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Interference alignment-and-cancellation scheme based on Alamouti code for the three-user multi-input–multi-output interference channel

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Abstract: An interference alignment (IA) scheme for interference channels was recently proposed to achieve the maximum degrees of freedom (DoF). Although most studies of IA focus on network throughput, that is, DoF, reliability in terms of diversity order is also an important performance measure. In this study, interference cancellation (IC) scheme based on Alamouti code in the multi-access scenario is applied to the K-user, multi-input–multi-output (MIMO) interference channel. This IC scheme gives the benefit of diversity order and requires no channel-state-information at the transmitters (CSIT). However, it requires more receive antennas than the IA scheme to achieve the same DoF. In order to reduce the number of receive antennas, especially for the three-user MIMO interference channel, an IA-and-cancellation (IAC) scheme based on Alamouti code is proposed. It keeps the same DoF as the IC scheme, but requires partial CSIT. It is analytically shown that the IC and IAC schemes enable symbol-by-symbol decoding and achieve diversity order of two, while the conventional IA scheme achieves diversity order of one.

1 Introduction

Wireless communication systems such as IMT-Advanced [1] require higher spectral efficiency and data rate, that is, 100 Mbps for high-speed mobility and 1 Gbps for low-speed mobility. With growing demands for higher spectral efficiency, the cell size is getting smaller and smaller, such as micro-, pico- and femto-cells. In the heterogeneous cellular networks [2] consisting of various sizes of cells, interference is a major barrier in achieving reliable high data-rate transmission.

To resolve the interference problem, an interference alignment (IA) scheme was recently proposed and has become a subject of special interest in the area of wireless communications. Cadambe and Jafar [3] showed that each user in a multi-user interference channel can utilise one half of all the network resources, which corresponds to the maximum degrees of freedom (DoF, also known as multiplexing gain). The key idea of this result is the IA which maximises the overlap of all interference signal spaces at each receiver so that the dimension of the interference-free space for the desired signal is maximised. Especially in the case of three-user, multi-input–multi-output (MIMO) interference channel, it is shown that an IA scheme can achieve the maximum DoF for the constant channel with a finite number of dimensions (number of antennas). The idea of IA was further developed by many researchers [4–8].

Although most of the studies for IA focus on network throughput, that is, DoF, reliability in terms of diversity order are also an important performance measure of communication systems. Since Alamouti code [9] has been simple and powerful approach to improve the diversity gain, in this paper, we study how to apply Alamouti code to the MIMO interference channels. We first show that the interference cancellation (IC) scheme based on Alamouti code in the multi-access scenario [10–12] can also be used for the K-user MIMO interference channel with $2M$ transmit antennas. The IC scheme based on quasi-orthogonal space-time block code was proposed for the MAC channel with more than two transmit antennas [12]. The purpose of our paper is to achieve the same DoF as the IA scheme and at the same time obtain diversity gain with single-symbol decoding. Note that there is no diversity gain for IA schemes. Therefore, Alamouti codes are considered, which is the only orthogonal STBC achieving rate 1. In this paper, it is analytically shown that the proposed IC scheme enables receivers to perform symbol-by-symbol decoding and achieves the diversity order of two. The proposed IC scheme takes advantage of the Alamouti structure in doing symbol-by-symbol decoding, whereas the scheme in [10] does not utilise the structure and performs multi-symbol decoding. Also, the proposed scheme does not require a matrix inversion for IC. Therefore, it has an advantage in terms of computational complexity over the scheme in [11], which requires inversion of large matrices. Furthermore, unlike IA schemes for K-user interference channel (except for a blind IA in [13]), the IC scheme does not require channel-state-information at the transmitters (CSIT). However, it requires more receive antennas than IA scheme to achieve the same DoF. As an effort to reduce the number of antennas at the receivers, especially for the three-user MIMO interference channel, we propose an IA-and-cancellation (IAC) scheme based on Alamouti code, which is motivated by the result in [14]. However, our IAC scheme and Li's scheme in [14] are quite different. Li's scheme designed beamforming matrices for alignment by casting interference into small dimensions in the received signal space. In our IAC scheme, by using the proper beamforming matrices, three Alamouti signals from three transmitters can be combined into two Alamouti signals. Then, the receivers separate the desired symbols to perform symbol-by-symbol decoding with less antennas. Further, Li's work focused on only the simplest multi-antenna case of the X channel, which consists of two double-antenna transmitters and two double-antenna receivers. In our IAC scheme considers the three-user interference channel, where each transmitter has arbitrary $2M$ antennas.

We will show that while the proposed IAC scheme achieves the same performance as the IC scheme in terms of DoF and diversity order together with the symbol-by-symbol decoding algorithm, it requires partial CSIT contrary to the IA scheme utilising full CSIT and less receive antennas than the IC scheme.

This paper is organised as follows. Section 2 describes the interference channel model and the IC scheme based on Alamouti code, where it is shown that a symbol-by-symbol decoding is possible for the K-user interference channel. In Section 3, we propose an IAC scheme based on Alamouti code for the three-user interference channel and derive its DoF and diversity order. The numerical analysis is performed in Section 4. Finally, concluding remarks are given in Section 5.

Throughout the paper, the following notations are used. The bold-face uppercase variables and the bold-face lowercase variables denote matrices and column vectors, respectively, and lowercase variables denote scalars. $X \sim \mathcal{CN}(0, \sigma^2)$ represents a complex Gaussian random variable with mean zero and variance $\sigma^2/2$ for each of real and imaginary parts. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\dagger}$ and $\|\cdot\|$ denote the transpose of a matrix or a vector, the conjugate transpose of a matrix or a vector, and the Frobenius norm of a matrix or a vector, respectively. $\mathbb{E}[\cdot]$ denotes the expectation of a random variable. $\boldsymbol{I}_k$ and $\mathbf{diag}(a_1, \ldots, a_k)$ represent the $k \times k$ identity matrix and the diagonal matrix with diagonal entries $a_1, \ldots, a_k$, respectively. Alamouti code (or matrix)

$$\begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix}$$

is denoted by $\boldsymbol{A}(a, b)$.

2 IC scheme based on Alamouti code

Consider the K-user interference channel in Fig. 1, where each transmitter and each receiver have N_{T} and N_{R} antennas, respectively. There are K transmitter–receiver pairs and each transmitter wishes to send an independent data stream to its corresponding receiver. The channel output at the kth receiver is described as

$$\boldsymbol{Y}^{[k]} = \sum_{t=1}^{K} \boldsymbol{H}^{[kt]}\boldsymbol{X}^{[t]} + \boldsymbol{N}^{[k]} \tag{1}$$

where $\boldsymbol{Y}^{[k]}$ is the received signal matrix at the receiver k, $k \in \{1, \ldots, K\}$, $\boldsymbol{X}^{[t]}$ is the transmitted signal matrix from the transmitter t, $t \in \{1, \ldots, K\}$, $\boldsymbol{H}^{[kt]}$ is the channel matrix from the transmitter t to the receiver k and $\boldsymbol{N}^{[k]}$ denotes the additive white complex Gaussian noise (AWGN) matrix with zero-mean and unit-variance entries at the receiver k. Entries of $\boldsymbol{H}^{[kt]}$ are assumed to be independently identically distributed complex Gaussian random variables. It is also assumed that channel is block fading (or constant), that is, the channel state does not change during the transmission of each code.

In the rest of this section, we will show that an IC scheme based on Alamouti code can be applied to the K-user MIMO interference channel. The system block diagram of the IC scheme is shown in Fig. 1.

Theorem 1: For the K-user interference channel, when $N_{\mathrm{T}} = 2M$ and $N_{\mathrm{R}} = KM$, $2M$ DoF and two diversity order are achieved using the IC method for the multi-access scenario [10–12] without CSIT.

Proof: (1) For $K = 2$: We first consider two-user interference channel, when $N_{\mathrm{T}} = N_{\mathrm{R}} = 2M$. Each transmitter sends $2M$ data symbols to its corresponding receiver in two channel uses. The transmitted $2M \times$

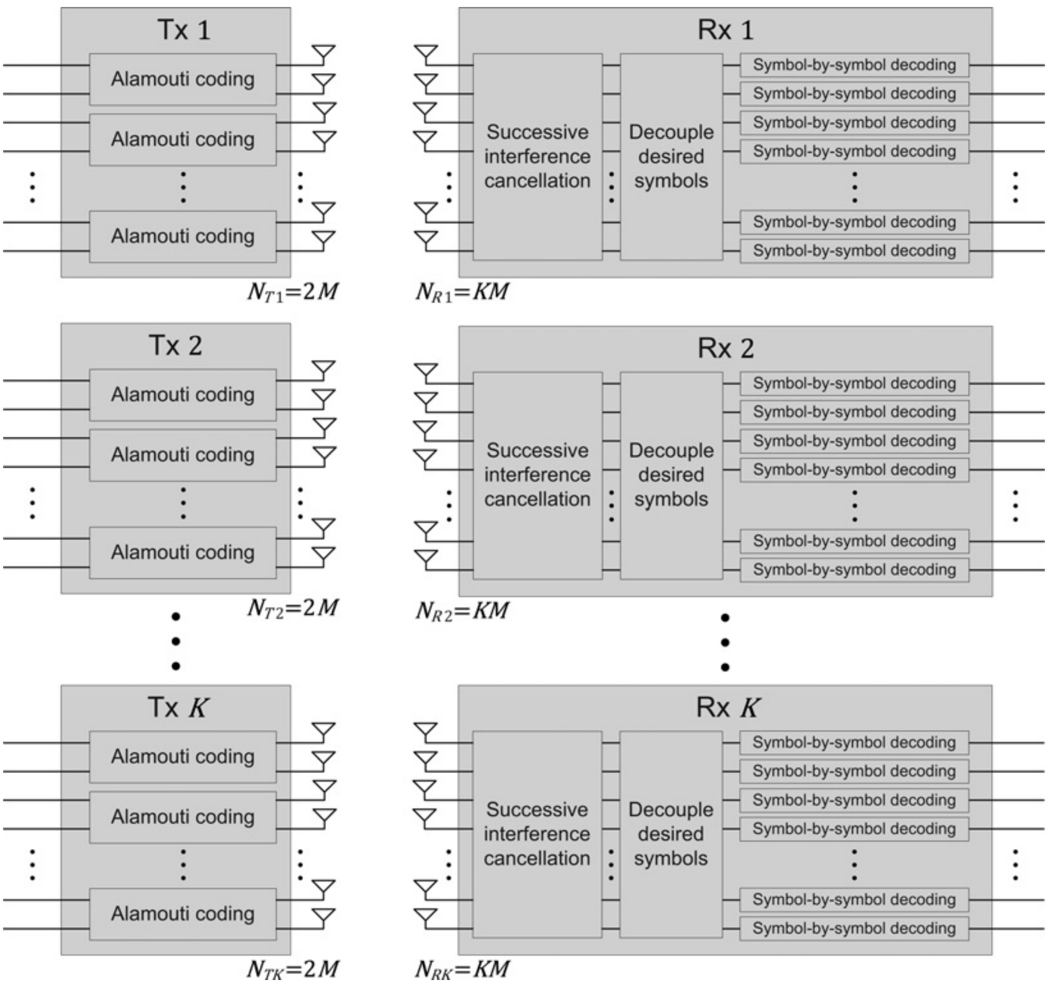

Fig. 1 *Block diagram of IC scheme for the K-user MIMO interference channel with 2M transmit antennas*

2 block codes are generated by using Alamouti codes as

$$\boldsymbol{X}^{[t]} = \begin{bmatrix} \boldsymbol{A}(x_1^{[t]}, x_2^{[t]}) \\ \boldsymbol{A}(x_3^{[t]}, x_4^{[t]}) \\ \vdots \\ \boldsymbol{A}(x_{2M-1}^{[t]}, x_{2M}^{[t]}) \end{bmatrix} \tag{2}$$

where $x_i^{[t]}$, $i \in \{1, \ldots, 2M\}$, $t \in \{1, 2\}$, is the ith data symbol transmitted from the transmitter t. As shown in (2), each transmitter encodes $2M$ data symbols using stacked M Alamouti matrices. It is assumed that every transmitter has the average power constraint per channel use P and thus we set $\mathbb{E}[|x_i^{[t]}|^2] = P/2M$. Without loss of generality, we concentrate on receiver 1, because receiver 2 operates similarly and shows the same performance as receiver 1 because of their symmetric structure. The received signal matrix at receiver 1 is given as

$$\boldsymbol{Y}^{[1]} = \boldsymbol{H}^{[11]}\boldsymbol{X}^{[1]} + \boldsymbol{H}^{[12]}\boldsymbol{X}^{[2]} + \boldsymbol{N}^{[1]} \tag{3}$$

where $\boldsymbol{N}^{[1]}$ is the AWGN matrix with zero-mean and unit-variance entries. We assume that $\boldsymbol{H}^{[1t]}$ is a $2M \times 2M$ matrix whose entries are $h_{i,j}^{[1t]}$, and $\boldsymbol{Y}^{[1]}$ and $\boldsymbol{N}^{[1]}$ are $2M \times 2$ matrices whose entries are $y_{i,j}^{[1]}$ and $n_{i,j}^{[1]}$, respectively. Then, (3) can be vectorised as

$$\begin{bmatrix} \boldsymbol{y}_1^{[1]} \\ \boldsymbol{y}_2^{[1]} \\ \vdots \\ \boldsymbol{y}_{2M}^{[1]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]} & \cdots & \boldsymbol{H}_{1,M}^{[11]} & \boldsymbol{H}_{1,1}^{[12]} & \cdots & \boldsymbol{H}_{1,M}^{[12]} \\ \boldsymbol{H}_{2,1}^{[11]} & \cdots & \boldsymbol{H}_{2,M}^{[11]} & \boldsymbol{H}_{2,1}^{[12]} & \cdots & \boldsymbol{H}_{2,M}^{[12]} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_{2M,1}^{[11]} & \cdots & \boldsymbol{H}_{2M,M}^{[11]} & \boldsymbol{H}_{2M,1}^{[12]} & \cdots & \boldsymbol{H}_{2M,M}^{[12]} \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^{[1]} \\ \vdots \\ \boldsymbol{s}_M^{[1]} \\ \boldsymbol{i}_1^{[1]} \\ \vdots \\ \boldsymbol{i}_M^{[1]} \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \vdots \\ \boldsymbol{n}_{2M}^{[1]} \end{bmatrix} \tag{4}$$

where

$$\boldsymbol{y}_i^{[1]} = \begin{bmatrix} y_{i,1}^{[1]} & y_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{H}_{i,j}^{[1t]} = \begin{bmatrix} h_{i,2j-1}^{[1t]} & h_{i,2j}^{[1t]} \\ h_{i,2j}^{[1t]*} & -h_{i,2j-1}^{[1t]*} \end{bmatrix}$$

$$\boldsymbol{n}_i^{[1]} = \begin{bmatrix} n_{i,1}^{[1]} & n_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}, \quad i \in \{1, \ldots, 2M\}, \quad j \in \{1, \ldots, M\}$$

and $\boldsymbol{s}_m^{[1]}$ and $\boldsymbol{i}_m^{[1]}$, $m \in \{1, \ldots, M\}$ are the desired symbol vector $\begin{bmatrix} x_{2m-1}^{[1]} & x_{2m}^{[1]} \end{bmatrix}^{\mathrm{T}}$ and the interfering symbol vector $\begin{bmatrix} x_{2m-1}^{[2]} & x_{2m}^{[2]} \end{bmatrix}^{\mathrm{T}}$, respectively. This vectorisation is extended from (10) in [15]. Note that $\boldsymbol{H}_{i,j}^{[1t]}$ is an Alamouti matrix.

Then the receiver successively removes the interfering symbol vectors $\boldsymbol{i}_m^{[1]}$ by using the IC method for the multi-access scenario [10–12], which utilises the following well-known properties of the Alamouti structure:

1. Alamouti Property 1: The Alamouti matrices are closed under conjugate transpose, matrix addition, matrix multiplication and real scalar multiplication.
2. Alamouti Property 2: For the Alamouti matrix $\boldsymbol{A}$, $\boldsymbol{A}\boldsymbol{A}^{\dagger} = \boldsymbol{A}^{\dagger}\boldsymbol{A} = (1/2)\|\boldsymbol{A}\|^2\boldsymbol{I}_2$.

Define $\boldsymbol{G}_i^{[1]}(l)$ as (see (5)) for $i = 1, \ldots, 2M - l$. Note that the 4×2 matrix $\boldsymbol{G}_i^{[1]}(l)$ is composed of two Alamouti matrices $\boldsymbol{I}_2$ and $\boldsymbol{F}_i^{[1]}(l)$ (because of Alamouti Property 1). Then, the interfering symbol vector $\boldsymbol{i}_{M-l+1}^{[1]}$ is removed successively by using $\boldsymbol{G}_i^{[1]}(l)$ starting from $l = 1$ to $l = M$ as (see (6)) where

$$\boldsymbol{y}_i^{[1]}(l) = \boldsymbol{G}_i^{[1]\dagger}(l) \begin{bmatrix} \boldsymbol{y}_i^{[1]}(l-1) \\ \boldsymbol{y}_{2M-l+1}^{[1]}(l-1) \end{bmatrix},$$

$$\boldsymbol{H}_{i,j}^{[1t]}(l) = \boldsymbol{G}_i^{[1]\dagger}(l) \begin{bmatrix} \boldsymbol{H}_{i,j}^{[1t]}(l-1) \\ \boldsymbol{H}_{2M-l+1,j}^{[1t]}(l-1) \end{bmatrix}$$

and

$$\boldsymbol{n}_i^{[1]}(l) = \boldsymbol{G}_i^{[1]\dagger}(l) \begin{bmatrix} \boldsymbol{n}_i^{[1]}(l-1) \\ \boldsymbol{n}_{2M-l+1}^{[1]}(l-1) \end{bmatrix}$$

and the index '(l)' in them indicates that they are the results after the lth IC. Note that $\boldsymbol{H}_{i,j}^{[1t]}(l)$'s are still Alamouti matrices because of Alamouti Property 1 and the last two rows and two columns of the equivalent channel matrix in (4) are removed after each cancellation. After the Mth IC, all the interfering symbols are

$$\boldsymbol{G}_i^{[1]}(l) = \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{F}_i^{[1]\dagger}(l) \end{bmatrix}^{\dagger} = \begin{cases} \begin{bmatrix} \boldsymbol{I}_2 & -\dfrac{2\boldsymbol{H}_{i,M-l+1}^{[12]}(l-1)\boldsymbol{H}_{2M-l+1,M-l+1}^{[12]\dagger}(l-1)}{\|\boldsymbol{H}_{2M-l+1,M-l+1}^{([12]}(l-1)\|^2} \end{bmatrix}^{\dagger}, & l = 0, \ldots, M \\ \begin{bmatrix} \boldsymbol{I}_2 & -\dfrac{2\boldsymbol{H}_{i,2M-l+1}^{[11]}(l-1)\boldsymbol{H}_{2M-l+1,2M-l+1}^{[11]\dagger}(l-1)}{\|\boldsymbol{H}_{2M-l+1,2M-l+1}^{[11]}(l-1)\|^2} \end{bmatrix}^{\dagger}, & l = M+1, \ldots, 2M-1 \end{cases} \tag{5}$$

$$\begin{bmatrix} \boldsymbol{y}_1^{[1]}(l) \\ \boldsymbol{y}_2^{[1]}(l) \\ \vdots \\ \boldsymbol{y}_{2M-l}^{[1]}(l) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]}(l) & \cdots & \boldsymbol{H}_{1,M}^{[11]}(l) & \boldsymbol{H}_{1,1}^{[12]}(l) & \cdots & \boldsymbol{H}_{1,M-l}^{[12]}(l) \\ \boldsymbol{H}_{2,1}^{[11]}(l) & \cdots & \boldsymbol{H}_{2,M}^{[11]}(l) & \boldsymbol{H}_{2,1}^{[12]}(l) & \cdots & \boldsymbol{H}_{2,M-l}^{[12]}(l) \\ & & & \vdots & & \\ \boldsymbol{H}_{2M-l,1}^{[11]}(l) & \cdots & \boldsymbol{H}_{2M-l,M}^{[11]}(l) & \boldsymbol{H}_{2M-l,1}^{[12]}(l) & \cdots & \boldsymbol{H}_{2M-l,M-l}^{[12]}(l) \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^{[1]} \\ \vdots \\ \boldsymbol{s}_M^{[1]} \\ \boldsymbol{i}_1^{[1]} \\ \vdots \\ \boldsymbol{i}_{M-l}^{[1]} \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_1^{[1]}(l) \\ \boldsymbol{n}_2^{[1]}(l) \\ \vdots \\ \boldsymbol{n}_{2M-l}^{[1]}(l) \end{bmatrix} \tag{6}$$

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removed and the received symbol equation is obtained as

$$\begin{bmatrix} \boldsymbol{y}_1^{[1]}(M) \\ \boldsymbol{y}_2^{[1]}(M) \\ \vdots \\ \boldsymbol{y}_M^{[1]}(M) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]}(M) & \boldsymbol{H}_{1,2}^{[11]}(M) & \cdots & \boldsymbol{H}_{1,M}^{[11]}(M) \\ \boldsymbol{H}_{2,1}^{[11]}(M) & \boldsymbol{H}_{2,2}^{[11]}(M) & \cdots & \boldsymbol{H}_{2,M}^{[11]}(M) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_{M,1}^{[11]}(M) & \boldsymbol{H}_{M,2}^{[11]}(M) & \cdots & \boldsymbol{H}_{M,M}^{[11]}(M) \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^{[1]} \\ \boldsymbol{s}_2^{[1]} \\ \vdots \\ \boldsymbol{s}_M^{[1]} \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_1^{[1]}(M) \\ \boldsymbol{n}_2^{[1]}(M) \\ \vdots \\ \boldsymbol{n}_M^{[1]}(M) \end{bmatrix} \tag{7}$$

In order to perform symbol-by-symbol decoding of the desired symbols, successive cancellation of $\boldsymbol{s}_{2M-l+1}^{[1]}$ should be performed starting from $l = M+1$ to $l = 2M-1$ similar to the above IC. That is, by using $\boldsymbol{G}_i^{[1]}(l)$ for $l = M+1, \ldots, 2M-1$ in (5), we have

$$\begin{bmatrix} \boldsymbol{y}_1^{[1]}(l) \\ \boldsymbol{y}_2^{[1]}(l) \\ \vdots \\ \boldsymbol{y}_{2M-l}^{[1]}(l) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]}(l) & \boldsymbol{H}_{1,2}^{[11]}(l) & \cdots & \boldsymbol{H}_{1,2M-l}^{[11]}(l) \\ \boldsymbol{H}_{2,1}^{[11]}(l) & \boldsymbol{H}_{2,2}^{[11]}(l) & \cdots & \boldsymbol{H}_{2,2M-l}^{[11]}(l) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_{2M-l,1}^{[11]}(l) & \boldsymbol{H}_{2M-l,2}^{[11]}(l) & \cdots & \boldsymbol{H}_{2M-l,2M-l}^{[11]}(l) \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^{[1]} \\ \boldsymbol{s}_2^{[1]} \\ \vdots \\ \boldsymbol{s}_{2M-l}^{[1]} \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_1^{[1]}(l) \\ \boldsymbol{n}_2^{[1]}(l) \\ \vdots \\ \boldsymbol{n}_{2M-l}^{[1]}(l) \end{bmatrix} \tag{8}$$

where

$$\boldsymbol{y}_i^{[1]}(l) = \boldsymbol{G}_i^{[1]\dagger}(l) \begin{bmatrix} \boldsymbol{y}_i^{[1]}(l-1) \\ \boldsymbol{y}_{2M-l+1}^{[1]}(l-1) \end{bmatrix},$$

$$\boldsymbol{H}_{i,j}^{[11]}(l) = \boldsymbol{G}_i^{[1]\dagger}(l) \begin{bmatrix} \boldsymbol{H}_{i,j}^{[11]}(l-1) \\ \boldsymbol{H}_{2M-l+1,j}^{[11]}(l-1) \end{bmatrix}$$

and

$$\boldsymbol{n}_i^{[1]}(l) = \boldsymbol{G}_i^{[1]\dagger}(l) \begin{bmatrix} \boldsymbol{n}_i^{[1]}(l-1) \\ \boldsymbol{n}_{2M-l+1}^{[1]}(l-1) \end{bmatrix}$$

After cancelling out $\boldsymbol{s}_M^{[1]}, \boldsymbol{s}_{M-1}^{[1]}, \ldots, \boldsymbol{s}_2^{[1]}$ from (7), receiver 1 can extract two desired symbols $\boldsymbol{s}_1^{[1]} = \begin{bmatrix} x_1^{[1]} & x_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ from

$$\boldsymbol{y}_1^{[1]}(2M-1) = \boldsymbol{H}_{1,1}^{[11]}(2M-1)\boldsymbol{s}_1^{[1]} + \boldsymbol{n}_1^{[1]}(2M-1) \tag{9}$$

where $\boldsymbol{H}_{1,1}^{[11]}(2M-1)$ still keeps the Alamouti structure because of Alamouti Property 1. Therefore symbol-by-symbol decoding is performed to estimate the desired symbols $x_1^{[1]}$ and $x_2^{[1]}$ and the other desired symbols can be decoded from (7) in a similar successive way. Note that Alamouti Property 2 makes each cancellation procedure possible and Alamouti Property 1 preserves the Alamouti structures of the equivalent channels. Owing to the symmetry, it is easy to show that the same method can be applied to receiver 2 to decode its $2M$ desired symbols. Consequently, the total DoF is $4M$ and the IC scheme achieves $2M$ DoF per channel use which is the maximum DoF for the two-user interference channel with $2M$ antennas. Moreover, the proposed IC scheme achieves diversity order of two, that is, its pairwise error probability (PEP) decays proportional to $1/P^2$, which will be shown in Section 3.

(2) For $K \geq 3$: Consider the $K(\geq 3)$-user interference channel, where each user has $2M$ antennas. Similar to (4), we have the equivalent system of the linear equations at receiver 1 as (see (10)) where

$$\boldsymbol{y}_i^{[1]} = \begin{bmatrix} y_{i,1}^{[1]} & y_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{H}_{i,j}^{[1t]} = \begin{bmatrix} h_{i,2j-1}^{[1t]} & h_{i,2j}^{[1t]} \\ h_{i,2j}^{[1t]*} & -h_{i,2j-1}^{[1t]*} \end{bmatrix},$$
$$\boldsymbol{n}_i^{[1]} = \begin{bmatrix} n_{i,1}^{[1]} & n_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}$$

and $\boldsymbol{s}_m^{[1]}$ and $\boldsymbol{i}_m^{[1]}$ are the desired symbol vector and the interfering symbol vector, respectively. After $2M-1$ cancellations similar to the two-user case, we have (see (11)) where the index '$(2M-1)$' is omitted for tractability. Equation (11) shows that receiver 1 cannot cancel the interference symbols anymore. Therefore the desired symbols cannot be obtained by symbol-by-symbol decoding. In order to apply symbol-by-symbol decoding to the IC scheme based on Alamouti code for the K-user interference channel, the number of rows of $\boldsymbol{H}_{\mathrm{eff}}$ in (10) should be the same as the number of elements of $\boldsymbol{S}_{\mathrm{eff}}$ in (10). Consequently, when each transmitter and each receiver have $2M$ and KM antennas, respectively, the IC scheme utilising symbol-by-symbol decoding can be applied to the K-user interference channel and KM DoF is achieved. In Section 3, it will be shown that the proposed IC scheme achieves diversity order of two. □

$$\begin{bmatrix} \boldsymbol{y}_1^{[1]} \\ \boldsymbol{y}_2^{[1]} \\ \vdots \\ \boldsymbol{y}_{2M}^{[1]} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]} & \cdots & \boldsymbol{H}_{1,M}^{[11]} & \boldsymbol{H}_{1,1}^{[12]} & \cdots & \boldsymbol{H}_{1,M}^{[12]} & \cdots\cdots & \boldsymbol{H}_{1,1}^{[1K]} & \cdots & \boldsymbol{H}_{1,M}^{[1K]} \\ \boldsymbol{H}_{2,1}^{[11]} & \cdots & \boldsymbol{H}_{2,M}^{[11]} & \boldsymbol{H}_{2,1}^{[12]} & \cdots & \boldsymbol{H}_{2,M}^{[12]} & \cdots\cdots & \boldsymbol{H}_{2,1}^{[1K]} & \cdots & \boldsymbol{H}_{2,M}^{[1K]} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_{2M,1}^{[11]} & \cdots & \boldsymbol{H}_{2M,M}^{[11]} & \boldsymbol{H}_{2M,1}^{[12]} & \cdots & \boldsymbol{H}_{2M,M}^{[12]} & \cdots\cdots & \boldsymbol{H}_{2M,1}^{[1K]} & \cdots & \boldsymbol{H}_{2M,M}^{[1K]} \end{bmatrix}}_{\boldsymbol{H}_{\mathrm{eff}}} \times \underbrace{\begin{bmatrix} \boldsymbol{s}_1^{[1]\mathrm{T}} & \cdots & \boldsymbol{s}_M^{[1]\mathrm{T}} & \boldsymbol{i}_1^{[1]\mathrm{T}} & \cdots & \boldsymbol{i}_M^{[1]\mathrm{T}} & \cdots\cdots & \boldsymbol{i}_{M(K-2)+1}^{[1]\mathrm{T}} & \cdots & \boldsymbol{i}_{M(K-1)}^{[1]\mathrm{T}} \end{bmatrix}^{\mathrm{T}}}_{\boldsymbol{S}_{\mathrm{eff}}} + \begin{bmatrix} \boldsymbol{n}_1^{[1]\mathrm{T}} & \boldsymbol{n}_2^{[1]\mathrm{T}} & \cdots & \boldsymbol{n}_{2M}^{[1]\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{10}$$

$$\boldsymbol{y}_1^{[1]} = \begin{bmatrix} \boldsymbol{H}_{11}^{[11]} & \cdots & \boldsymbol{H}_{1M}^{[11]} & \boldsymbol{H}_{11}^{[12]} & \cdots & \boldsymbol{H}_{1M}^{[12]} & \cdots & \boldsymbol{H}_{11}^{[1(K-2)]} & \cdots & \boldsymbol{H}_{1M}^{[1(K-2)]} & \boldsymbol{H}_{11}^{[1(K-1)]} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{s}_1^{[1]\mathrm{T}} & \cdots & \boldsymbol{s}_M^{[1]\mathrm{T}} & \boldsymbol{i}_1^{[1]\mathrm{T}} & \cdots & \boldsymbol{i}_M^{[1]\mathrm{T}} & \cdots & \boldsymbol{i}_{M(K-4)+1}^{[1]\mathrm{T}} & \cdots & \boldsymbol{i}_{M(K-3)}^{[1]\mathrm{T}} & \boldsymbol{i}_{M(K-3)+1}^{[1]\mathrm{T}} \end{bmatrix}^{\mathrm{T}} + \boldsymbol{n}_1^{[1]} \tag{11}$$

3 IAC scheme for the three-user MIMO interference channel

In this section, we consider the three-user interference channel where each transmitter has $2M$ antennas to utilise Alamouti codes. In the study of IA, the three-user MIMO interference channel is interesting because $3M$ DoF is achieved with constant channel matrices and finite number of dimensions (number of antennas). The result in Section 2 implies that if each receiver has $3M$ antennas, the IC scheme utilising symbol-by-symbol decoding can be applied to the three-user interference channel and $3M$ DoF is achieved. However, it is well known that IA scheme can achieve the same DoF of $3M$ when each transmitter and each receiver have $2M$ antennas. Even though the IC scheme achieves diversity order of two and requires no CSIT, it requires more receive antennas than the IA scheme. In this section, an IAC scheme based on Alamouti code for the three-user interference channel is proposed as a part of efforts to reduce the number of antennas at the receivers while keeping the advantages of IC scheme.

3.1 Transmission and reception schemes

The system block diagram of the IAC scheme is shown in Fig. 2. In order to reduce the number of antennas at the receivers, the IAC scheme utilises beamforming matrices. The transmitted $2M \times 2$ block code at each transmitter is designed as

$$X^{[t]} = P^{[t]} \underbrace{\begin{bmatrix} A(x_1^{[t]}, x_2^{[t]}) \\ A(x_3^{[t]}, x_4^{[t]}) \\ \vdots \\ A(x_{2M-1}^{[t]}, x_{2M}^{[t]}) \end{bmatrix}}_{A_t} \tag{12}$$

where $x_i^{[t]}$ is the ith data symbol transmitted at the transmitter t and $P^{[t]}$ is the beamforming matrix of the transmitter t. As shown in (12), each transmitter encodes data symbols using stacked M Alamouti codes followed by $P^{[t]}$. Then, the received signal matrix at the receiver k, $k \in \{1, 2, 3\}$, is given as

$$Y^{[k]} = H^{[k1]}P^{[1]}A_1 + H^{[k2]}P^{[2]}A_2 + H^{[k3]}P^{[3]}A_3 + N^{[k]} \tag{13}$$

In (13), the term $H^{[kk]}P^{[k]}A_k$ is the desired part and the other two terms are interfering parts at the receiver k. If two interfering parts can be aligned without breaking the Alamouti structure, we can regard this situation as that the receiver receives two Alamouti codes from two transmitters. Then, similar to the IC scheme in Section 2, the receiver can cancel interference and perform symbol-by-symbol decoding with less antennas.

Suppose that the beamforming matrices $P^{[t]}$ are designed to satisfy the following conditions

$$H^{[12]}P^{[2]} = H^{[13]}P^{[3]} \tag{14}$$

$$H^{[21]}P^{[1]} = H^{[23]}P^{[3]} \tag{15}$$

$$H^{[31]}P^{[1]} = H^{[32]}P^{[2]} \tag{16}$$

Then, (13) can be rewritten as

$$Y^{[k]} = H^{[kk]}P^{[k]}A_k + H^{[kt]}P^{[t]}\sum_{t \neq k} A_t + N^{[k]} \tag{17}$$

Since $\sum_{t \neq k} A_t$ has the Alamouti structure because of Alamouti Property 1, (17) has the same structure as (3). Then the receiver can decode by using the same method in the previous section with $2M$ antennas. However, the conditions (14)–(16) cannot have non-trivial exact solutions $P^{[t]}$ simultaneously. Therefore, we propose an IAC scheme which uses the beamforming matrices satisfying just two of the three conditions. We choose the identity matrix I_{2M} as $P^{[1]}$, and $P^{[2]}$ and $P^{[3]}$ are determined to satisfy (15) and (16). By using these beamforming matrices, receivers 2 and 3 satisfy (17) and thus the number of antennas at each of them is decreased from $3M$ to $2M$. Note that the channel state information at receiver 1 is not required to the transmitters to design the above beamforming matrices. It can be interpreted that, in the proposed IAC scheme, the receivers with less antennas must feedback their channel state information to all transmitters.

Now, the decoding scheme at each receiver will be proposed. First, for receiver 1, the IC decoding scheme as shown in Section 2 is used. The received signal matrix at receiver 1 is given as

$$Y^{[1]} = H^{[11]}P^{[1]}A_1 + H^{[12]}P^{[2]}A_2 + H^{[13]}P^{[3]}A_3 + N^{[1]} \tag{18}$$

Note that receiver 1 has $3M$ antennas. We assume that $H^{[1t]}P^{[t]}$ is a $3M \times 2M$ matrix whose entries are $h_{i,j}^{[1t]}$, and $Y^{[1]}$ and $N^{[1]}$ are $3M \times 2$ matrices whose entries are $y_{i,j}^{[1]}$ and $n_{i,j}^{[1]}$, respectively. Then the equivalent system of the linear equations in (18) can be vectorised as where (see (19))

$$y_i^{[1]} = \begin{bmatrix} y_{i,1}^{[1]} & y_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}, \quad H_{i,j}^{[1t]} = \begin{bmatrix} h_{i,2j-1}^{[1t]} & h_{i,2j}^{[1t]} \\ h_{i,2j}^{[1t]*} & -h_{i,2j-1}^{[1t]*} \end{bmatrix}$$

and

$$n_i^{[1]} = \begin{bmatrix} n_{i,1}^{[1]} & n_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}},$$
$$i \in \{1, \ldots, 3M\}, j \in \{1, \ldots, M\}, t \in \{1, 2, 3\}$$

The vectors s_m and i_m are the desired symbol vector and interfering symbol vector given as

$$s_m^{[1]} = \begin{bmatrix} x_{2m-1}^{[1]} & x_{2m}^{[1]} \end{bmatrix}^{\mathrm{T}}, \quad m \in \{1, \ldots, M\}$$

$$i_m^{[1]} = \begin{cases} [x_{2m-1}^{[2]} \quad x_{2m}^{[2]}]^{\mathrm{T}}, & m \in \{1, \ldots, M\} \\ \begin{bmatrix} x_{2(m-M)-1}^{[3]} & x_{2(m-M)}^{[3]} \end{bmatrix}^{\mathrm{T}}, & m \in \{M+1, \ldots, 2M\} \end{cases}$$

Since the number of rows of H_{eff} in (19) is the same as the number of elements of S_{eff} in (19), receiver 1 can cancel all interfering symbols and symbol-by-symbol decoding is possible.

$$\begin{bmatrix} y_1^{[1]} \\ y_2^{[1]} \\ \vdots \\ y_{3M}^{[1]} \end{bmatrix} = \underbrace{\begin{bmatrix} H_{1,1}^{[11]} & \cdots & H_{1,M}^{[11]} & H_{1,1}^{[12]} & \cdots & H_{1,M}^{[12]} & H_{1,1}^{[13]} & \cdots & H_{1,M}^{[13]} \\ H_{2,1}^{[11]} & \cdots & H_{2,M}^{[11]} & H_{2,1}^{[12]} & \cdots & H_{2,M}^{[12]} & H_{2,1}^{[13]} & \cdots & H_{2,M}^{[13]} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & & & \\ H_{3M,1}^{[11]} & \cdots & H_{3M,M}^{[11]} & H_{3M,1}^{[12]} & \cdots & H_{3M,M}^{[12]} & H_{3M,1}^{[13]} & \cdots & H_{3M,M}^{[13]} \end{bmatrix}}_{H_{\mathrm{eff}}} \underbrace{\begin{bmatrix} s_1^{[1]} \\ \vdots \\ s_M^{[1]} \\ i_1^{[1]} \\ \vdots \\ i_{2M}^{[1]} \end{bmatrix}}_{S_{\mathrm{eff}}} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{3M} \end{bmatrix} \tag{19}$$

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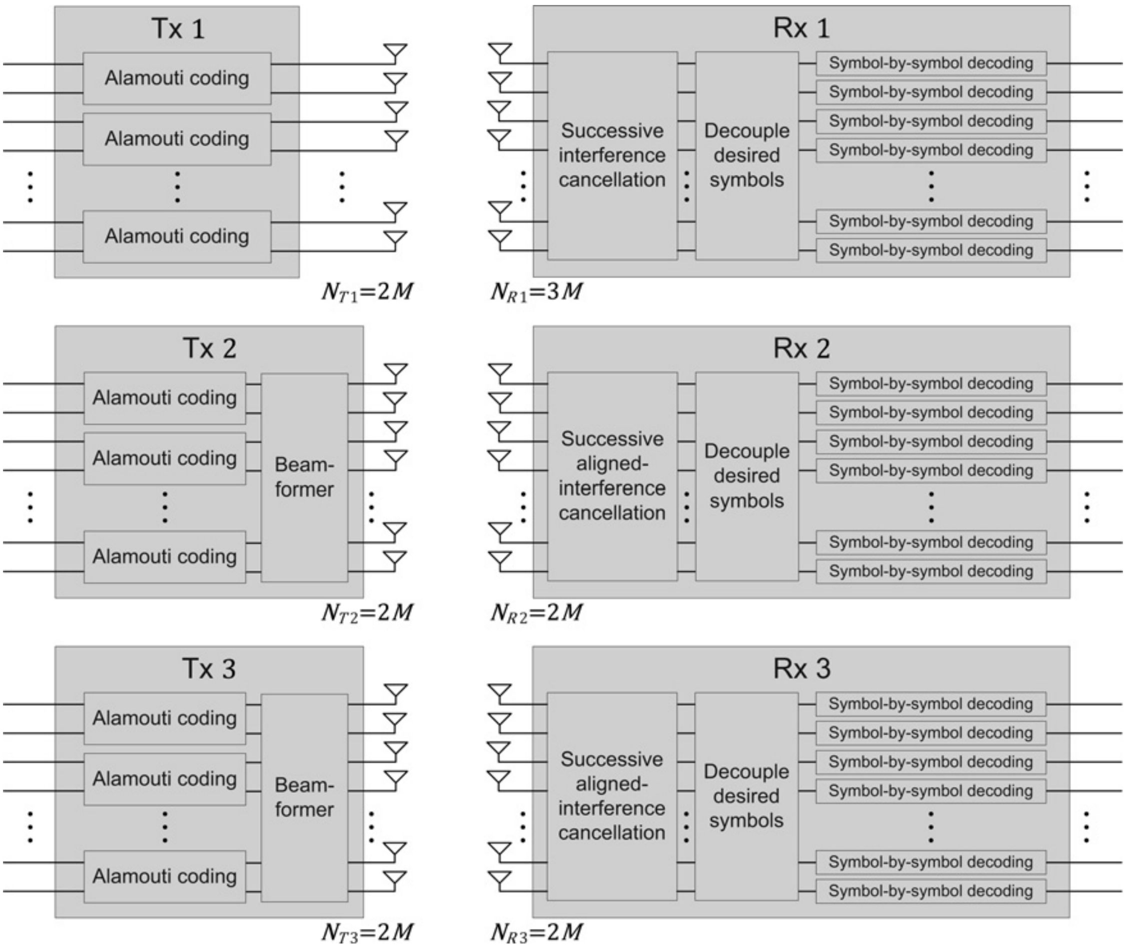

Fig. 2 *Block diagram of IAC scheme for the three-user MIMO interference channel with 2M transmit antennas*

Each of receivers 2 and 3 receives the signal in the form of (17). Therefore, by using $2M$ antennas, they can separate the desired symbols from the interfering symbols to perform symbol-by-symbol decoding, which is similar to the IC scheme for the two-user interference channel in the previous section. Consequently, the total DoF is $6M$ and the DoF per channel use is $3M$ when one receiver has $3M$ antennas and each of the other two receivers has $2M$ antennas.

3.2 Diversity analysis

In this subsection, the diversity order of the proposed IAC scheme is derived by using PEP. We show the achievable diversity order of the proposed IAC scheme in the following theorem.

Theorem 2: For the three-user interference channel, when one receiver has $3M$ antennas and each of the other two receivers has $2M$ antennas, the proposed IAC scheme achieves a diversity order of two for each symbol.

Proof: (1) For receiver 1 when $M = 1$: First, consider the case of $M = 1$, that is, the three-user interference channel with three transmitters and three receivers, where each is equipped with two antennas except that receiver 1 has three antennas. At receiver 1 with three antennas, (19) can be rewritten as

$$\begin{bmatrix} \boldsymbol{y}_1^{[1]} \\ \boldsymbol{y}_2^{[1]} \\ \boldsymbol{y}_3^{[1]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]} & \boldsymbol{H}_{1,1}^{[12]} & \boldsymbol{H}_{1,1}^{[13]} \\ \boldsymbol{H}_{2,1}^{[11]} & \boldsymbol{H}_{2,1}^{[12]} & \boldsymbol{H}_{2,1}^{[13]} \\ \boldsymbol{H}_{3,1}^{[11]} & \boldsymbol{H}_{3,1}^{[12]} & \boldsymbol{H}_{3,1}^{[13]} \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^{[1]} \\ \boldsymbol{i}_1^{[1]} \\ \boldsymbol{i}_2^{[1]} \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \boldsymbol{n}_3^{[1]} \end{bmatrix} \tag{20}$$

Note that each entry of $\boldsymbol{H}_{ij}^{[1t]}$ in (20) might not be a complex Gaussian random variable, since it is the result of multiplication of Gaussian channel matrix with the beamforming matrix. However, the authors in [16] derived that approximating each entry of $\boldsymbol{H}_{ij}^{[1t]}$ as a complex Gaussian random variable does not change the diversity order. Therefore we approximate $\boldsymbol{H}_{ij}^{[1t]}$ as a complex Gaussian matrix to derive the diversity order of the proposed IAC scheme.

Using

$$\boldsymbol{G}_i^{[1]}(1) = \left[\boldsymbol{I}_2 \quad \boldsymbol{F}_i^{[1]\dagger}(1)\right]^\dagger = \left[\boldsymbol{I}_2 \quad -\frac{2\boldsymbol{H}_{i,1}^{[13]}\boldsymbol{H}_{3,1}^{[13]\dagger}}{\|\boldsymbol{H}_{3,1}^{[13]}\|^2}\right]^\dagger, \quad i \in \{1, 2\}$$

and Alamouti Property 2, receiver 1 removes the interfering symbol vector $\boldsymbol{i}_2$ as

$$\begin{bmatrix} \boldsymbol{y}_1^{[1]}(1) \\ \boldsymbol{y}_2^{[1]}(1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]}(1) & \boldsymbol{H}_{1,1}^{[12]}(1) \\ \boldsymbol{H}_{2,1}^{[11]}(1) & \boldsymbol{H}_{2,1}^{[12]}(1) \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^{[1]} \\ \boldsymbol{i}_1^{[1]} \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_1^{[1]}(1) \\ \boldsymbol{n}_2^{[1]}(1) \end{bmatrix} \tag{21}$$

where

$$\boldsymbol{y}_i^{[1]}(1) = \boldsymbol{G}_i^{[1]\dagger}(1)\begin{bmatrix} \boldsymbol{Y}_i^{[1]} \\ \boldsymbol{Y}_3^{[1]} \end{bmatrix}, \quad \boldsymbol{H}_{i,1}^{[1t]}(1) = \boldsymbol{G}_i^{[1]\dagger}(1)\begin{bmatrix} \boldsymbol{H}_{i,1}^{[1t]} \\ \boldsymbol{H}_{3,1}^{[1t]} \end{bmatrix},$$
$$\boldsymbol{n}_i^{[1]}(1) = \boldsymbol{G}_i^{[1]\dagger}(1)\begin{bmatrix} \boldsymbol{n}_i^{[1]} \\ \boldsymbol{n}_3^{[1]} \end{bmatrix} \tag{22}$$

Let

$$\boldsymbol{G}_1^{[1]}(2) = \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{F}_1^{[1]\dagger}(2) \end{bmatrix}^{\dagger} = \begin{bmatrix} \boldsymbol{I}_2 & -\dfrac{2\boldsymbol{H}_{1,1}^{[12]}(1)\boldsymbol{H}_{2,1}^{[12]\dagger}(1)}{\|\boldsymbol{H}_{2,1}^{[12]}(1)\|^2} \end{bmatrix}^{\dagger}$$

Then, the second IC is performed to obtain

$$\boldsymbol{y}_1^{[1]}(2) = \boldsymbol{H}_{1,1}^{[11]}(2)\boldsymbol{s}_1^{[1]} + \boldsymbol{n}_1^{[1]}(2) \tag{23}$$

which is equivalent to

$$\begin{aligned} &\boldsymbol{G}_1^{[1]\dagger}(2)\begin{bmatrix} \boldsymbol{y}_1^{[1]}(1) \\ \boldsymbol{y}_2^{[1]}(1) \end{bmatrix} = \boldsymbol{G}_1^{[1]\dagger}(2)\begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]}(1) \\ \boldsymbol{H}_{2,1}^{[11]}(1) \end{bmatrix}\boldsymbol{s}_1^{[1]} + \boldsymbol{G}_1^{[1]\dagger}(2)\begin{bmatrix} \boldsymbol{n}_1^{[1]}(1) \\ \boldsymbol{n}_2^{[1]}(1) \end{bmatrix} \\ &\overset{(a)}{\Leftrightarrow} \begin{bmatrix} \boldsymbol{G}_1^{[1]}(1) \\ \boldsymbol{G}_2^{[1]}(1)\boldsymbol{F}_1^{[1]}(2) \end{bmatrix}^{\dagger}\begin{bmatrix} \boldsymbol{y}_1^{[1]} \\ \boldsymbol{y}_3^{[1]} \\ \boldsymbol{y}_2^{[1]} \\ \boldsymbol{y}_3^{[1]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_1^{[1]}(1) \\ \boldsymbol{G}_2^{[1]}(1)\boldsymbol{F}_1^{[1]}(2) \end{bmatrix}^{\dagger}\begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]} \\ \boldsymbol{H}_{3,1}^{[11]} \\ \boldsymbol{H}_{2,1}^{[11]} \\ \boldsymbol{H}_{3,1}^{[11]} \end{bmatrix}\boldsymbol{s}_1^{[1]} \\ &\quad + \begin{bmatrix} \boldsymbol{G}_1^{[1]}(1) \\ \boldsymbol{G}_2^{[1]}(1)\boldsymbol{F}_1^{[1]}(2) \end{bmatrix}^{\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_3^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \boldsymbol{n}_3^{[1]} \end{bmatrix} \end{aligned} \tag{24}$$

where (a) uses the definition of $\boldsymbol{G}_1^{[1]}(2)$ and (22). Equation (24) shows that all the interfering signals are removed after two successive cancellations and $\boldsymbol{H}_{1,1}^{[11]}(2)$ in (23) still keeps the Alamouti structure because of Alamouti Property 1. From the definition of $\boldsymbol{G}_i^{[1]}(l)$, (24) can be rearranged as

$$\begin{aligned} \widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{y}_1^{[1]} \\ \boldsymbol{y}_2^{[1]} \\ \boldsymbol{y}_3^{[1]} \end{bmatrix} &= \widehat{\boldsymbol{G}}^{[1]\dagger}\underbrace{\begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]} \\ \boldsymbol{H}_{2,1}^{[11]} \\ \boldsymbol{H}_{3,1}^{[11]} \end{bmatrix}}_{\boldsymbol{H}_{\text{eff}}^{[1]}}\boldsymbol{s}_1^{[1]} + \widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \boldsymbol{n}_3^{[1]} \end{bmatrix} \\ &= \underbrace{\widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{h}_1^{[11]} & \boldsymbol{h}_2^{[11]} \end{bmatrix}}_{\boldsymbol{H}_{\text{eff}}^{[1]}}\boldsymbol{s}_1^{[1]} + \widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \boldsymbol{n}_3^{[1]} \end{bmatrix} \end{aligned} \tag{25}$$

where

$$\widehat{\boldsymbol{G}}^{[1]} = \begin{bmatrix} \boldsymbol{I}_2 \\ \boldsymbol{F}_1^{[1]}(2) \\ \boldsymbol{F}_1^{[1]}(1) + \boldsymbol{F}_2^{[1]}(1)\boldsymbol{F}_1^{[1]}(2) \end{bmatrix} \tag{26}$$

and $\boldsymbol{h}_i^{[11]}, i \in \{1, 2\}$, is defined as the ith column vector of $\begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]\dagger} & \boldsymbol{H}_{2,1}^{[11]\dagger} & \boldsymbol{H}_{3,1}^{[11]\dagger} \end{bmatrix}^{\dagger}$. Alamouti Property 1 shows that $\widehat{\boldsymbol{G}}^{[1]}$ is a 6×2 matrix stacked with three Alamouti matrices and the equivalent channel $\boldsymbol{H}_{\text{eff}}^{[1]}$ in (25) is also an Alamouti matrix. Therefore (25) can be divided into two equations for two symbols of $\boldsymbol{s}_1^{[1]} = \begin{bmatrix} x_1^{[1]} & x_2^{[1]} \end{bmatrix}$ by multiplying the conjugate transpose of the equivalent channel matrix.

Receiver 1 can decode its desired symbols $x_1^{[1]}$ and $x_2^{[1]}$ by using the symbol-by-symbol decoding as

$$\boldsymbol{H}_{\text{eff}}^{[1]\dagger}\widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{y}_1^{[1]} \\ \boldsymbol{y}_2^{[1]} \\ \boldsymbol{y}_3^{[1]} \end{bmatrix} = \boldsymbol{H}_{\text{eff}}^{[1]\dagger}\boldsymbol{H}_{\text{eff}}^{[1]}\boldsymbol{s}_1^{[1]} + \boldsymbol{H}_{\text{eff}}^{[1]\dagger}\widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \boldsymbol{n}_3^{[1]} \end{bmatrix} \tag{27}$$

Owing to the symmetry, we focus only on the desired symbol $x_1^{[1]}$. Since $\boldsymbol{H}_{\text{eff}}^{[1]\dagger}\boldsymbol{H}_{\text{eff}}^{[1]}$ is the scaled identity matrix from Alamouti Property 2, we can extract an equation in only $x_1^{[1]}$ from (27) as

$$\tilde{r}_1^{[1]} = \boldsymbol{h}_1^{[11]\dagger}\widehat{\boldsymbol{G}}^{[1]}\widehat{\boldsymbol{G}}^{[1]\dagger}\boldsymbol{h}_1^{[11]}x_1^{[1]} + \boldsymbol{h}_1^{[11]\dagger}\widehat{\boldsymbol{G}}^{[1]}\widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \boldsymbol{n}_3^{[1]} \end{bmatrix} \tag{28}$$

where $\tilde{r}_1^{[1]}$ denotes the first element of

$$\boldsymbol{H}_{\text{eff}}^{[1]\dagger}\widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{y}_1^{[1]\dagger} & \boldsymbol{y}_2^{[1]\dagger} & \boldsymbol{y}_3^{[1]\dagger} \end{bmatrix}^{\dagger}$$

in (27).

Let $\mathbb{E}[|x_1^{[1]}|^2] = P_1^{[1]}$ and the covariance matrix of $\begin{bmatrix} \boldsymbol{n}_1^{[1]\dagger} & \boldsymbol{n}_2^{[1]\dagger} & \boldsymbol{n}_3^{[1]\dagger} \end{bmatrix}^{\dagger}$ is the identity matrix. Then, the normalised instantaneous receive signal-to-noise ratio (SNR) is expressed as

$$\begin{aligned} \text{SNR}^{[1]} &= P_1^{[1]}\frac{(\boldsymbol{h}_1^{[11]\dagger}\widehat{\boldsymbol{G}}^{[1]}\widehat{\boldsymbol{G}}^{[1]\dagger}\boldsymbol{h}_1^{[11]})^2}{\boldsymbol{h}_1^{[11]\dagger}\widehat{\boldsymbol{G}}^{[1]}\widehat{\boldsymbol{G}}^{[1]\dagger}\widehat{\boldsymbol{G}}^{[1]}\widehat{\boldsymbol{G}}^{[1]\dagger}\boldsymbol{h}_1^{[11]}} \\ &\overset{(b)}{=} P_1^{[1]}\left(\frac{\|\widehat{\boldsymbol{G}}^{[1]}\|^2}{2}\right)^{-1}\boldsymbol{h}_1^{[11]\dagger}\widehat{\boldsymbol{G}}^{[1]}\widehat{\boldsymbol{G}}^{[1]\dagger}\boldsymbol{h}_1^{[11]} \end{aligned} \tag{29}$$

where (b) uses

$$\widehat{\boldsymbol{G}}^{[1]\dagger}\widehat{\boldsymbol{G}}^{[1]} = \left(\frac{\|\widehat{\boldsymbol{G}}^{[1]}\|^2}{2}\right)\boldsymbol{I}_2$$

Let

$$\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}} = \sqrt{\left(\frac{\|\widehat{\boldsymbol{G}}^{[1]}\|^2}{2}\right)^{-1}}\widehat{\boldsymbol{G}}^{[1]\dagger}\boldsymbol{h}_1^{[11]}$$

Then, (29) is rewritten as

$$\text{SNR}^{[1]} = P_1^{[1]}\begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix}^{\dagger}\begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} \tag{30}$$

In order to easily analyse the diversity order of the proposed IAC scheme using PEP, we fix $\boldsymbol{H}^{[kt]}$ for distinct k, $t \in \{1, 2, 3\}$, and allow $\boldsymbol{H}^{[kk]}$ to change for all $k \in \{1, 2, 3\}$. Then, $\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ is a jointly circular complex Gaussian random vector. Let

$$\begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} = \underbrace{\begin{bmatrix} u_{1\text{R}}^{[1]} \\ u_{2\text{R}}^{[1]} \end{bmatrix}}_{\boldsymbol{u}_{\text{R}}} + \text{j}\underbrace{\begin{bmatrix} u_{1\text{I}}^{[1]} \\ u_{2\text{I}}^{[1]} \end{bmatrix}}_{\boldsymbol{u}_{\text{I}}} \tag{31}$$

where $\boldsymbol{u}_{\text{R}}$ and $\boldsymbol{u}_{\text{I}}$ are the real part and imaginary part of $\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ with covariance matrices $\boldsymbol{K}_{\text{RR}}$ and $\boldsymbol{K}_{\text{II}}$, respectively, and cross-covariance matrix $\boldsymbol{K}_{\text{RI}}$, that is

$$\mathbb{E}[\boldsymbol{u}_{\text{R}}\boldsymbol{u}_{\text{R}}^{\mathrm{T}}] = \boldsymbol{K}_{\text{RR}}, \quad \mathbb{E}[\boldsymbol{u}_{\text{I}}\boldsymbol{u}_{\text{I}}^{\mathrm{T}}] = \boldsymbol{K}_{\text{II}}, \quad \mathbb{E}[\boldsymbol{u}_{\text{R}}\boldsymbol{u}_{\text{I}}^{\mathrm{T}}] = \boldsymbol{K}_{\text{RI}} \tag{32}$$

Since $\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ is a jointly circular complex Gaussian random vector, by using (2b) and (17) in [17], we have the following relations

$$\boldsymbol{K}_{\text{RR}} = \boldsymbol{K}_{\text{II}}, \quad \boldsymbol{K}_{\text{RI}}^{\mathrm{T}} = -\boldsymbol{K}_{\text{RI}} \quad (\boldsymbol{K}_{\text{RI}})_{jj} = 0, \quad j = 1, 2 \tag{33}$$

The average PEP conditioned on interfering channels at receiver 1 can be written as

$$
\begin{aligned}
P(x_1^{[1]} \to \hat{x}_1^{[1]} | \boldsymbol{H}^{[ij]}) &= \mathbb{E}_{\boldsymbol{H}^{[11]}}\left[Q\left(\sqrt{\frac{d_1^{[1]}\mathrm{SNR}^{[1]}}{2}}\right)\right] \\
&\leq \mathbb{E}_{\boldsymbol{H}^{[11]}}\left[\frac{1}{2}\exp\left(-\frac{d_1^{[1]}\mathrm{SNR}^{[1]}}{4}\right)\right] \\
&= \mathbb{E}_{\boldsymbol{H}^{[11]}}\left[\frac{1}{2}\exp\left(-\frac{d_1^{[1]}P_1^{[1]}}{4}(|u_1^{[1]}|^2 + |u_2^{[1]}|^2)\right)\right]
\end{aligned} \tag{34}
$$

where $d_1^{[1]} = |x_1^{[1]} - \hat{x}_1^{[1]}|^2$. The characteristic function (CF) of $\bar{\gamma} = \left[\, |u_1^{[1]}|^2 \quad |u_2^{[1]}|^2 \,\right]^{\mathrm{T}}$ will be used to analyse the PEP in (34), the joint CF of which is derived by using (75d) in [17] as

$$
\begin{aligned}
&\Psi_{\bar{\gamma}}(\mathrm{j}\boldsymbol{w}) \\
&= \mathbb{E}[\exp(\mathrm{j}\bar{\gamma}^{\mathrm{T}}\boldsymbol{w})] \\
&= \{\det(\boldsymbol{I}_2 - 2\mathrm{j}\,\mathrm{diag}(\boldsymbol{w})\boldsymbol{K}_{\mathrm{II}})\}^{-1/2} \\
&\quad \times \{\det(\boldsymbol{I}_2 - 2\mathrm{j}\,\mathrm{diag}(\boldsymbol{w})\boldsymbol{K}_{\mathrm{RR}} \\
&\quad + 4\,\mathrm{diag}(\boldsymbol{w})\boldsymbol{K}_{\mathrm{RI}}[\boldsymbol{I}_2 - 2\mathrm{j}\,\mathrm{diag}(\boldsymbol{w})\boldsymbol{K}_{\mathrm{II}}]^{-1}\mathrm{diag}(\boldsymbol{w})\boldsymbol{K}_{\mathrm{RI}}^{\mathrm{T}})\}^{-1/2}
\end{aligned} \tag{35}
$$

where $\boldsymbol{w} = [w_1 w_2]^{\mathrm{T}}$. Then the last line of (34) can be rewritten using the CF in (35) as

$$
\begin{aligned}
&\mathbb{E}_{\boldsymbol{H}^{[11]}}\left[\frac{1}{2}\exp\left(-\frac{d_1^{[1]}P_1^{[1]}}{4}(|u_1^{[1]}|^2 + |u_2^{[1]}|^2)\right)\right] \\
&= \mathbb{E}_{\boldsymbol{H}^{[11]}}\left[\frac{1}{2}\Psi_{\bar{\gamma}}(j\boldsymbol{w})|_{\boldsymbol{w}=((\mathrm{j}d_1^{[1]}P_1^{[1]})/4)[1 \quad 1]^{\mathrm{T}}}\right] \\
&\overset{(c)}{\simeq} \frac{1}{2}\det\left(\frac{d_1^{[1]}P_1^{[1]}}{2}\boldsymbol{K}_{\mathrm{II}}\right)^{-1/2} \\
&\quad \det\left(\frac{d_1^{[1]}P_1^{[1]}}{2}\boldsymbol{K}_{\mathrm{RR}} - \frac{d_1^{[1]}P_1^{[1]}}{2}\boldsymbol{K}_{\mathrm{RI}}\boldsymbol{K}_{\mathrm{II}}^{-1}\boldsymbol{K}_{\mathrm{RI}}^{\mathrm{T}}\right)^{-1/2} \\
&= \frac{2}{(d_1^{[1]}P_1^{[1]})^2}\det(\boldsymbol{K}_{\mathrm{II}})^{-1}\det(\boldsymbol{I}_2 - \boldsymbol{K}_{\mathrm{II}}^{-1}\boldsymbol{K}_{\mathrm{RI}}\boldsymbol{K}_{\mathrm{II}}^{-1}\boldsymbol{K}_{\mathrm{RI}}^{\mathrm{T}})^{-1/2}
\end{aligned} \tag{36}
$$

where (c) uses high SNR approximation. Since $\boldsymbol{K}_{\mathrm{RR}}$, $\boldsymbol{K}_{\mathrm{RI}}$ and $\boldsymbol{K}_{\mathrm{II}}$ are full rank matrices with probability 1 and $\boldsymbol{H}^{[kt]}$'s are drawn from a continuous distribution, we have

$$
\Pr\left(\det(\boldsymbol{K}_{\mathrm{II}})^{-1}\det(\boldsymbol{I}_2 - \boldsymbol{K}_{\mathrm{II}}^{-1}\boldsymbol{K}_{\mathrm{RI}}\boldsymbol{K}_{\mathrm{II}}^{-1}\boldsymbol{K}_{\mathrm{RI}}^{\mathrm{T}})^{-1/2} = 0\right) = 0 \tag{37}
$$

Since (37) is satisfied, that is

$$
\det(\boldsymbol{K}_{\mathrm{II}})^{-1}\det(\boldsymbol{I}_2 - \boldsymbol{K}_{\mathrm{II}}^{-1}\boldsymbol{K}_{\mathrm{RI}}\boldsymbol{K}_{\mathrm{II}}^{-1}\boldsymbol{K}_{\mathrm{II}}^{\mathrm{T}})^{-1/2}
$$

can be regarded as non-zero and $\boldsymbol{K}_{\mathrm{II}}$, $\boldsymbol{K}_{\mathrm{RI}}$ and $\boldsymbol{I}_2$ are independent of $P_1^{[1]}$, PEP decays proportional to $1/(P_1^{[1]})^2$, that is, receiver 1 achieves diversity order of two for $x_1^{[1]}$. It can be similarly shown that diversity order of two is achieved for the other desired symbol $x_2^{[1]}$. Note that diversity analysis for receiver 1 in the proposed IAC scheme can be directly applied to the IC scheme in Section 2 and thus the IC scheme in Section 2 achieves diversity order of two as well.

(2) For receivers 2 and 3 when $M = 1$: Now let us analyse the diversity order of receivers 2 and 3 with two antennas. Without loss of generality, we only consider receiver 2. Owing to the symmetry, receiver 3 operates similarly and has the same performance as receiver 2. The received signal matrix at receiver 2 is given as

$$
\begin{aligned}
\boldsymbol{Y}^{[2]} &= \boldsymbol{H}^{[21]}\boldsymbol{P}^{[1]}\boldsymbol{A}_1 + \boldsymbol{H}^{[22]}\boldsymbol{P}^{[2]}\boldsymbol{A}_2 + \boldsymbol{H}^{[23]}\boldsymbol{P}^{[3]}\boldsymbol{A}_3 + \boldsymbol{N}^{[2]} \\
&\overset{(d)}{=} \boldsymbol{H}^{[22]}\boldsymbol{P}^{[2]}\boldsymbol{A}_2 + \boldsymbol{H}^{[21]}\boldsymbol{P}^{[1]}(\boldsymbol{A}_1 + \boldsymbol{A}_3) + \boldsymbol{N}^{[2]}
\end{aligned} \tag{38}
$$

where $\boldsymbol{A}_t = \boldsymbol{A}(x_1^{[t]}, x_2^{[t]})$ and the equality (d) follows from the alignment condition in (15). Note that $\boldsymbol{A}_1 + \boldsymbol{A}_3$ is also an Alamouti matrix because of Alamouti Property 1. The received signal matrix in (38) can be viewed as a received signal through a multi-access channel (MAC), where two transmitters transmit Alamouti codes to the same receiver. As before, we convert (38) to the following vectorised form

$$
\begin{bmatrix} \boldsymbol{y}_1^{[2]} \\ \boldsymbol{y}_2^{[2]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{1,1}^{[22]} & \boldsymbol{H}_{1,1}^{[21]} \\ \boldsymbol{H}_{2,1}^{[22]} & \boldsymbol{H}_{2,1}^{[21]} \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_1^{[2]} \\ \boldsymbol{i}_1^{[2]} \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_1^{[2]} \\ \boldsymbol{n}_2^{[2]} \end{bmatrix} \tag{39}
$$

where

$$
\boldsymbol{y}_i^{[2]} = \left[\, y_{i,1}^{[2]} \quad y_{i,2}^{[2]*} \,\right]^{\mathrm{T}}, \quad \boldsymbol{H}_{i,1}^{[2t]} = \begin{bmatrix} h_{i,1}^{[2t]} & h_{i,2}^{[2t]} \\ h_{i,2}^{[2t]*} & -h_{i,1}^{[2t]*} \end{bmatrix}
$$

$h_{i,j}^{[2t]}$'s are entries of $\boldsymbol{H}^{[2t]}\boldsymbol{P}^{[t]}$, and $\boldsymbol{n}_i^{[2]} = \left[\, n_{i,1}^{[2]} \quad n_{i,2}^{[2]*} \,\right]^{\mathrm{T}}$. The vectors $\boldsymbol{s}_1^{[2]}$ and $\boldsymbol{i}_1^{[2]}$ denote the desired symbol vector $\left[\, x_1^{[2]} \quad x_2^{[2]} \,\right]^{\mathrm{T}}$ and interfering symbol vector $\left[\, x_1^{[1]} + x_1^{[3]} \quad x_2^{[1]} + x_2^{[3]} \,\right]^{\mathrm{T}}$ for receiver 2, respectively. Then we can remove the interference by multiplying the conjugate transpose of the 4×2 matrix

$$
\widehat{\boldsymbol{G}}^{[2]} = \boldsymbol{G}_1^{[2]}(1) = \left[\, \boldsymbol{I}_2 \quad \boldsymbol{F}_1^{[2]\dagger}(1) \,\right]^{\dagger} = \left[\, \boldsymbol{I}_2 \quad -\frac{2\boldsymbol{H}_{1,1}^{[21]}\boldsymbol{H}_{2,1}^{[21]\dagger}}{\|\boldsymbol{H}_{2,1}^{[21]}\|^2} \,\right]^{\dagger}
$$

as

$$
\widehat{\boldsymbol{G}}^{[2]\dagger}\begin{bmatrix} \boldsymbol{y}_1^{[2]} \\ \boldsymbol{y}_2^{[2]} \end{bmatrix} \overset{(e)}{=} \underbrace{\widehat{\boldsymbol{G}}^{[2]\dagger}\begin{bmatrix} \boldsymbol{H}_{1,1}^{[22]} \\ \boldsymbol{H}_{2,1}^{[22]} \end{bmatrix}}_{\boldsymbol{H}_{\mathrm{eff}}^{[2]}} \boldsymbol{s}_1^{[2]} + \widehat{\boldsymbol{G}}^{[2]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[2]} \\ \boldsymbol{n}_2^{[2]} \end{bmatrix} \tag{40}
$$

where the equality (e) uses the fact that $\widehat{\boldsymbol{G}}^{[2]\dagger}\left[\, \boldsymbol{H}_{1,1}^{[21]\dagger} \quad \boldsymbol{H}_{2,1}^{[21]\dagger} \,\right]^{\dagger}$ is a 2×2 zero matrix because of Alamouti Property 2. The equivalent channel after cancelling the interference, $\boldsymbol{H}_{\mathrm{eff}}^{[2]}$ in (40) is also an Alamouti matrix because of Alamouti Property 1. Therefore the desired symbols $x_1^{[2]}$ and $x_2^{[2]}$ can be derived by symbol-by-symbol decoding.

Define column vectors $\boldsymbol{h}_i^{[22]}$, $i \in \{1, 2\}$, as

$$
\left[\, \boldsymbol{h}_1^{[22]} \quad \boldsymbol{h}_2^{[22]} \,\right] = \left[\, \boldsymbol{H}_{1,1}^{[22]\dagger} \quad \boldsymbol{H}_{2,1}^{[22]\dagger} \,\right]^{\dagger}
$$

in (40). As before, receiver 2 can estimate its desired symbols $x_1^{[2]}$ and $x_2^{[2]}$ by symbol-by-symbol decoding as

$$
\boldsymbol{H}_{\mathrm{eff}}^{[2]\dagger}\widehat{\boldsymbol{G}}^{[2]\dagger}\begin{bmatrix} \boldsymbol{Y}_1^{[2]} \\ \boldsymbol{Y}_2^{[2]} \end{bmatrix} = \boldsymbol{H}_{\mathrm{eff}}^{[2]\dagger}\boldsymbol{H}_{\mathrm{eff}}^{[2]}\boldsymbol{s}_1^{[2]} + \boldsymbol{H}_{\mathrm{eff}}^{[2]\dagger}\widehat{\boldsymbol{G}}^{[2]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[2]} \\ \boldsymbol{n}_2^{[2]} \end{bmatrix} \tag{41}
$$

Now, we focus only on the desired symbol $x_1^{[2]}$. Since $\boldsymbol{H}_{\mathrm{eff}}^{[2]\dagger}\boldsymbol{H}_{\mathrm{eff}}^{[2]}$ is a scaled identity matrix from Alamouti Property 2, we can extract the equation of $x_1^{[2]}$ from (41) as

$$
\tilde{r}_1^{[2]} = \boldsymbol{h}_1^{[22]\dagger}\widehat{\boldsymbol{G}}^{[2]}\widehat{\boldsymbol{G}}^{[2]\dagger}\boldsymbol{h}_1^{[22]}x_1^{[2]} + \boldsymbol{h}_1^{[22]\dagger}\widehat{\boldsymbol{G}}^{[2]}\widehat{\boldsymbol{G}}^{[2]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[2]} \\ \boldsymbol{n}_2^{[2]} \end{bmatrix} \tag{42}
$$

Let $\mathbb{E}[|x_1^{[2]}|^2] = P_1^{[2]}$ and assume that the covariance matrix of $\begin{bmatrix} \boldsymbol{n}_1^{[2]\dagger} & \boldsymbol{n}_2^{[2]\dagger} \end{bmatrix}^\dagger$ is the identity matrix. Then, the normalised instantaneous receive SNR is expressed as

$$\begin{aligned} \mathrm{SNR}^{[2]} &= P_1^{[2]} \frac{(\boldsymbol{h}_1^{[22]\dagger}\widehat{\boldsymbol{G}}^{[2]}\widehat{\boldsymbol{G}}^{[2]\dagger}\boldsymbol{h}_1^{[22]})^2}{\boldsymbol{h}_1^{[22]\dagger}\widehat{\boldsymbol{G}}^{[2]}\widehat{\boldsymbol{G}}^{[2]\dagger}\widehat{\boldsymbol{G}}^{[2]}\widehat{\boldsymbol{G}}^{[2]\dagger}\boldsymbol{h}_1^{[22]}} \\ &\overset{(f)}{=} P_1^{[2]}\left(\frac{\|\widehat{\boldsymbol{G}}^{[2]}\|^2}{2}\right)^{-1}\boldsymbol{h}_1^{[22]\dagger}\widehat{\boldsymbol{G}}^{[2]}\widehat{\boldsymbol{G}}^{[2]\dagger}\boldsymbol{h}_1^{[22]} \end{aligned} \tag{43}$$

where the equality (f) uses

$$\widehat{\boldsymbol{G}}^{[2]\dagger}\widehat{\boldsymbol{G}}^{[2]} = \frac{\|\widehat{\boldsymbol{G}}^{[2]}\|^2}{2}\boldsymbol{I}_2$$

Let

$$\begin{bmatrix} u_1^{[2]} & u_2^{[2]} \end{bmatrix}^{\mathrm{T}} = \sqrt{\left(\frac{\|\widehat{\boldsymbol{G}}^{[2]}\|^2}{2}\right)^{-1}}\widehat{\boldsymbol{G}}^{[2]\dagger}\boldsymbol{h}_1^{[22]}$$

Then, (43) can be rewritten as

$$\mathrm{SNR}^{[2]} = P_1^{[2]}\begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix}^\dagger \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix} \tag{44}$$

Since $\begin{bmatrix} u_1^{[2]} & u_2^{[2]} \end{bmatrix}^{\mathrm{T}}$ is a jointly circular complex Gaussian random vector, PEP can be analysed similar to (31)–(37). It is easy to show that receiver 2 achieves diversity order of two for $x_1^{[2]}$ and diversity order of the other desired symbol $x_2^{[2]}$ can also be derived in the same way, which is also two.

(3) For $M \geq 2$: Let us consider the general case, that is, an interference channel with three transmitters and three receivers, where each of them is equipped with $2M$ antennas except that receiver 1 has $3M$ antennas. Similar to the previous case, the interference is cancelled at each receiver by using $\widehat{\boldsymbol{G}}^{[k]}$ as

$$\widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{y}_1^{[1]} \\ \boldsymbol{y}_2^{[1]} \\ \vdots \\ \boldsymbol{y}_{3M}^{[1]} \end{bmatrix} = \widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{H}_{1,1}^{[11]} \\ \boldsymbol{H}_{2,1}^{[11]} \\ \vdots \\ \boldsymbol{H}_{3M,1}^{[11]} \end{bmatrix}\boldsymbol{s}_1^{[1]} + \widehat{\boldsymbol{G}}^{[1]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[1]} \\ \boldsymbol{n}_2^{[1]} \\ \vdots \\ \boldsymbol{n}_{3M}^{[1]} \end{bmatrix} \tag{45}$$

$$\widehat{\boldsymbol{G}}^{[k]\dagger}\begin{bmatrix} \boldsymbol{y}_1^{[k]} \\ \boldsymbol{y}_2^{[k]} \\ \vdots \\ \boldsymbol{y}_{2M}^{[k]} \end{bmatrix} = \widehat{\boldsymbol{G}}^{[k]\dagger}\begin{bmatrix} \boldsymbol{H}_{1,1}^{[kk]} \\ \boldsymbol{H}_{2,1}^{[kk]} \\ \vdots \\ \boldsymbol{H}_{2M,1}^{[kk]} \end{bmatrix}\boldsymbol{s}_1^{[k]} + \widehat{\boldsymbol{G}}^{[k]\dagger}\begin{bmatrix} \boldsymbol{n}_1^{[k]} \\ \boldsymbol{n}_2^{[k]} \\ \vdots \\ \boldsymbol{n}_{2M}^{[k]} \end{bmatrix}, \quad k = 2, 3 \tag{46}$$

Note that the matrices $\widehat{\boldsymbol{G}}^{[k]}$ in (45) and (46) are obtained through tedious derivation similar to the previous case. Let

$$\widehat{\boldsymbol{G}}^{[k]} = \begin{bmatrix} \boldsymbol{A}_1^{[k]\dagger} & \boldsymbol{A}_2^{[k]\dagger} & \cdots \end{bmatrix}^\dagger$$

where $\boldsymbol{A}_j^{[k]}$'s are 2×2 matrices. Define $\mathcal{Z}^{(n)}(\boldsymbol{A}_j^{[k]})$ as the operation which replaces all $\boldsymbol{F}_i^{[k]}(l)$ in $\boldsymbol{A}_j^{[k]}$ by $\boldsymbol{F}_{i+n}^{[k]}(l-n)$, where $\mathcal{Z}^{(n)}(\boldsymbol{I}_2) = \boldsymbol{I}_2$. For example, $\widehat{\boldsymbol{G}}^{[1]}$ in (26) is rewritten as

$$\widehat{\boldsymbol{G}}^{[1]} = \begin{bmatrix} \boldsymbol{A}_1^{[1]} \\ \boldsymbol{A}_2^{[1]} \\ \boldsymbol{A}_3^{[1]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_2 \\ \boldsymbol{F}_1^{[1]}(2) \\ \boldsymbol{F}_1^{[1]}(1) + \boldsymbol{F}_2^{[1]}(1)\boldsymbol{F}_1^{[1]}(2) \end{bmatrix}$$

and $\mathcal{Z}^{(1)}(\boldsymbol{A}_3^{[1]}) = \boldsymbol{F}_2^{[1]}(0) + \boldsymbol{F}_3^{[1]}(0)\boldsymbol{F}_2^{[1]}(1)$. Then we have $\widehat{\boldsymbol{G}}^{[1]}$, $\widehat{\boldsymbol{G}}^{[2]}$ and $\widehat{\boldsymbol{G}}^{[3]}$ in (45) and (46) as (see (47) and (48)). Since $\boldsymbol{F}_i^{[k]}(l)$'s in (47) and (48) are all Alamouti matrices, all $\boldsymbol{A}_j^{[k]}$'s are also Alamouti matrices because of Alamouti Property 1 and the equivalent channels in (45) and (46) are Alamouti matrices as well. Therefore it can be easily verified that the proposed IAC scheme for the general case with $2M$ antennas achieves diversity order of two in a similar way to (27)–(37). Until now, it is shown that the proposed IAC scheme for the three-user interference channel with one $3M$-antenna receiver and two $2M$-antenna receivers achieves both $3M$ DoF and diversity order of two. □

$$\widehat{\boldsymbol{G}}^{[1]} = \begin{bmatrix} \boldsymbol{A}_1^{[1]} \\ \boldsymbol{A}_2^{[1]} \\ \boldsymbol{A}_3^{[1]} \\ \boldsymbol{A}_4^{[1]} \\ \vdots \\ \boldsymbol{A}_j^{[1]} \\ \vdots \\ \boldsymbol{A}_{3M}^{[1]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_2 \\ \boldsymbol{F}_1^{[1]}(3M-1) \\ \boldsymbol{F}_1^{[1]}(3M-2) + \mathcal{Z}^{(1)}(\boldsymbol{A}_2^{[1]})\boldsymbol{F}_1^{[1]}(3M-1) \\ \boldsymbol{F}_1^{[1]}(3M-3) + \mathcal{Z}^{(2)}(\boldsymbol{A}_2^{[1]})\boldsymbol{F}_1^{[1]}(3M-2) + \mathcal{Z}^{(1)}(\boldsymbol{A}_3^{[1]})\boldsymbol{F}_1^{[1]}(3M-1) \\ \vdots \\ \sum_{l=1}^{j-1}\mathcal{Z}^{(j-l)}(\boldsymbol{A}_l^{[1]})\boldsymbol{F}_1^{[1]}(3M-j+l) \\ \vdots \\ \sum_{l=1}^{3M-1}\mathcal{Z}^{(3M-l)}(\boldsymbol{A}_l^{[1]})\boldsymbol{F}_1^{[1]}(l) \end{bmatrix} \tag{47}$$

$$\widehat{\boldsymbol{G}}^{[k]} = \begin{bmatrix} \boldsymbol{A}_1^{[k]} \\ \boldsymbol{A}_2^{[k]} \\ \boldsymbol{A}_3^{[k]} \\ \boldsymbol{A}_4^{[k]} \\ \vdots \\ \boldsymbol{A}_j^{[k]} \\ \vdots \\ \boldsymbol{A}_{2M}^{[k]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_2 \\ \boldsymbol{F}_1^{[k]}(2M-1) \\ \boldsymbol{F}_1^{[k]}(2M-2) + \mathcal{Z}^{(1)}(\boldsymbol{A}_2^{[k]})\boldsymbol{F}_1^{[k]}(2M-1) \\ \boldsymbol{F}_1^{[k]}(2M-3) + \mathcal{Z}^{(2)}(\boldsymbol{A}_2^{[k]})\boldsymbol{F}_1^{[k]}(2M-2) + \mathcal{Z}^{(1)}(\boldsymbol{A}_3^{[k]})\boldsymbol{F}_1^{[k]}(2M-1) \\ \vdots \\ \sum_{l=1}^{j-1}\mathcal{Z}^{(j-l)}(\boldsymbol{A}_l^{[k]})\boldsymbol{F}_1^{[k]}(2M-j+l) \\ \vdots \\ \sum_{l=1}^{2M-1}\mathcal{Z}^{(2M-l)}(\boldsymbol{A}_l^{[k]})\boldsymbol{F}_1^{[k]}(l) \end{bmatrix}, \quad k = 2, 3 \tag{48}$$

3.3 Extension to K-user MIMO interference channel

To apply the proposed IAC scheme to the K-user interference channel, consider the K-user interference channel where each transmitter has $2M$ antennas. Note that KM antennas are required for all receivers to apply the IC scheme achieving KM DoF per channel use. To reduce the number of receive antennas by using IAC scheme, the beamforming matrices should satisfy the alignment conditions, for example

$$\boldsymbol{H}^{[kt]}\boldsymbol{P}^{[t]} = \boldsymbol{H}^{[kt']}\boldsymbol{P}^{[t']}, \quad t, t' \neq k, \quad 1 \le t, t' \le K, \quad 2 \le k \le K \tag{49}$$

It is clear that the linear equations (49) have no solutions if the channel matrices are not $2M \times 2M$ matrices. In other words, the number of antennas at the receivers 2, …, K should be reduced to $2M$. For this, the beamforming matrices are designed to satisfy the following conditions

$$\begin{aligned} &\boldsymbol{H}^{[21]}\boldsymbol{P}^{[1]} = \boldsymbol{H}^{[23]}\boldsymbol{P}^{[3]} = \cdots = \boldsymbol{H}^{[2\,K-1]}\boldsymbol{P}^{[K-1]} = \boldsymbol{H}^{[2\,K]}\boldsymbol{P}^{[K]} \\ &\vdots \\ &\boldsymbol{H}^{[K-1\,1]}\boldsymbol{P}^{[1]} = \boldsymbol{H}^{[K-1\,2]}\boldsymbol{P}^{[2]} \\ &\quad = \cdots = \boldsymbol{H}^{[K-1\,K-2]}\boldsymbol{P}^{[K-2]} = \boldsymbol{H}^{[K-1\,K]}\boldsymbol{P}^{[K]} \\ &\boldsymbol{H}^{[K\,1]}\boldsymbol{P}^{[1]} = \boldsymbol{H}^{[K\,2]}\boldsymbol{P}^{[2]} \\ &\quad = \cdots = \boldsymbol{H}^{[K\,K-2]}\boldsymbol{P}^{[K-2]} = \boldsymbol{H}^{[K\,K-1]}\boldsymbol{P}^{[K-1]} \end{aligned} \tag{50}$$

However, the above conditions cannot be satisfied simultaneously and we have no choice but to determine the beamforming matrices satisfying the alignment conditions for only one receiver. Therefore our IAC scheme for the three-user interference channel may be directly generalised to the K-user interference channel by reducing the number of antennas at only one receiver from KM to $2M$. In fact, there are other methods which can reduce the number of antennas at each receiver differently for the above K-user interference channel or achieve more diversity gain. For example, IA over lattice (IA-L) with beamformer selection was proposed in [18]. The simulation results in [18] showed that the IA-L achieves the approximately the diversity order of 2.4 at receivers 2 and 3, and the diversity order of 1.4 at receiver 1, while our IC and IAC schemes achieve the diversity order of two at each receiver. Since the beamformer selection for precoder design provides diversity gain, it will be an interesting future work to apply beamformer selection to our schemes or general IA schemes and analytically derive the diversity order.

4 Simulation results

In this section, average symbol error rate (SER) performance of the IC and IAC schemes are compared with that of the IA scheme for the three-user MIMO interference channel. It is assumed that the channel is Rayleigh block fading, that is, the channel state does not change during transmission of each code but varies independently from block to block. All channel coefficients and noise at the receivers are assumed to be complex Gaussian random variables $\mathcal{CN}(0, 1)$. Quadrature phase-shift keying is used and the average transmit power per symbol at each transmitter is set to P. Fig. 3 compares the SER performance of IA, IC and IAC schemes for the three-user interference channel when $M = 1, 2$. Note that when $M = 1$ (or 2), that is, each transmitter has two (or four) antennas for IA, IC and IAC schemes, they achieve three (or six) DoF per channel use, respectively. Fig. 3 shows that the SERs of IC and IAC schemes are almost identical regardless of the number of transmit antennas ($2M = 2$, 4). The IA scheme achieves diversity order of one for all cases, which can be verified by the slope of the SER curve in high SNR region. When $M = 1$, although the IC scheme requires three antennas at each receiver, Fig. 3 shows that it achieves diversity order of two. The IAC scheme also achieves diversity order of two when one receiver has three antennas and the others have two antennas. When $M = 2$, the IC scheme using six antennas at each receiver achieves diversity order of two. The

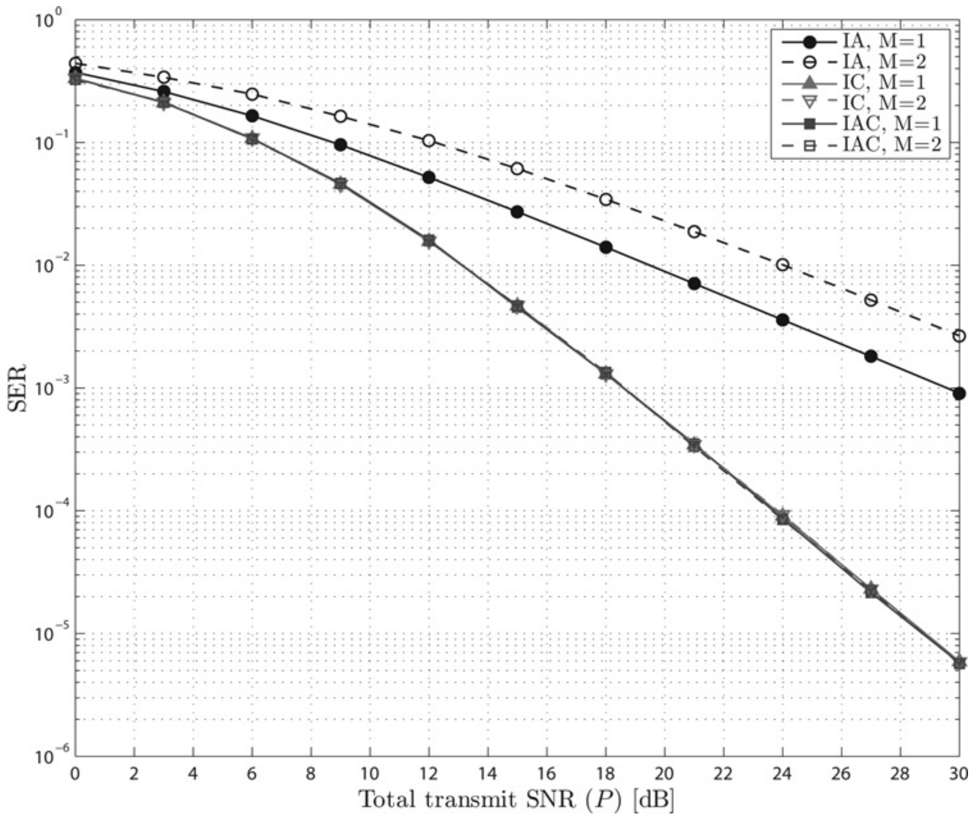

Fig. 3 *SER performance comparison of IA, IC and IAC schemes for the three-user MIMO interference channel when M = 1, 2*

IAC scheme achieves diversity order of two by using six antennas at one receiver and four antennas at each of the other two receivers. It is clear that the diversity orders shown in Fig. 3 match well with the analytical results in the previous sections.

5 Conclusions

In this paper, we propose a method on how to apply Alamouti code to MIMO interference channels. The IC method based on Alamouti code for the multi-access scenario can be used for the *K*-user interference channel, which enables the receivers to perform symbol-by-symbol decoding by cancelling interfering signals by utilising Alamouti structure and achieve diversity order of two. Moreover, it does not require CSIT unlike the IA scheme.

However, the IC scheme requires more receive antennas than the IA scheme to achieve the same DoF. In order to reduce the number of receive antennas, especially for the three-user interference channel, we propose an IAC scheme based on Alamouti code which utilises beamforming matrices with partial CSIT to align interfering signals. We also show that while the proposed IAC scheme requires less antennas at the receivers than the IC scheme, it achieves the same performance in terms of DoF and diversity order. In fact, there may be other methods which can achieve more diversity order or reduce the number of receive antennas differently for the *K*-user interference channel. We leave these as a future work.

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7 References

1 ITU-R WP 5D: 'Requirements related to technical system performance for IMT-Advanced radio interface(s) [IMT.TECH]', R07-WP5D-080128-TD-0028, 2008
2 Peng, J., Hong, J., Xue, K.: 'Performance analysis of switching strategy in LTE-A heterogeneous networks', *J. Commun. Netw.*, 2013, **15**, (3), pp. 292–300
3 Cadambe, V.R., Jafar, S.A.: 'Interference alignment and degrees of freedom of the *K*-user interference channel', *IEEE Trans. Inf. Theory*, 2008, **54**, (8), pp. 3425–3441
4 Jafar, S.A., Shamai, S.: 'Degrees of freedom region of the MIMO *X* channel', *IEEE Trans. Inf. Theory*, 2008, **54**, (1), pp. 151–170
5 Maddah-Ali, M.A., Motahari, A.S., Khandani, A.K.: 'Communication over MIMO *X* channel: interference alignment, decomposition, and performance analysis', *IEEE Trans. Inf. Theory*, 2008, **54**, (8), pp. 3457–3470
6 Gomadam, K.S., Cadambe, V.R., Jafar, S.A.: 'A distributed numerical approach to interference alignment and applications to wireless interference networks', *IEEE Trans. Inf. Theory*, 2011, **57**, (6), pp. 3309–3322
7 Yetis, C.M., Gou, T., Jafar, S.A., Kayran, A.H.: 'On feasibility of interference alignment in MIMO interference networks', *IEEE Trans. Signal Process.*, 2010, **58**, (9), pp. 4771–4782
8 Jafar, S.A., Fakhereddin, M.: 'Degrees of freedom for the MIMO interference channel', *IEEE Trans. Inf. Theory*, 2007, **53**, (7), pp. 2637–2642
9 Alamouti, S.M.: 'A simple transmit diversity technique for wireless communications', *IEEE J. Sel. Areas Commun.*, 1998, **16**, (8), pp. 1451–1458
10 Naguib, A.F., Seshadri, N., Calderbank, A.R.: 'Applications of space-time block codes and interference suppression for high capacity and high data rate wireless systems'. Proc. 32nd Asilomar Conf. Signals, Systems, Comput., Pacific Grove, CA, USA, November 1998, pp. 1803–1810
11 Al-Dhahir, N., Calderbank, A.R.: 'Further results on interference cancellation and space-time block codes'. Proc. 35th Asilomar Conf. Signals, Systems, Comput., Pacific Grove, CA, USA, November 2001, pp. 257–262
12 Kazemitabar, J., Jafarkhani, H.: 'Multiuser interference cancellation and detection for users with more than two transmit antennas', *IEEE Trans. Commun.*, 2008, **56**, (4), pp. 574–583
13 Jafar, S.A.: 'Blind interference alignment', *IEEE J. Sel. Top. Signal Process.*, 2012, **6**, (3), pp. 216–227
14 Li, L., Jafarkhani, H., Jafar, S.A.: 'When Alamouti codes meet interference alignment: Transmission schemes for two-user *X* channel'. Proc. IEEE Int. Symp. Inf. Theory (ISIT), Saint Petersburg, Russia, August 2011, pp. 2577–2581
15 Hassibi, B., Hochwald, B.M.: 'High rate codes that are linear in space and time', *IEEE Trans. Inf. Theory*, 2002, **48**, (7), pp. 1804–1824
16 Ning, H., Ling, C., Leung, K.K.: 'Feasibility condition for interference alignment with diversity', *IEEE Trans. Inf. Theory*, 2011, **57**, (5), pp. 2902–2912
17 Mallik, R.K.: 'On multivariate Rayleigh and exponential distributions', *IEEE Trans. Inf. Theory*, 2003, **49**, (6), pp. 1499–1515
18 Choi, J.: 'Interference alignment over lattices for MIMO interference channels', *IEEE Commun. Lett.*, 2011, **15**, (4), pp. 3425–3441