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Minimum number of antennas and degrees of freedom of multiple-input–multiple-output multi-user two-way relay X channels

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Abstract: In this study, the minimum number of antennas at each user is derived to obtain $2M^2$ degrees of freedom (DOF) in the $M \times 1 \times M$ multiple-input–multiple-output (MIMO) multi-user two-way relay X channels. Based on the design of beamforming vectors for the conventional signal space alignment scheme, it is shown that each user needs at least $\lceil M^2 - (M/2) \rceil$ antennas to obtain DOF $2M^2$ with the relay having M^2 antennas. As the number of users increases, the number of antennas also increases, which makes the implementation difficult. In an effort to reduce the number of antennas, the authors propose a new beamforming scheme of MIMO multi-user two-way relay X channels using the time extension. Through the numerical analysis, it is confirmed that the proposed scheme with time extension is a good alternative scheme to replace the conventional scheme.

1 Introduction

In wireless communication systems, various transmit and receive strategies have been studied to overcome the limitation of radio resources (i.e. time and frequency). Owing to the broadcast nature of the wireless systems, concurrent transmissions over the same frequency band naturally cause interference. Recently, many researchers have investigated into various signalling schemes to successfully deal with the interference problem so that network capacity may be increased. Two signalling schemes have received increasing attention to handle the interference problem: interference alignment and network coding. Interference alignment has been proposed to achieve the optimal multiplexing gain in the interference channel. The main idea of interference alignment was proposed in [1–3]. Cadambe and Jafar [1] shown that the capacity of the K-user time-varying interference channel is characterised as

$$C(\text{SNR}) = \frac{K}{2} \log(\text{SNR}) + o(\log(\text{SNR})) \tag{1}$$

by using interference alignment. It is a surprising result since before the idea of interference alignment, it was believed that the sum capacity for the K-user interference channel is [4]

$$C(\text{SNR}) = \log(\text{SNR}) + o(\log(\text{SNR})) \tag{2}$$

which is achievable by cake-cutting approaches such as orthogonal access schemes, that is, time-division multiple access (TDMA) and frequency-division multiple access.

Network coding is a technique that, instead of simply relaying the messages they receive, the nodes of a network take several messages and combine them together for efficient transmission. The fundamental concept of network coding was first introduced in [5] and then fully developed in [6]. Since then, network coding has generated huge interest in information and coding areas, networking, switching and wireless communications. Ahlswede *et al.* [6] studied the information flow in an arbitrary network with a single source whose data is multicast to a collection of destinations, called sinks. Li *et al.* [7] showed that the multicast network capacity is the minimum of the max-flow of the information from the source to the multicast sinks.

Relaying system in wireless networks provides energy efficiency, robustness and extended capacity. Capacity in relay channel was studied in [8]. Recently, two-way relay channels have attracted increasing research interest because of their high spectral efficiency [9–13]. Sezgin *et al.* [9] characterised the capacity of multi-pair two-way relay channels in the Gaussian channel. In [10, 11], the idea that two users transmit data sequentially and the relay broadcasts an XORed version of two users' data after decoding both of them is proposed. In the multiple-input–multiple-output (MIMO) two-way relay channels, the diversity gain can also be achieved by several different methods in [12, 13]. Furthermore, some novel transmission schemes combining the interference alignment and network coding for the multi-way relay channels have been developed in [14, 15].

In this paper, the $M \times 1 \times M$ MIMO multi-user two-way relay X channels are considered where M users with K antennas exist in each side and the relay has M^2 antennas. And it is also assumed that the global channel state information (CSI) is available at each node and every channel is quasi-static fading one. In order to obtain $2M^2$ degrees of freedom (DOF), we derive the least number of antennas at each user based on the scheme for $M \times 1 \times M$ MIMO multi-user two-way relay X channels in [16]. Long *et al.* [17] proposed an efficient transmission scheme for the multi-user two-way relay X channel, which requires less antennas than our proposed scheme to achieve the same DOF. However, our proposed scheme has several benefits. In the broadcast channel (BC) phase of the scheme in [17], the relay utilises the ZF-based linear precoding (also known as the 'channel inversion') to eliminate all the co-channel interferences at each node. A channel inversion incurs a power penalty at the relay, that is, a large amount of transmit power is required for inverting eigenmodes of the channel matrix having very small power gains [18]. Since our proposed scheme utilises interference nulling beamforming (INB) instead of channel inversion beamforming, it achieves a higher signal-to-noise ratio and better bit error rate performance than the scheme in [17]. Further, the scheme in [17] aligns each user pair's received signal space during the BC phase. Then, each user requires not only its own CSI at the receiver (CSIR) but also other group users' CSIR to design the decoding alignment vectors in the BC phase. That requires lots of channel overheads to be transmitted. However, since our scheme does not utilise the signal space alignment in the BC phase, each user requires only its own

CSIR. Thus, our proposed scheme does not require the extra CSIR transmission in the BC phase.

As the number of users increases, the number of antennas for also increases, which makes the implementation difficult. Thus, a new beamforming scheme for $M \times 1 \times M$ multi-user MIMO two-way relay X channels with time extension is proposed, which requires the reduced number of antennas but achieves the same DOF as that of the previous scheme. In brief, the scheme in [16] is generalised in an environment with $M > 2$ users and a new alternative scheme using time extension is proposed in this paper. In the proposed scheme, the received signal spaces are reserved to facilitate the alignment for the desired signal and interference signal.

This paper is organised as follows. In Section 2, we investigate into the $M \times 1 \times M$ MIMO multi-user two-way relay X channels and then we derive the minimum number of antennas at each user of MIMO multi-user two-way relay X channels achieving DOF $2M^2$. In Section 3, a new beamforming scheme with M-time extension is proposed for $M \times 1 \times M$ MIMO multi-user two-way relay X channels and we show how it works in the multiple access channel (MAC) and BC phases and its DOF is the same as that of the previous scheme in Section 2. In Section 4, the sum-rate of the proposed scheme is shown through the numerical analysis. Concluding remarks are given in Section 5.

2 Minimum number of antennas for $M \times 1 \times M$ MIMO multi-user two-way relay X channels

In this section, we investigate into $M \times 1 \times M$ MIMO multi-user two-way relay X channels with M users in each side and one relay. Especially, we derive the minimum number of antennas for each user to achieve DOF $2M^2$ in this network for the relay having M^2 antennas.

For the signal space alignment, the relay should provide M^2 dimensions because two independent signals can be aligned along one vector and we have $2M^2$ independent signals to be received at the relay. It implies that the received signal spaces should be an M^2-dimensional space, therefore M^2 antennas are required at the relay. In fact, $2M^2$ is not the maximum DOF for $M \times 1 \times M$ two-way relay X channel. However, if we assume that each node in one side needs M DOF in order to communicate with M nodes in the other side, $2M^2$ DOF is the minimum DOF that is required in the $M \times 1 \times M$ two-way relay X channel. Therefore we focus on achieving $2M^2$ DOF.

2.1 System model

Consider the $M \times 1 \times M$ MIMO multi-user two-way relay X channel shown in Fig. 1, which comprises of M users in each side and a relay. Users T_i, $i \in \{1, 2, \ldots, M\}$, and T_j, $j \in \{M+1, M+2, \ldots, 2M\}$ with K ($K > M$) antennas want to exchange their messages through the relay with M^2 antennas. Every user has M messages, one for each user in the opposite side. It is assumed that there are no direct links between any two users because of the large direct path loss and thus the system can be divided into two phases; MAC phase and BC phase.

During the MAC phase, users T_i in the left-hand side and T_j in the right-hand side transmit M data streams to the relay, simultaneously. Let $W^{[j,i]}$ and $x^{[j,i]}$ be the message and data symbol transmitted from user i to user j, respectively. Users T_i and T_j encode the messages $W^{[j,i]}$ and $W^{[i,j]}$ into the data symbols $x^{[j,i]}$ and $x^{[i,j]}$ and transmit them to the relay by using beamforming vectors $\boldsymbol{v}^{[j,i]}$ and $\boldsymbol{v}^{[i,j]}$, respectively. The received signal at the relay $\boldsymbol{Y}^{[r]}$ is given as

$$\boldsymbol{Y}^{[r]} = \sum_{i=1}^{M} \boldsymbol{H}^{[r,i]}\boldsymbol{X}^{[i]} + \sum_{j=M+1}^{2M} \boldsymbol{H}^{[r,j]}\boldsymbol{X}^{[j]} + \boldsymbol{N}^{[r]} \quad (3)$$

where

$$\boldsymbol{X}^{[i]} = \sum_{j=M+1}^{2M} \boldsymbol{v}^{[j,i]}x^{[j,i]} \quad (4)$$

and

$$\boldsymbol{X}^{[j]} = \sum_{i=1}^{M} \boldsymbol{v}^{[i,j]}x^{[i,j]} \quad (5)$$

represent the $K \times 1$ transmit signal vectors for users T_i and T_j, respectively. $\boldsymbol{H}^{[r,l]}$ is the $M^2 \times K$ channel matrix from users T_l to the relay, respectively. $\boldsymbol{N}^{[r]}$ is an $M^2 \times 1$ additive noise vector with the components from complex Gaussian distribution with zero mean and unit variance. Each user has the average power constraint, $E\{\mathrm{tr}[\boldsymbol{X}^{[i]}\boldsymbol{X}^{[i]\dagger}]\} \le P_i$ and $E\{\mathrm{tr}[\boldsymbol{X}^{[j]}\boldsymbol{X}^{[j]\dagger}]\} \le P_j$, where † denotes the conjugate transpose.

During the BC phase, after receiving data symbols from all users, the relay transmits network coded data symbols to the all users. Let $\boldsymbol{N}^{[l]}$ be the additive white Gaussian noise (AWGN) vector at user T_l having the components generated from complex Gaussian distribution with zero mean and unit variance and $\boldsymbol{H}^{[l,r]}$ be the $K \times M^2$ channel matrix from the relay to user T_l. Then the received signal at user T_l is given as

$$\boldsymbol{Y}^{[l]} = \boldsymbol{H}^{[l,r]}\boldsymbol{X}^{[r]} + \boldsymbol{N}^{[l]} \quad (6)$$

where $l \in \{1, 2, \ldots, 2M\}$ and $\boldsymbol{X}^{[r]}$ denotes $M^2 \times 1$ transmit signal

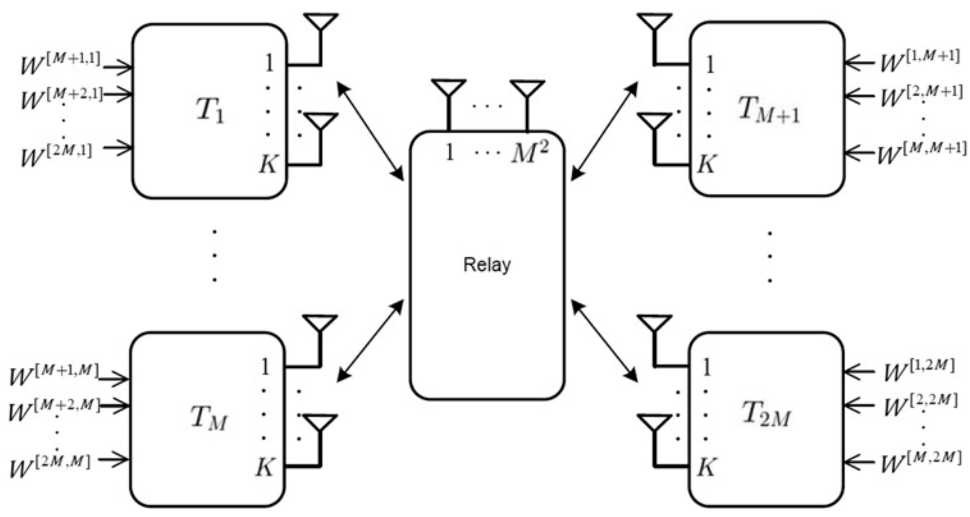

Fig. 1 *M × 1 × M MIMO multi-user two-way relay X network*

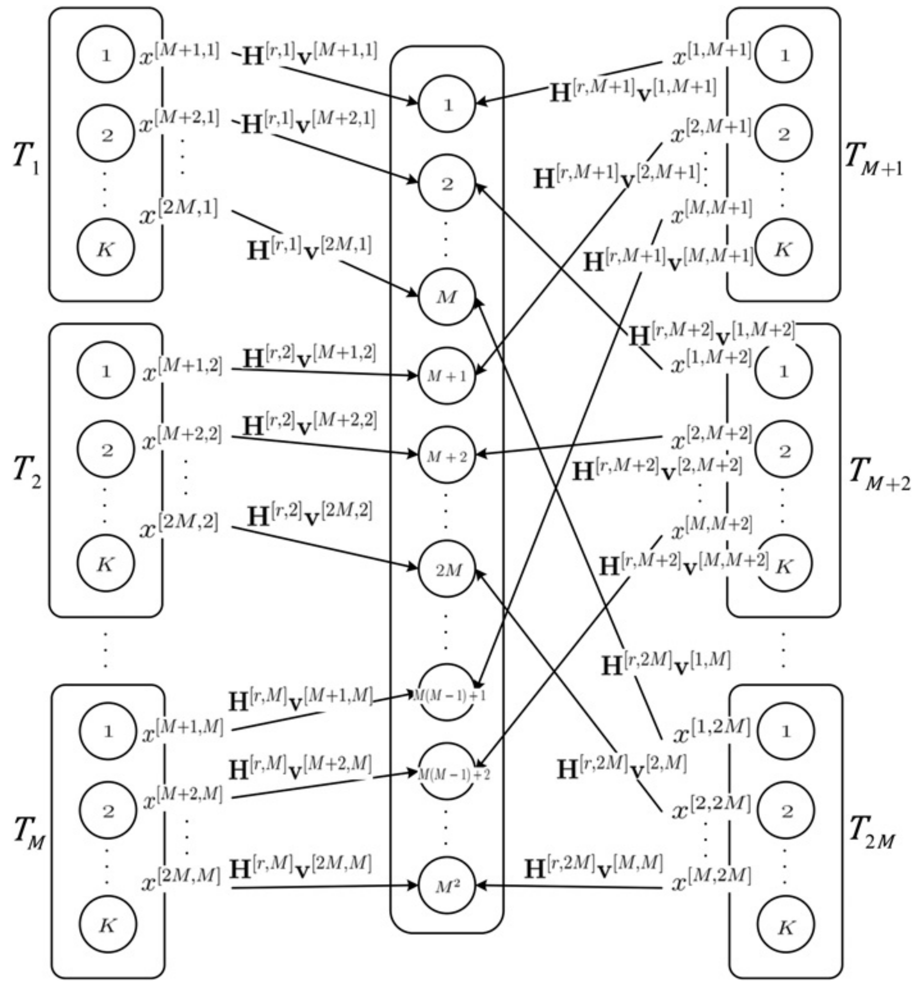

Fig. 2 *$M \times 1 \times M$ MIMO multi-user two-way relay X network during the MAC phase*

vectors at the relay, which has the average power constraint $E\{\text{tr}[\boldsymbol{X}^{[r]}\boldsymbol{X}^{[r]\dagger}]\} \le P_r$. It is assumed that the channel coefficients are independent and identically distributed (i.i.d.) continuous complex Gaussian random variables with zero mean and unit variance and all CSIs are perfectly known to all nodes. It is also assumed that every channel is quasi-static Rayleigh faded.

2.2 Achievability scheme

In [16], DOF 8 is achieved for $2 \times 1 \times 2$ MIMO multi-user two-way relay X channel by using signal space alignment [19] and nulling beamforming in the MAC and BC phases, respectively. And each user uses three antennas and the relay uses four antennas.

In this section, we extend the scheme in [16] for the general case of MIMO multi-user two-way relay X channels with M users in each side and then we will derive the minimum number of each user's antennas to achieve DOF $2M^2$ in this network.

2.2.1 MAC phase: signal space alignment for network coding: During the MAC phase, each user transmits M data symbols to the relay by using M $K \times 1$ beamforming vectors. Note that the received signal space at the relay is of M^2-dimension, whereas the number of beamforming vectors transmitted from $2M$ users in both sides are $2M^2$. As suggested in [16], one of the simple ways to resolve this situation is to align each pair of two beamforming vectors $\boldsymbol{v}^{[i,j]}$ and $\boldsymbol{v}^{[j,i]}$ for the data symbols $x^{[i,j]}$ and $x^{[j,i]}$ in the same subspace of the received signal space as depicted in Fig. 2, that is

$$\text{span}\left(\boldsymbol{H}^{[r,i]}\boldsymbol{v}^{[j,i]}\right) = \text{span}\left(\boldsymbol{H}^{[r,j]}\boldsymbol{v}^{[i,j]}\right) \tag{7}$$

Let $\boldsymbol{r}^{[M(i-2)+j]}$ be a basis vector of span$(\boldsymbol{H}^{[r,i]}\boldsymbol{v}^{[j,i]})$. It is always possible to make all the vectors $\boldsymbol{H}^{[r,i]}\boldsymbol{v}^{[j,i]}$'s linearly independent since the entries of the channel matrices are generated from continuous random variable and they almost surely have full rank [20]. Therefore $\boldsymbol{r}^{[M(i-2)+j]}$'s for all i and j are linearly independent with probability one and all $\boldsymbol{v}^{[i,j]}$'s can be determined directly from (7). Consequently, the relay receives $2M^2$ data symbols and every pair of the two symbols $x^{[i,j]}$ and $x^{[j,i]}$ are aligned in the same subspace of the received signal space. Thus there are M^2 network coded data symbols in the relay given as

$$\tilde{x}^{[i,j]} = \alpha^{[j,i]}x^{[j,i]} + \alpha^{[i,j]}x^{[i,j]} \tag{8}$$

where $\alpha^{[i,j]}$ is the normalised channel gain. In fact, $\tilde{x}^{[i,j]}$ is broadcasted to users T_i and T_j in the BC phase from the relay. The received signal at the relay becomes

$$\boldsymbol{Y}^{[r]} = \begin{bmatrix} \boldsymbol{r}^{[1]} & \boldsymbol{r}^{[2]} & \cdots & \boldsymbol{r}^{[M^2]} \end{bmatrix} \times \begin{bmatrix} \alpha^{[M+1,1]}x^{[M+1,1]} + \alpha^{[1,M+1]}x^{[1,M+1]} \\ \alpha^{[M+2,1]}x^{[M+2,1]} + \alpha^{[1,M+2]}x^{[1,M+2]} \\ \vdots \\ \alpha^{[2M,M]}x^{[2M,M]} + \alpha^{[M,2M]}x^{[M,2M]} \end{bmatrix} + \boldsymbol{N}^{[r]}$$

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$$= \left[\boldsymbol{r}^{[1]} \quad \boldsymbol{r}^{[2]} \quad \cdots \quad \boldsymbol{r}^{[M^2]} \right] \begin{bmatrix} \tilde{x}^{[1,M+1]} \\ \tilde{x}^{[1,M+2]} \\ \vdots \\ \tilde{x}^{[M,2M]} \end{bmatrix} + \boldsymbol{N}^{[r]}$$
$$= \sum_{i=1}^{M} \sum_{j=M+1}^{2M} \boldsymbol{r}^{[M(i-2)+j]} \tilde{x}^{[i,j]} + \boldsymbol{N}^{[r]} \quad (9)$$

2.2.2 BC phase: INB: During the BC phase, the relay broadcasts M^2 network coded data symbols along the M^2 beamforming vectors $\boldsymbol{f}_{i,j}$ for network coded data symbols $\tilde{x}^{[i,j]}$. Among these M^2 symbols, the desired symbols for each user are only M and the remainders are interference. Note that each user wants to receive M desired signals and it is desirable to use as few antennas as possible. Then we have to use INB method at the relay [16].

2.3 Minimum number of each user's antennas to achieve DOF $2M^2$

The least number of each user's antennas in the $M \times 1 \times M$ MIMO multi-user two-way relay X channels with DOF $2M^2$ is given in the following theorem.

Theorem 1: In $M \times 1 \times M$ MIMO multi-user two-way relay X channels, the minimum number K of antennas at each user for achieving DOF $2M^2$ is given as

$$K = \left\lceil M^2 - \frac{M}{2} \right\rceil \quad (10)$$

where $\lceil \cdot \rceil$ denotes the ceiling.

Proof: For the signal space alignment in the MAC phase as in (7), the column spaces of $M^2 \times K$ channel matrices $\boldsymbol{H}^{[r,i]}$ and $\boldsymbol{H}^{[r,j]}$ must have a non-empty intersection. This implies that some column vectors of two channel matrices are linearly dependent. Each channel matrix consists of K M^2-tuple random column vectors generated independently and thus, in order for two channel matrices to have a non-empty intersection, the sum of the number of columns of $\boldsymbol{H}^{[r,i]}$ and $\boldsymbol{H}^{[r,j]}$ should be larger than M^2. Thus, the condition $2K > M^2$ should be satisfied for the signal space alignment at the relay.
The condition $2K > M^2$ of signal space alignment for network coding in the MAC phase prohibits the existence of the intersection of any two null spaces of $\boldsymbol{H}^{[l1,r]}$ and $\boldsymbol{H}^{[l2,r]}$, $1 \le l_1,\ l_2 \le 2M$, since the dimension $M^2 - K$ of these null spaces is less than $M^2/2$. Consequently, any beamforming vector from the relay can be eliminated by at most one channel matrix $\boldsymbol{H}^{[l,r]}$. Therefore total M^2 interference signal can be eliminated at best.

On the other hand, since each user has K antennas, $(M^2 - K)$ signals should be eliminated while they are broadcasted along the null space. Therefore, for all users, $2M(M^2 - K)$ interference signals should be eliminated.

Thus, we have

$$M^2 \ge 2M(M^2 - K) \quad (11)$$

and the minimum number of antennas in the $M \times 1 \times M$ MIMO multi-user two-way relay X channels to achieve DOF $2M^2$ is derived as

$$K = \left\lceil M^2 - \frac{M}{2} \right\rceil \quad (12)$$

2.4 Examples

2.4.1 Signal beamforming of the BC phase with $M = 3$ and $K = 8$: Table 1 shows how the beamforming vectors at the relay are formed when $M = 3$ and $K = 8$. T_i denotes the ith user and $\boldsymbol{f}_{i,j}$ denotes the beamforming vectors at the relay for users T_i and T_j. In the first column of Table 1, the network coded data symbol $\tilde{x}^{[i,j]}$, which is the desired signal for T_1 and T_4 denoted by (O), is broadcasted along the beamforming vector $\boldsymbol{f}_{1,4}$ from the relay. Each user has $K = 8$ dimensional signal spaces and thus can handle three desired signals denoted by (O) and five interfering signals denoted by (I). This means that at least one interfering signal at each row should be eliminated by nulling denoted by (X) while it is broadcasted. Note that each 8×9 channel matrix from the relay to each user has one dimensional null space. The 9×1 beamforming vector $\boldsymbol{f}_{1,4}$ for network coded data symbol $\tilde{x}^{[1,4]}$ can be chosen from the null space of one of the channel matrices other than $\boldsymbol{H}^{[1,r]}$ and $\boldsymbol{H}^{[4,r]}$. In the first column of Table 1, the beamforming vector $\boldsymbol{f}_{1,4}$ is selected from the null space of channel matrix $\boldsymbol{H}^{[2,r]}$ from the relay to T_2. Similarly, we can choose the other five beamforming vectors denoted by (X) in Table 1 as

$$\begin{aligned} &\boldsymbol{f}_{1,4} \in \text{null}\left(\boldsymbol{H}^{[2,r]}\right), \boldsymbol{f}_{1,6} \in \text{null}\left(\boldsymbol{H}^{[3,r]}\right), \\ &\boldsymbol{f}_{2,5} \in \text{null}\left(\boldsymbol{H}^{[4,r]}\right), \boldsymbol{f}_{2,6} \in \text{null}\left(\boldsymbol{H}^{[1,r]}\right), \\ &\boldsymbol{f}_{3,5} \in \text{null}\left(\boldsymbol{H}^{[6,r]}\right), \boldsymbol{f}_{3,6} \in \text{null}\left(\boldsymbol{H}^{[5,r]}\right) \end{aligned} \quad (13)$$

where null(·) denotes the null space of the matrix. The other three beamforming vectors $\boldsymbol{f}_{1,5}$, $\boldsymbol{f}_{2,4}$ and $\boldsymbol{f}_{3,4}$ need not to be chosen from null space of channel matrices, but they have to be linearly independent each other. By using the above nine beamforming vectors, one of the interfering signals is eliminated at each user in the BC phase. For example, in the first row of Table 1, the signal from $\boldsymbol{f}_{2,6}$ is eliminated at user T_1 by nulling. T_1 receives the signals along the vectors $\{\boldsymbol{f}_{1,4}, \boldsymbol{f}_{1,5}, \boldsymbol{f}_{1,6}\}$, which contain three desired network coded data symbols $\{\tilde{x}^{[1,4]}, \tilde{x}^{[1,5]}, \tilde{x}^{[1,6]}\}$. It also receives the signals along the vectors $\{\boldsymbol{f}_{2,4}, \boldsymbol{f}_{2,5}, \boldsymbol{f}_{3,4}, \boldsymbol{f}_{3,5}, \boldsymbol{f}_{3,6}\}$, which contain five interfering symbols $\{\tilde{x}^{[2,4]}, \tilde{x}^{[2,5]}, \tilde{x}^{[3,4]}, \tilde{x}^{[3,5]}, \tilde{x}^{[3,6]}\}$. Let $\boldsymbol{x}^{[r]}$ be the 9×1 signal vector from relay given as

$$\boldsymbol{x}^{[r]} = [\tilde{x}^{[1,4]}, \tilde{x}^{[1,5]}, \tilde{x}^{[1,6]}, \tilde{x}^{[2,4]}, \tilde{x}^{[2,5]}, \tilde{x}^{[2,6]}, \tilde{x}^{[3,4]}, \tilde{x}^{[3,5]}, \tilde{x}^{[3,6]}]^T \quad (14)$$

and $\boldsymbol{x}^{[1,r]}$ the 8×1 signal vector given as

$$\boldsymbol{x}^{[1,r]} = [\tilde{x}^{[1,4]}, \tilde{x}^{[1,5]}, \tilde{x}^{[1,6]}, \tilde{x}^{[2,4]}, \tilde{x}^{[2,5]}, \tilde{x}^{[3,4]}, \tilde{x}^{[3,5]}, \tilde{x}^{[3,6]}]^{\mathrm{T}} \quad (15)$$

where the network coded data symbol $\tilde{x}^{[2,6]}$ is eliminated from $\boldsymbol{x}^{[r]}$. $\boldsymbol{Y}^{[1]}$ is 8×1 received signal vector of user T_1 and $\boldsymbol{N}^{[1]}$ is 8×1

Table 1 Signal beamforming with $M = 3$ and $K = 8$

	$f_{1,4}$	$f_{1,5}$	$f_{1,6}$	$f_{2,4}$	$f_{2,5}$	$f_{2,6}$	$f_{3,4}$	$f_{3,5}$	$f_{3,6}$
T_1	O	O	O	I	I	X	I	I	I
T_2	X	I	I	O	O	O	I	I	I
T_3	I	I	X	I	I	I	O	O	O
T_4	O	I	I	O	X	I	O	I	I
T_5	I	O	I	I	O	I	I	O	X
T_6	I	I	O	I	I	O	I	X	O

Desired signal (O), nulling signal (X) and interfering signal (I) broadcasted from relay to users T_1–T_6

Table 2 Signal beamforming with $M = 3$ and $K = 7$

	$f_{1,4}$	$f_{1,5}$	$f_{1,6}$	$f_{2,4}$	$f_{2,5}$	$f_{2,6}$	$f_{3,4}$	$f_{3,5}$	$f_{3,6}$
T_1	O	O	O	I	I	X	X	I	I
T_2	X	X	I	O	O	O	I	I	I
T_3	I	X	X	I	I	I	O	O	O
T_4	O	I	X	O	X	I	O	I	I
T_5	I	O	I	I	O	I	I	O	X
T_6	I	I	O	I	X	O	I	X	O

Desired signal (O), nulling signal (X) and interfering signal (I) broadcasted from relay to users T_1–T_6

AWGN vector. Then the received signal at T_1 is expressed as

$$\begin{aligned}\boldsymbol{Y}^{[1]} &= \boldsymbol{H}^{[1,r]}\left[\boldsymbol{f}_{1,4},\boldsymbol{f}_{1,5},\boldsymbol{f}_{1,6},\boldsymbol{f}_{2,4},\boldsymbol{f}_{2,5},\boldsymbol{f}_{2,6},\boldsymbol{f}_{3,4},\boldsymbol{f}_{3,5},\boldsymbol{f}_{3,6}\right]\boldsymbol{x}^{[r]} + \boldsymbol{N}^{[1]} \\ &= \boldsymbol{H}^{[1,r]}\left[\boldsymbol{f}_{1,4},\boldsymbol{f}_{1,5},\boldsymbol{f}_{1,6},\boldsymbol{f}_{2,4},\boldsymbol{f}_{2,5},\boldsymbol{f}_{3,4},\boldsymbol{f}_{3,5},\boldsymbol{f}_{3,6}\right]\boldsymbol{x}^{[1,r]} + \boldsymbol{N}^{[1]}\end{aligned} \tag{16}$$

Since the matrix

$$\boldsymbol{H}^{[1,r]}\left[\boldsymbol{f}_{1,4},\boldsymbol{f}_{1,5},\boldsymbol{f}_{1,6},\boldsymbol{f}_{2,4},\boldsymbol{f}_{2,5},\boldsymbol{f}_{3,4},\boldsymbol{f}_{3,5},\boldsymbol{f}_{3,6}\right] \tag{17}$$

has full rank almost surely [20], T_1 can detect its desired network coded data symbols. That is, T_1 and T_4 can detect the network coded data symbol $\tilde{x}^{[1,4]}$ and since they know their own symbols $x^{[4,1]}$ and $x^{[1,4]}$, they can extract desired symbols $x^{[1,4]}$ and $x^{[4,1]}$, respectively.

2.4.2 Signal beamforming of the BC phase with $M = 3$ and $K = 7$:

In this part, we want to check whether the DOF 9 in the previous example can be achievable with $K = 7$.

Table 2 shows an example with $M = 3$ and $K = 7$ of designing beamforming vectors at the relay. In the second column, the network coded data symbol $\tilde{x}^{[1,5]}$ broadcasted along the beamforming vector $\boldsymbol{f}_{1,5}$ in the relay is the desired signal for T_1 and T_5. Since each user has $K = 7$ dimensional signal spaces, it should have three desired signals and four interfering signals, which means that two interfering signals among nine signals should be eliminated while they are broadcasted from the relay. Note that the 7×9 channel matrix from the relay to each user has two-dimensional (2D) null spaces. The 9×1 beamforming vector $\boldsymbol{f}_{1,5}$ for network coded data symbol $\tilde{x}^{[1,5]}$ can be chosen from the intersection of two null spaces other than $\boldsymbol{H}^{[1,r]}$ and $\boldsymbol{H}^{[5,r]}$. The second column of Table 2 says that the beamforming vector $\boldsymbol{f}_{1,5}$ should be selected from the null spaces of channel matrices $\boldsymbol{H}^{[2,r]}$ and $\boldsymbol{H}^{[3,r]}$, that is

$$\boldsymbol{f}_{1,5} \in \text{null}\left(\boldsymbol{H}^{[2,r]}\right) \cap \text{null}\left(\boldsymbol{H}^{[3,r]}\right) \tag{18}$$

Note that any nine rows selected from the rows of $\boldsymbol{H}^{[2,r]}$ and $\boldsymbol{H}^{[3,r]}$ are almost surely linearly independent since the entries of every channel matrix are generated from continuous distribution. This implies that it is almost sure that the only vector $\boldsymbol{f}_{1,5}$ satisfying (18) is a zero vector. Consequently, we cannot design the beamforming vectors when $M = 3$ and $K = 7$.

3 MIMO multi-user two-way relay X channels with time extension

In the previous section, we derived the minimum number of antennas at each user for $M \times 1 \times M$ MIMO multi-user two-way relay X channels with maximum DOF $2M^2$. As the number of users increases, the number of antennas at each user should be increased, which may not be practical in the viewpoint of implementation. In order to reduce the number of antennas at each user, we use the idea of time extension. Actually, in MAC or BC, if the base station has M antennas and uses L time slots, it can serve ML users. This idea can be applied to the $M \times 1 \times M$ MIMO multi-user two-way relay X channels directly. We will see how the new scheme works in the $M \times 1 \times M$ MIMO multi-user two-way relay X channels and show that the DOF of the proposed scheme with time extension is the same as that of the previous scheme.

3.1 System model

In the proposed scheme for $M \times 1 \times M$ MIMO multi-user two-way relay X channels, each user and the relay have only M antennas but use M time slots. In this system, users T_i, $i \in \{1, 2, \ldots, M\}$, and T_j, $j \in \{M+1, M+2, \ldots, 2M\}$, with M antennas want to send M independent messages $W^{[j,\ i]}$ and $W^{[i,\ j]}$ via a relay with M antennas. Overall system and procedure are similar to the scheme in the previous section and the proposed scheme also achieves DOF $2M^2$, but it does not use INB during the BC phase.

3.2 MAC phase: signal space alignment for network coding

In the MAC phase, two methods for the $M \times 1 \times M$ MIMO multi-user two-way relay X channels with M time extension are proposed as:

(i) Simultaneous transmission: All users simultaneously transmit beamforming vectors in M time slots.
(ii) TDMA transmission: In the ith time slot, T_i in the left-hand side transmits its own M beamforming vectors and each user in the right-hand side transmits its ith beamforming vector.

3.2.1 Simultaneous transmission:

During the MAC phase, similarly to the scheme in Section 2, users T_i and T_j in each side encode the messages $W^{[j,i]}$ and $W^{[i,j]}$ into the data symbols $x^{[j,i]}$ and $x^{[i,j]}$ and transmit them to the relay by using beamforming vectors for M time slots.

Let $\boldsymbol{Y}^{[r]}(t)$ and $\boldsymbol{N}^{[r]}(t)$ be an $M \times 1$ received signal vector and AWGN vector in time slot t, respectively, and $\boldsymbol{X}^{[i]}(t)$ and $\boldsymbol{X}^{[j]}(t)$ be $M \times 1$ transmitted vectors from users T_i and T_j in time slot t. The received signal in time slot t at the relay is given as

$$\begin{aligned}\boldsymbol{Y}^{[r]}(t) &= \sum_{i=1}^{M} \boldsymbol{H}^{[r,i]}(t)\boldsymbol{X}^{[i]}(t) + \sum_{j=M+1}^{2M} \boldsymbol{H}^{[r,j]}(t)\boldsymbol{X}^{[j]}(t) \\ &\quad + \boldsymbol{N}^{[r]}(t),\ t \in \{1, 2, \ldots, M\}\end{aligned} \tag{19}$$

Let us assume that $\boldsymbol{H}^{[r,i]}(t) = \boldsymbol{H}^{[r,i]}(t')$, $t \neq t'$ and each user has the average power constraint $E\{\text{tr}[\boldsymbol{X}^{[i]}(t)\boldsymbol{X}^{[i]\dagger}(t)]\} \leq P_i$ and $E\{\text{tr}[\boldsymbol{X}^{[j]}(t)\boldsymbol{X}^{[j]\dagger}(t)]\} \leq P_j$. For M time slots, the $M^2 \times 1$ vectorised forms of the transmitted signal at the ith user and the received signal at the relay are given as

$$\begin{aligned}\boldsymbol{X}^{[l]} &= \left[\boldsymbol{X}^{[l]}(1)^{\text{T}} \quad \boldsymbol{X}^{[l]}(2)^{\text{T}} \quad \cdots \boldsymbol{X}^{[l]}(M)^{\text{T}}\right]^{\text{T}} \\ \boldsymbol{Y}^{[r]} &= \left[\boldsymbol{Y}^{[r]}(1)^{\text{T}} \quad \boldsymbol{Y}^{[r]}(2)^{\text{T}} \quad \cdots \quad \boldsymbol{Y}^{[r]}(M)^{\text{T}}\right]^{\text{T}}\end{aligned} \tag{20}$$

Since the quasi-static fading is assumed, the $M^2 \times M^2$ equivalent channel matrix $\boldsymbol{H}^{[r,i]}$ for M time slots can be given as

$$\boldsymbol{H}^{[r,i]} = \begin{bmatrix}\boldsymbol{H}^{[r,i]}(1) & & & \\ & \boldsymbol{H}^{[r,i]}(2)(=\boldsymbol{H}^{[r,i]}(1)) & & \\ & & \ddots & \\ & & & \boldsymbol{H}^{[r,i]}(M)(=\boldsymbol{H}^{[r,i]}(1))\end{bmatrix} \tag{21}$$

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Therefore the received signal $\boldsymbol{Y}^{[r]}$ can be expressed as

$$\boldsymbol{Y}^{[r]} = \sum_{i=1}^{M} \boldsymbol{H}^{[r,i]}\boldsymbol{X}^{[i]} + \sum_{i=M+1}^{2M} \boldsymbol{H}^{[r,i]}\boldsymbol{X}^{[i]} + \boldsymbol{N}^{[r]} \quad (22)$$

where $\boldsymbol{N}^{[r]}$ denotes the AWGN vector at the relay and $\boldsymbol{X}^{[i]}$ and $\boldsymbol{X}^{[j]}$ are obtained similar to (4) and (5).

The beamforming vectors for users T_i and T_j are designed so that they can be aligned in the same subspace of the received signal space in order that total $2M^2$ signals from all users can be aligned in the M^2-dimensional signal spaces at the relay. Let $\boldsymbol{v}^{[r,i]}$ be the $M^2 \times 1$ beamforming vector at the ith user and $\boldsymbol{U}$ be the $M^2 \times M^2$ matrix such that columns are orthogonal to each other given as

$$\boldsymbol{U} = \left[\boldsymbol{u}^{[1]}\boldsymbol{u}^{[2]}, \ldots, \boldsymbol{u}^{[M^2]}\right] \quad (23)$$

where M^2 columns represent the signal space at the relay. Then we can design the beamforming vectors $\boldsymbol{v}^{[r,i]}$ and $\boldsymbol{v}^{[r,j]}$ such that $x^{[j,i]}$ and $x^{[i,j]}$ are aligned in the same direction in the received signal space as

$$\boldsymbol{H}^{[r,i]}\boldsymbol{v}^{[r,M(i-2)+j]} = \boldsymbol{H}^{[r,j]}\boldsymbol{v}^{[r,M(i-2)+j]} = \boldsymbol{u}^{[M(i-2)+j]} \quad (24)$$

Note that channel matrices are generated from the continuous random variable. Consequently, relay receives $2M^2$ symbols and two symbols $x^{[j,i]}$ and $x^{[i,j]}$ are aligned in the same direction. Thus there are M^2 independent network coded data symbols at the relay and each symbol is represented as

$$\tilde{x}^{[i,j]} = x^{[j,i]} + x^{[i,j]} \quad (25)$$

In fact, $\tilde{x}^{[i,j]}$ is broadcasted to users T_i and T_j in the BC phase from the relay. Thus, the received signal at the relay is given as (9).

3.2.2 TDMA transmission: During the MAC phase, in the first time slot, user T_1 transmits M data symbols for M users in the right-hand side and each user in the right-hand side transmits its first data symbol for T_1 to the relay. From the second to the Mth time slot, they transmit the data symbols in the same manner in turn, as shown in Fig. 3.

Consequently, the received signal at the relay can be written as

$$\boldsymbol{Y}^{[r]}(t) = \boldsymbol{H}^{[r,t]}\boldsymbol{X}^{[t]}(t) + \sum_{j=M+1}^{2M} \boldsymbol{H}^{[r,j]}\boldsymbol{X}^{[j]}(t) + \boldsymbol{N}^{[r]}(t) \quad (26)$$

where

$$\boldsymbol{X}^{[t]}(t) = \sum_{j=M+1}^{2M} \boldsymbol{v}^{[j,t]}x^{[j,t]}, \quad \boldsymbol{X}^{[j]}(t) = \boldsymbol{v}^{[t,j]}x^{[t,j]} \quad (27)$$

in time slot t, $t \in \{1, 2, \ldots, M\}$. As shown in Fig. 3, two

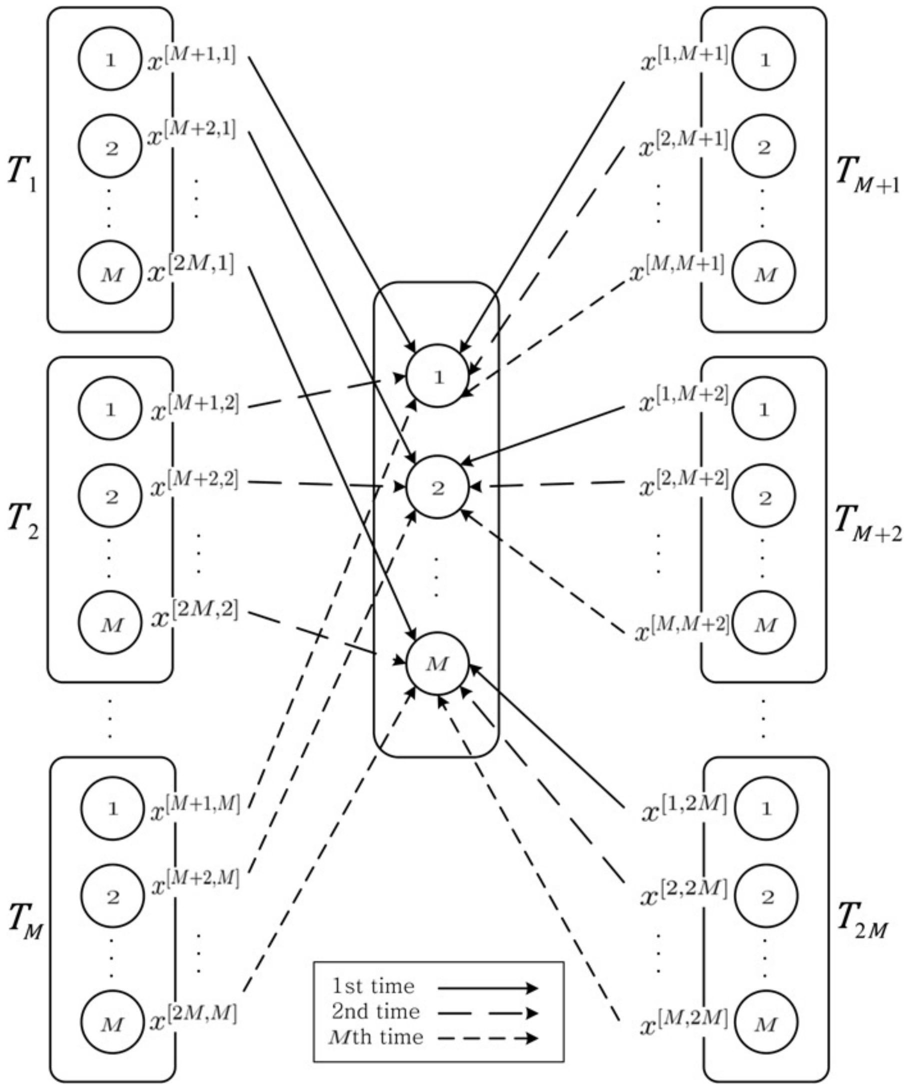

Fig. 3 *TDMA in MIMO multi-user two-way relay X network during the MAC phase*

beamforming vector for $x^{[j,i]}$ and $x^{[i,j]}$ should be aligned in the same direction at the relay. If the beamforming vectors are designed to satisfy the following conditions

$$\begin{aligned} \boldsymbol{H}^{[r,t]}\boldsymbol{v}^{[M+1,t]} &= \boldsymbol{H}^{[r,M+1]}\boldsymbol{v}^{[t,M+1]} \\ \boldsymbol{H}^{[r,t]}\boldsymbol{v}^{[M+2,t]} &= \boldsymbol{H}^{[r,M+2]}\boldsymbol{v}^{[t,M+2]} \\ &\vdots \\ \boldsymbol{H}^{[r,t]}\boldsymbol{v}^{[2M,t]} &= \boldsymbol{H}^{[r,2M]}\boldsymbol{v}^{[t,2M]} \end{aligned} \tag{28}$$

then two symbols $x^{[j,i]}$ and $x^{[i,j]}$ can be aligned in the same dimension of the received signal space at the relay.

3.3 BC phase

During the BC phase, the relay broadcasts M^2 network coded data symbols to all $2M$ users for M time slots. Each user receives M^2 network coded data symbols from the relay for M times, where M network coded data symbols are desired data symbols to each user. Since each user has M antennas with M time extension, M^2-dimensional signal spaces are available. $\boldsymbol{H}^{[l,r]}$ is the channel matrix from relay to user T_l. Since all the channels are assumed to be quasi-static, all the channel coefficients are fixed, that is, $\boldsymbol{H}^{[t,r]}(t) = \boldsymbol{H}^{[t,r]}$ for all t. Let $\boldsymbol{X}^{[r]}(t)$ be the $M \times 1$ transmitted signal vector from the relay in time slot t and let $\boldsymbol{Y}^{[l]}(t)$ and $\boldsymbol{N}^{[l]}(t)$ be the $M \times 1$ received signal vector and $M \times 1$ AWGN noise vector of the user T_l in time slot t and

$$\boldsymbol{X}^{[r]}(t) = \sum_{j=M+1}^{2M} \boldsymbol{v}^{[t,j]}\, \tilde{x}^{[t,j]} \tag{29}$$

where $\boldsymbol{v}^{[t,j]}$ is the $M \times 1$ beamforming vector for the network coded data symbol to be transmitted from the relay to users T_t and T_j. Then the received signal vector of user T_l in time slot t is expressed as

$$\boldsymbol{Y}^{[l]}(t) = \boldsymbol{H}^{[l,r]}\boldsymbol{X}^{[r]}(t) + \boldsymbol{N}^{[l]}(t) \tag{30}$$

where $t \in \{1, 2, \ldots, M\}$ and $l \in \{1, 2, \ldots, 2M\}$. It is assumed that the relay has the average power constraint $E\{\mathrm{tr}[\boldsymbol{X}^{[r]}\boldsymbol{X}^{[r]\dagger}]\} \le P_r$. Since the entries of $\boldsymbol{H}^{[t,r]}$ are drawn from continuous random variable, $\boldsymbol{H}^{[t,r]}$ almost surely has full rank. Consequently, all the columns in the received dimensional signal space of each user are almost surely linearly independent and each user can detect the desired network coded data symbols. For example, the desired network coded data symbol $\tilde{x}^{[i,j]}$, which is for users T_i and T_j, is the sum of

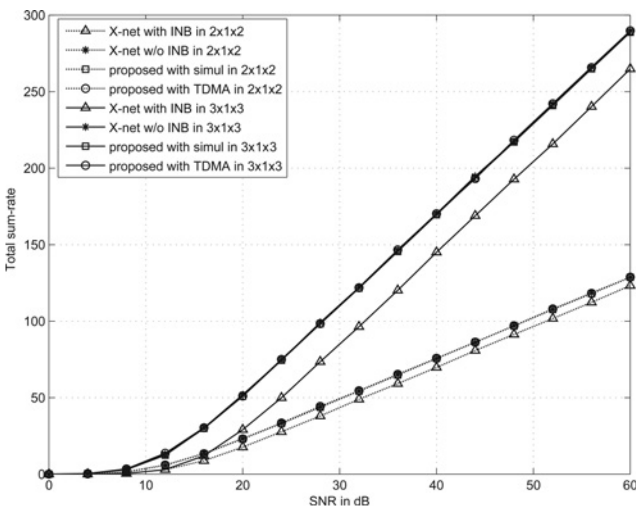

Fig. 4 *Total sum-rate of the four schemes in 2 × 1 × 2 and 3 × 1 × 3 networks*

Table 3 Number of antennas and time slots of four schemes in 2 × 1 × 2 and 3 × 1 × 3 networks

	Scheme	Time-slot	Number of user's antennas	Number of relay antennas
2 × 1 × 2 system	1. X networks with INB [16]	1	3	4
	2. X networks without INB	1	4	4
	3. proposed with simul	2	2	2
	4. proposed with TDMA	2	2	2
3 × 1 × 3 system	1. X networks with INB	1	8	9
	2. X networks without INB	1	9	9
	3. proposed with simul	3	3	3
	4. proposed with TDMA	3	3	3

the data symbols $x^{[j,i]}$ and $x^{[i,j]}$ and thus they can extract the desired symbols $x^{[i,j]}$ and $x^{[j,i]}$, respectively. During the MAC phase and BC phase, total $2M^2$ data symbols are transmitted and received. Thus, its DOF is $2M^2$.

4 Numerical analysis

In this section, we provide numerical analysis to assess the sum-rate performance of the proposed scheme in Section 3. It is assumed that equal power allocation is employed for each transmitted data stream, that is, $P_i = P_j = P_r$ and noise variance at each user and relay is $\sigma_i^2 = \sigma_j^2 = \sigma_r^2 = 1$. All the channels are assumed to be independent quasi-static Rayleigh faded and the sum-rate is calculated by using the result in [21].

We compare four different schemes, MIMO multi-user two-way relay X channels with INB and without INB, the proposed scheme with simultaneous-beamforming transmission and the proposed scheme with TDMA transmission in Section 3. They are called 'X-net with INB', 'X-net without INB', 'proposed with simul' and 'proposed with TDMA', respectively, in Fig. 4. The total sum-rate performance of the above four schemes is presented with respect to overall communication resources, that is, antennas and time, for a fair comparison. It is assumed that each user and relay uses the same power, the bases of the received signal space in the relay are orthonormal each other, and beamforming vectors of all the schemes in the BC phase are orthonormal except for MIMO multi-user two-way relay X channels with INB.

Table 3 lists the number of antennas of the above four schemes for 2 × 1 × 2 and 3 × 1 × 3 channels. As shown in Fig. 4, schemes 2–4 in 2 × 1 × 2 and 3 × 1 × 3 channels have almost the same sum-rate performance. Scheme 1 has lower sum-rate performance than the other schemes, because in the BC phase, INB is used. Scheme 1 reduces the number of antennas of each user, but it limits selection of beamforming vector because beamforming vector has been chosen among the null spaces of the channel matrices. The other schemes 2–4, in the BC phase beamforming vectors can be chosen such that beamforming vectors are orthonormal each other. Through the simulation results, we can see that the proposed scheme shows better performance. In fact, it is caused by using more antennas or time slots than the scheme in [16], that is, the gain is obtained from using a little more resources. Therefore the proposed scheme using time extension can be a good alternative with less antennas compared to the scheme in [16].

5 Conclusion

In this paper, motivated by the early works of signal space alignment for network coding and INB in MIMO multi-user twoway relay X channels [16], the minimum number of antennas at each user for $M \times 1 \times M$ MIMO multiuser twoway relay X channels was derived to obtain DOF $2M^2$. From this result, we confirmed that as the number of users increases, the number of antennas increases proportionally to M^2. Therefore a new beamforming scheme with M time extension is proposed. Through the numerical analysis, it is shown that the proposed scheme is a good alternative to that in [16].

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