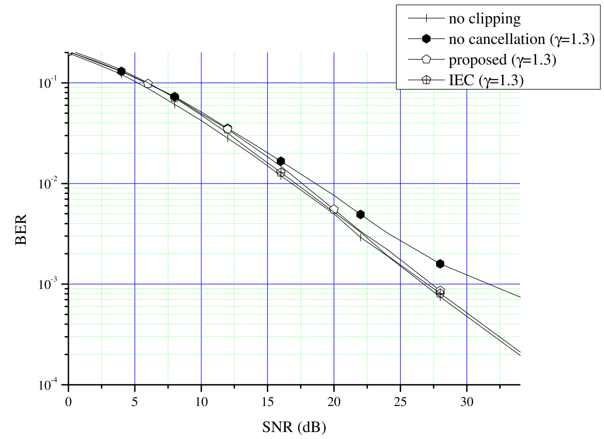

Fig. 7. BER comparison of the proposed scheme and the IEC scheme [17] over a frequency-selective fading channel when $L = 4$, $N = 128$, and 16-QAM are used.

modulations. In Fig. 7, only the BER performance of the IEC scheme and the proposed scheme are given because the others do not work when clipping and filtering at an oversampling rate is used. Also, in TR-CS, the OMP algorithm is used for CS reconstruction for a fair comparison. In Fig. 6, quadrature phase shift keying (QPSK) and severe clipping ($\gamma = 1.0$) are used in order to show the performance gap clearly. Although the nonzero components of the clipping noise c becomes too large to keep the sparsity of c, the proposed scheme works well because the dominant pulses of the clipping noise are recovered by CS technique.

Firstly, the tone reservation by CS (TR-CS) scheme in [25] shows poor BER performance due to the weakness of CS reconstruction to the channel noise, although it uses 41 reserved tones out of 128 tones, which seems to be sufficient. Meanwhile, the scheme in [26] uses compressed observations of pilot tones. Thus, from the result, one can expect that the scheme in [26] also shows poor BER performance unless the number of pilot tones is larger than 41, which becomes unpractical. As we discussed, the absence of the selecting process of reliable observations seems to result in poor BER performance of the previous works [25], [26] using CS.

Secondly, the DAR scheme in [18] is the most frequently cited scheme among the clipping noise cancelation schemes outside CS regime. The DAR scheme shows good BER performance, but it works only for the case when clipping at the Nyquist sampling rate is used. This limitation is a major drawback of the DAR scheme in a practical sense.

Thirdly, the BER performance of the IEC scheme in [17] is also given. The scheme in [17] is based on iterative estimation and cancelation of the clipping noise. In Figs. 5–7, the IEC scheme works well for two sampling rates similarly as the proposed scheme. But, in Fig. 6, the proposed scheme shows better BER performance than the IEC scheme. This is because of hard clipping with 0dB. The fundamental of IEC scheme is to recreate the clipping process at the receiver. The IEC scheme has no process to select low noise or reliable observations. Then severely contaminated received symbols due to hard clipping usually induce error propagation in the recreated clipping process at the receiver. However, our scheme exploits only low noise observations by selecting reliable tones.

In terms of computational complexity, the IEC scheme requires two LN-point FFT / IFFT pairs to recreate the clipping process regardless of the clipping ratio. Thus, the proposed scheme has a computational benefit over the IEC scheme for a large clipping ratio and small N as described in Section III-D.

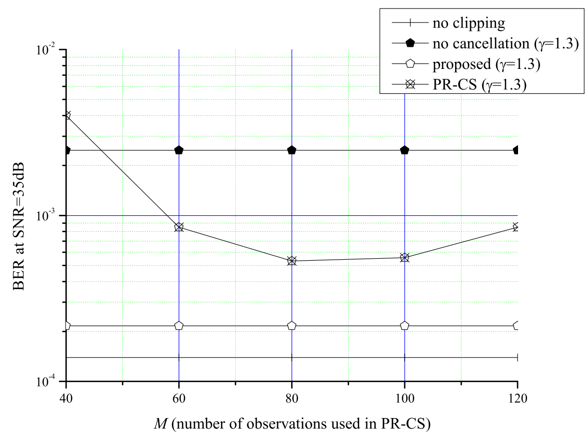

Fig. 8. BER comparison of the proposed scheme and the PR-CS scheme [35] over a frequency-selective fading channel when $L = 1$, $N = 128$, and 16-QAM are used.

In Fig. 6, the proposed scheme shows the best BER performance compared to the other schemes. Moreover, it is worth noting that the proposed scheme shows better BER performance than the no clipping case. This is because the clipping procedure can reduce the average transmission power compared to the no clipping case, and this results in BER performance gain, which is called the shaping gain.

Fig. 8 compares the BER performance of the proposed scheme and the PR-CS scheme in [35]. In Fig. 8, clipping at the Nyquist sampling is used and $\gamma = 1.3$ is used. In PR-CS scheme [35], the number of compressed observations M needs to be optimized. Thus, BER at SNR=35dB is plotted versus the number of compressed observations of PR-CS [35]. Note that our scheme adaptively sets the number of compressed observations according to (45), which is more intelligent than the PR-CS scheme. In PR-CS, the OMP algorithm is used for CS reconstruction for a fair comparison. Clearly, the proposed scheme shows superior BER performance compared to the PR-CS scheme [35].

V. Conclusion

In this paper, a new clipping noise cancelation scheme using CS for OFDM systems is proposed. To reconstruct the clipping noise, the proposed scheme exploits its compressed observations underlying in the data tones less contaminated by channel noise. To do this, an observation noise level in each data tone is estimated by exploiting the statistical model of a clipped signal. The proposed clipping noise cancelation scheme cancels out the clipping noise well over an AWGN channel and a frequency-selective fading channel, which is verified through simulations. Compared with the previous schemes, the proposed scheme substantially improves the reconstruction performance by adopting the selection of reliable observations.

References

[1] D.-W. Lim, S.-J. Heo, and J.-S. No, "An overview of peak-to-average power ratio reduction schemes for OFDM signals," *J. Commun. Netw.*, vol. 11, no. 3, pp. 229–239, Jun. 2009.

[2] G. E. Arrobo and R. D. Gitlin, "Improving the performance of OFDM-based vehicular systems through diversity coding," *J. Commun. Netw.*, vol. 15, no. 2, pp. 132–141, Apr. 2013.

[3] G. Wunder, R. F. H. Fischer, H. Boche, S. Litsyn, and J.-S. No, "The PAPR problem in OFDM transmission: New directions for a long-lasting problem," *IEEE Signal Process. Mag.*, vol. 30, no. 6, pp. 130–144, Nov. 2013.

[4] H.-B. Jeon, K.-H. Kim, J.-S. No, and D.-J. Shin, "Bit-based SLM schemes for PAPR reduction in QAM modulated OFDM signals," *IEEE Trans. Broadcast.*, vol. 55, no. 3, pp. 679–685, Sep. 2009.

[5] S. H. Muller, R. W. Bauml, R. F. H. Fischer, and J. B. Huber, "OFDM with reduced peak-to-average power ratio by multiple signal representation," *Ann. Telecommun.*, vol. 52, nos. 1–2, pp. 58–67, Feb. 1997.

[6] S.-J. Heo, H.-S. Noh, J.-S. No, and D.-J. Shin, "Modified SLM scheme with low complexity for PAPR reduction of OFDM systems," *IEEE Trans. Broadcast.*, vol. 53, no. 4, pp. 804–808, Dec. 2007.

[7] S.-H. Wang, J.-C. Sie, C.-P. Li, and Y.-F. Chen, "A low-complexity PAPR reduction scheme for OFDMA uplink systems," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1242–1251, Apr. 2011.

[8] X. Li and L. J. Cimini, Jr., "Effects of clipping and filtering on the performance of OFDM," *IEEE Commun. Lett.*, vol. 2, no. 5, pp. 131–133, May 1998.

[9] P. Banelli and S. Cacopardi, "Theoretical analysis and performance of OFDM signals in nonlinear AWGN channels," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 430–441, Mar. 2000.

[10] H. Ochiai and H. Imai, "Performance analysis of deliberately clipped OFDM signals," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 89–101, Jan. 2002.

[11] E. Costa, M. Midrio, and S. Pupolin, "Impact of amplifier nonlinearities on OFDM transmission system performance," *IEEE Commun. Lett.*, vol. 3, no. 2, pp. 37–39, Feb. 1999.

[12] L. Wang and C. Tellambura, "Analysis of clipping noise and tone-reservation algorithms for peak reduction in OFDM systems," *IEEE Trans. Veh. Technol.*, vol. 57, no. 3, pp. 1675–1694, May 2008.

[13] U.-K. Kwon, D. Kim, and G.-H. Im, "Amplitude clipping and iterative reconstruction of MIMO-OFDM signals with optimum equalization," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 268–277, Jan. 2009.

[14] R. J. Baxely, C. Zhao, and G. T. Zhou, "Constrained clipping for crest factor reduction in OFDM," *IEEE Trans. Broadcast.*, vol. 52, no. 4, pp. 570–575, Dec. 2006.

[15] Y. Wang, W. Chen, and C. Tellambura, "Genetic algorithm based nearly optimal peak reduction tone set selection for adaptive amplitude clipping PAPR reduction," *IEEE Trans. Broadcast.*, vol. 58, no. 3, pp. 462–471, Sep. 2012.

[16] H. Saeedi, M. Sharif, and F. Marvasti, "Clipping noise cancellation in OFDM systems using oversampled signal reconstruction," *IEEE Commun. Lett.*, vol. 6, no. 2, pp. 73–75, Feb. 2002.

[17] H. Chen and A. M. Haimovich, "Iterative estimation and cancellation of clipping noise for OFDM signals," *IEEE Commun. Lett.*, vol. 7, no. 7, pp. 305–307, Jul. 2003.

[18] D. Kim and G. L. Stuber, "Clipping noise mitigation for OFDM by decision-aided reconstruction," *IEEE Commun. Lett.*, vol. 3, no. 1, pp. 4–6, Jan. 1999.

[19] Y. Chen, J. Zhang, and A. D. S. Jayalath, "Estimation and compensation of clipping noise in OFDMA systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 523–527, Feb. 2010.

[20] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[21] E. J. Candes, J. K. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Commun. Pure Appl. Math.*, vol. 59, no. 8, pp. 1207–1223, Aug. 2006.

[22] E. J Candes and T. Tao, "Near-optimal signal recovery from random projections: Universal encoding strategies?" *IEEE Trans. Inf. Theory*, vol. 52, no. 12, pp. 5406–5425, Dec. 2006.

[23] S. J. Wright, R. D. Nowak, and M. A. T. Figueiredo, "Sparse reconstruction by separable approximation," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2479–2493, Jul. 2009.

[24] E. B. Al-Safadi and T. Y. Al-Naffouri, "On reducing the complexity of tone reservation based PAPR reduction schemes by compressive sensing," in *Proc. IEEE Conf. Global Telecommun. (GLOBECOM)*, Honolulu, HI, USA, Nov. 2009, pp. 1905–1910.

[25] E. B. Al-Safadi and T. Y. Al-Naffouri, "Peak reduction and clipping mitigation in OFDM by augmented compressive sensing," *IEEE Trans. Signal Process.*, vol. 60, no. 7, pp. 3834–3839, Jul. 2012.

[26] M. Mohammadnia-Avval, A. Ghassemi, and L. Lampe, "Compressive sensing recovery of nonlinearity distorted OFDM signals," in *Proc. IEEE Int. Conf. Commun.*, Kyoto, Japan, Jun. 2011, pp. 1–5.

[27] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4655–4666, Dec. 2007.

[28] M. Sharif, M. Gharavi-Alkansari, and B. H. Khalaj, "On the peak-to-average power of OFDM signals based on oversampling," *IEEE Trans. Commun.*, vol. 51, no. 1, pp. 72–78, Jan. 2003.

[29] J. G. Proakis and M. Salehi, *Communication Systems Engineering*, 2nd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[30] D. Gordon. (Aug. 1, 2011). *La Jolla Difference Set Repository*. [Online]. Available: http://www.ccrwest.org/diffsets/diff_sets/baumert.html

[31] T. T. Cai and L. Wang, "Orthogonal matching pursuit for sparse signal recovery with noise," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4680–4688, Jul. 2011.

[32] R. AliHemmati and P. Azmi, "Iterative reconstruction-based method for clipping noise suppression in OFDM systems," *IEE Proc. Commun.*, vol. 152, no. 4, pp. 452–456, Aug. 2005.

[33] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ, USA: Prentice-Hall, 1993.

[34] K.-H. Kim, H. Park, J.-S. No, and H. Chung, "Clipping noise cancellation for OFDM and OFDMA systems using compressed sensing," *IEEE Trans. Signal Process.* Oct. 2011. [Online]. Available: http://arxiv.org/pdf/1110.4174v1.pdf

[35] E. B. Al-Safadi and T. Y. Al-Naffouri, "Pilotless recovery of nonlinearly distorted OFDM signals by compressive sensing over reliable data carriers," in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Cesme, Turkey, Jun. 2012, pp. 580–584.

[36] E. B. Al-Safadi and T. Y. Al-Naffouri, "Pilotless recovery of clipped OFDM signals by compressive sensing over reliable data carriers," arXiv:1111.3274 [cs.IT], Nov. 2011. [Online]. Available: http://arxiv.org/pdf/1111.3274v1.pdf

[37] S. Kunis and H. Rauhut, "Random sampling of sparse trigonometric polynomials II: Orthogonal matching pursuit versus basis pursuit," *Found. Comput. Math.*, vol. 8, no. 6, pp. 737–763, 2008.