PAPR Analysis of Class-III SLM Scheme Based on Variance of Correlation of Alternative OFDM Signal Sequences

Jun-Young Woo, Hyun Seung Joo, Kee-Hoon Kim, Jong-Seon No, and Dong-Joon Shin

***Abstract*—Selected mapping (SLM) is a well-known peak-to-average power ratio (PAPR) reduction technique for orthogonal frequency division multiplexing (OFDM) systems. Recently, a low-complexity SLM scheme, called Class-III SLM scheme, was proposed, which performs only one inverse fast Fourier transform (IFFT) to generate alternative OFDM signal sequences. By randomly selecting the cyclic shift and rotation values, Class-III SLM scheme can generate up to N^3 alternative OFDM signal sequences, where N is the IFFT size. However, all N^3 alternative OFDM signal sequences do not achieve good PAPR reduction performances. Therefore, an efficient selection method of good rotation and cyclic shift values is needed, which results in good PAPR reduction performance. In this letter, a selection method of cyclic shift values is proposed, which is optimal in terms of minimizing the variance of correlation values between alternative OFDM signal sequences. It is also shown that rotation values are useless when $U \leq N/8$, where U is the number of alternative OFDM signal sequences. Also, a selection method of proper rotation values when $U > N/8$ is proposed. Simulation results show that the proposed method achieves the optimal PAPR reduction performance. In addition, the proposed scheme requires less memory and side information than random scheme.**

***Index Terms*—Class-III selected mapping (SLM), correlation, orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is a popular multicarrier modulation technique. Because of the orthogonality of its subcarriers, a receiver can recover the transmitted data without interference. Due to its robustness against multipath fading, OFDM has been adopted as a standard technique for various wireless communication systems such as IEEE 802.11 (WLAN), IEEE 802.16 (WiMAX), and long term evolution (LTE). However, it has a high peak-to-average power ratio (PAPR) problem. When OFDM signals with high PAPR pass through nonlinear high power amplifier, they experience in-band distortion and out-of-band radiation. Thus, in order to reduce PAPR, many schemes have been proposed such as selected mapping (SLM) [1], partial transmit sequence [2], and tone reservation [3].

Because of large computational complexity, the conventional SLM scheme in [1] has been modified to many low-complexity SLM schemes. A low-complexity SLM scheme using a set of conversion matrices was proposed [4]. Finally, three low-complexity SLM schemes using conversion vectors were proposed [5]. Among them, it is known that the one using Class-III conversion vector (which we will refer to as Class-III SLM scheme) shows better PAPR reduction performance than Class-I and Class-II SLM schemes [5, Sec. 4.3]. Thus, in this letter, we focus on Class-III SLM scheme and propose a selection method of optimal cyclic shift values by analyzing the correlation between alternative OFDM signal sequences. The proposed analytic method can also be applied to Class-I and Class-II SLM schemes.

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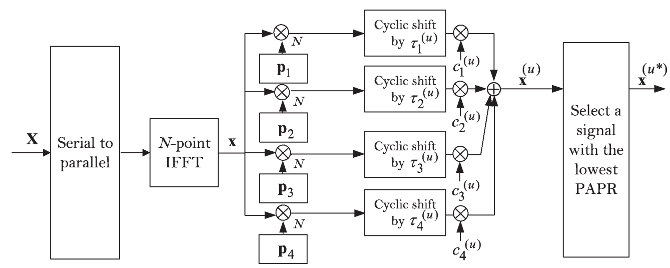

Fig. 1. A block diagram of Class-III SLM scheme.

II. Overview of Class-III SLM Scheme

Fig. 1 shows a block diagram of Class-III SLM scheme. Clearly, it requires only one inverse fast Fourier transform (IFFT) to generate all alternative OFDM signal sequences. The input symbol sequence $\mathbf{X} = [X_0, X_1, X_2, \cdots, X_{N-1}]$ to the IFFT module is usually modulated by M-ary phase-shift keying (MPSK) or M-ary quadrature amplitude modulation (M-QAM), where N is the IFFT size and $N \geq 4$. The OFDM signal sequence $\mathbf{x} = [x_0, x_1, x_2, \cdots, x_{N-1}]$ is obtained by N-point IFFT of $\mathbf{X}$ and then, transformed by N-point circular convolution (denoted by $\otimes_N$) with each of four base vectors $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, and $\mathbf{p}_4$ defined as

$$\begin{aligned}
\mathbf{p}_1 &= \Big[\underbrace{1,0,\cdots,0}_{\frac{N}{4}},\underbrace{1,0,\cdots,0}_{\frac{N}{4}},\underbrace{1,0,\cdots,0}_{\frac{N}{4}},\underbrace{1,0,\cdots,0}_{\frac{N}{4}}\Big] \\
\mathbf{p}_2 &= \Big[\underbrace{1,0,\cdots,0}_{\frac{N}{4}},\underbrace{j,0,\cdots,0}_{\frac{N}{4}},\underbrace{-1,0,\cdots,0}_{\frac{N}{4}},\underbrace{-j,0,\cdots,0}_{\frac{N}{4}}\Big] \\
\mathbf{p}_3 &= \Big[\underbrace{1,0,\cdots,0}_{\frac{N}{4}},\underbrace{-1,0,\cdots,0}_{\frac{N}{4}},\underbrace{1,0,\cdots,0}_{\frac{N}{4}},\underbrace{-1,0,\cdots,0}_{\frac{N}{4}}\Big] \\
\mathbf{p}_4 &= \Big[\underbrace{1,0,\cdots,0}_{\frac{N}{4}},\underbrace{-j,0,\cdots,0}_{\frac{N}{4}},\underbrace{-1,0,\cdots,0}_{\frac{N}{4}},\underbrace{j,0,\cdots,0}_{\frac{N}{4}}\Big].
\end{aligned} \tag{1}$$

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The ith sequence of four generated sequences is cyclically right shifted by $0 \le \tau_i^{(u)} < N/4$ and rotated by multiplying $c_i^{(u)} \in \{\pm 1, \pm j\}$, where $1 \le i \le 4$ and u is the index of alternative OFDM signal sequence. Note that without loss of generality, we can set $\tau_1^{(u)} = 0$ and $c_1^{(u)} = 1$. By summing the resulting four sequences, the uth alternative OFDM signal sequences $\mathbf{x}^{(u)}$ are generated and the one with the lowest PAPR is transmitted.

Let $\mathbf{p}^{(u)}$ be the uth Class-III conversion vector to generate $\mathbf{x}^{(u)}$, which is given as

$$\mathbf{p}^{(u)} = \sum_{i=1}^{4} c_i^{(u)} \mathbf{p}_{i\left\langle \tau_i^{(u)} \right\rangle} \tag{2}$$

where $\mathbf{p}_{i\langle \tau_i^{(u)} \rangle}$ denotes the cyclic-shifted version of $\mathbf{p}_i$ to the right by $\tau_i^{(u)}$. It is clear that $\mathbf{x}^{(u)}$ is generated by $\mathbf{x}^{(u)} = \mathbf{p}^{(u)} \otimes_N \mathbf{x}$.

Class-III SLM scheme can generate up to $(N/4)^3 4^3 = N^3$ alternative OFDM signal sequences by varying $\tau_i^{(u)}$ and $c_i^{(u)}$. Since N^3 is a large number in practice, a deterministic selection method of a good subset of $\tau_i^{(u)}$'s and $c_i^{(u)}$'s is needed, which results in good PAPR reduction performance. In the next section, such selection method will be proposed.

III. Selection of Optimal Alternative OFDM Signal Sequences for Class-III SLM Scheme

A. Correlation Analysis

The magnitude of the correlation $R_{st}(m)$ between the sth and the tth alternative OFDM signal sequences is defined as

$$\begin{aligned} |R_{st}(m)| &= \left| E\left\{ x_n^{(s)} x_{n+m}^{(t)*} \right\} \right| \\ &= \frac{1}{N^2} \left| E\left\{ \sum_{k=0}^{N-1} |X(k)|^2 P^{(s)}(k) P^{(t)*}(k) e^{-j\frac{2\pi}{N}km} \right\} \right| \\ &= \frac{1}{N^2} \left| \sum_{k=0}^{N-1} P^{(s)}(k) P^{(t)*}(k) e^{-j\frac{2\pi}{N}km} \right| \end{aligned} \tag{3}$$

where $x_n^{(s)}$ denotes the nth element of the sth alternative OFDM signal sequence, $P^{(s)}(k)$ denotes the kth element of the sth phase sequence $\mathbf{P}^{(s)} = \mathrm{FFT}\{\mathbf{p}^{(s)}\}$, $(\cdot)^*$ denotes the complex conjugation, and $-(N-1) \le m \le N-1$. $X(k)$'s are assumed to be independent and identically distributed with $E\{|X(k)|^2\} = 1$. Therefore, $E\{X(k)X^*(e)\} = 1$ if $k = e$, or $E\{X(k)X^*(e)\} = 0$. Since $|R_{st}(m)|$ is symmetric about m, analyzing $|R_{st}(m)|$ over $0 \le m \le N-1$ is enough [6].

The uth alternative OFDM signal sequence of Class-III SLM scheme is expressed as

$$\begin{aligned} \mathbf{x}^{(u)} &= \mathbf{p}^{(u)} \otimes_N \mathbf{x} \\ &= \left(\sum_{i=1}^{4} c_i^{(u)} \mathbf{p}_{i\left\langle \tau_i^{(u)} \right\rangle} \right) \otimes_N \mathbf{x} \\ &= \mathrm{IFFT}\left\{ \left(\sum_{i=1}^{4} c_i^{(u)} \mathbf{P}_i^{\left(\tau_i^{(u)}\right)} \right) \odot \mathbf{X} \right\} \end{aligned} \tag{4}$$

where $\mathbf{P}_i^{(\tau_i^{(u)})} \equiv \mathrm{FFT}\{\mathbf{p}_{i\langle \tau_i^{(u)} \rangle}\}$ and $\odot$ denotes the component-wise multiplication of vectors. It is clear that $\sum_{i=1}^{4} c_i^{(u)} \mathbf{P}_i^{\left(\tau_i^{(u)}\right)}$ can be regarded as a phase sequence. Therefore, Class-III SLM scheme is a conventional SLM scheme using the following $\mathbf{P}^{(u)}$ as the uth phase sequence

$$\mathbf{P}^{(u)} = \sum_{i=1}^{4} c_i^{(u)} \mathbf{P}_i^{\left(\tau_i^{(u)}\right)}. \tag{5}$$

More specifically, $\mathbf{P}_i = \mathrm{FFT}\{\mathbf{p}_i\}$ and $c_i^{(u)} \mathbf{P}_i^{\left(\tau_i^{(u)}\right)}$ are

$$\begin{aligned} \mathbf{P}_i &= 4\Bigg[\underbrace{0, \cdots, 0}_{i-1}, \underbrace{1, 0, 0, 0}_{4}, \underbrace{1, 0, 0, 0}_{4}, \cdots, \underbrace{0, \cdots, 0}_{4-i}\Bigg] \\ c_i^{(u)} \mathbf{P}_i^{\left(\tau_i^{(u)}\right)} &= 4\Bigg[\underbrace{0, \cdots, 0}_{i-1}, \cdots, \underbrace{c_i^{(u)} e^{-j\frac{2\pi}{N}\tau_i^{(u)}}, 0, 0, 0}_{4}, \cdots, \underbrace{0, \cdots, 0}_{4-i}\Bigg]. \end{aligned}$$

Now, we obtain the uth phase sequence in (5) as

$$\mathbf{P}^{(u)} = 4\left[P^{(u)}(0), P^{(u)}(1), \cdots, P^{(u)}(N-1)\right] \tag{6}$$

where

$$P^{(u)}(k) = \begin{cases} c_1^{(u)} e^{-j\frac{2\pi}{N}k\tau_1^{(u)}}, & k = 0 \mod 4 \\ c_2^{(u)} e^{-j\frac{2\pi}{N}k\tau_2^{(u)}}, & k = 1 \mod 4 \\ c_3^{(u)} e^{-j\frac{2\pi}{N}k\tau_3^{(u)}}, & k = 2 \mod 4 \\ c_4^{(u)} e^{-j\frac{2\pi}{N}k\tau_4^{(u)}}, & k = 3 \mod 4. \end{cases} \tag{7}$$

By plugging (7) into (3), the inner part $\mathrm{A}(m)$ of the magnitude $|\cdot|$ in (3) is expressed as

$$\begin{aligned} \mathrm{A}(m) &= \sum_{k=0}^{N-1} P^{(s)}(k) P^{(t)*}(k) e^{-j\frac{2\pi}{N}km} \\ &= 16 \sum_{v=0}^{\frac{N}{4}-1} \sum_{i=1}^{4} c_i^{(s)} c_i^{(t)*} e^{-j\frac{2\pi}{N}(4v+i-1)\left(m-\left(\tau_i^{(t)}-\tau_i^{(s)}\right)\right)} \\ &= 16 \sum_{i=1}^{4} \mathrm{A}_i(m) \end{aligned} \tag{8}$$

where $\mathrm{A}_i(m)$ is defined as

$$\begin{aligned} \mathrm{A}_i(m) &= c_i^{(s)} c_i^{(t)*} \sum_{v=0}^{\frac{N}{4}-1} e^{-j\frac{2\pi}{N}(4v+i-1)\left(m-\left(\tau_i^{(t)}-\tau_i^{(s)}\right)\right)} \\ &= c_i^{(s)} c_i^{(t)*} \bar{\mathrm{A}}_i(m). \end{aligned} \tag{9}$$

If $m - (\tau_i^{(t)} - \tau_i^{(s)})$ is a multiple of $N/4$, then $|\bar{\mathrm{A}}_i(m)| = N/4$. Otherwise, it is zero. Let $d_{st}(\tau_i) = \tau_i^{(t)} - \tau_i^{(s)} \bmod N/4$. Then, the values of $\bar{\mathrm{A}}_i(m)$ for all m are listed in Table I.

B. Selection of Optimal Cyclic Shift Values

In [6], it is shown that a set of U phase sequences with low variance of correlation (VC) in SLM scheme gives good PAPR reduction performance. VC is defined as

$$\mathrm{VC} = \left(\sum_{0 \le s < t \le U-1} \mathrm{Var}\left\{ |R_{st}(m)|^2 \right\}_{m=0}^{N-1} \right) \Big/ \binom{U}{2} \tag{10}$$

TABLE I
$\bar{\mathrm{A}}_i(m)$ FOR ALL m

m	$d_{st}(\tau_i)$	$\frac{N}{4}+d_{st}(\tau_i)$	$\frac{2N}{4}+d_{st}(\tau_i)$	$\frac{3N}{4}+d_{st}(\tau_i)$	Otherwise
$\bar{\mathrm{A}}_1(m)$	$\frac{N}{4}$	$\frac{N}{4}$	$\frac{N}{4}$	$\frac{N}{4}$	0
$\bar{\mathrm{A}}_2(m)$	$\frac{N}{4}$	$-\frac{N}{4}j$	$\frac{N}{4}$	$\frac{N}{4}j$	0
$\bar{\mathrm{A}}_3(m)$	$\frac{N}{4}$	$-\frac{N}{4}$	$\frac{N}{4}$	$-\frac{N}{4}$	0
$\bar{\mathrm{A}}_4(m)$	$\frac{N}{4}$	$\frac{N}{4}j$	$\frac{N}{4}$	$-\frac{N}{4}j$	0

TABLE II
SELECTION OF OPTIMAL CYCLIC SHIFT VALUES

u	1	2	3	$\cdots$	k
$\tau_1^{(u)}$	0	0	0	$\cdots$	0
$\tau_2^{(u)}$	1	2	3	$\cdots$	$k \bmod \frac{N}{4}$
$\tau_3^{(u)}$	2	4	6	$\cdots$	$2k \bmod \frac{N}{4}$
$\tau_4^{(u)}$	3	6	9	$\cdots$	$3k \bmod \frac{N}{4}$

where $\mathrm{Var}\{\cdot\}$ denotes the variance. Low VC means that alternative OFDM signal sequences are low correlated. Since the conventional SLM scheme shows good PAPR reduction performance when alternative OFDM signal sequences are low correlated, VC can be a good criterion for PAPR reduction by Class-III SLM scheme. Based on VC, we derive the optimal condition for cyclic shift values of Class-III SLM.

Theorem 1: If cyclic shift values satisfy $d_{st}(\tau_i) \neq d_{st}(\tau_{i'})$ mod $N/4$ for all $\binom{U}{2}$ pairs of alternative OFDM signal sequences, where $1 \leq s < t \leq U$ and $1 \leq i \neq i' \leq 4$, then the optimal PAPR reduction performance in terms of minimizing VC can be achieved.

Proof: Note that each $\bar{\mathrm{A}}_i(m)$ has four nonzero values for all $0 \leq m \leq N-1$ (see Table I). Suppose that $d_{st}(\tau_1) = d_{st}(\tau_2) = d_{st}(\tau_3) \neq d_{st}(\tau_4)$. Then, when $m = d_{st}(\tau_1)$, $\bar{\mathrm{A}}_1(m) = \bar{\mathrm{A}}_2(m) = \bar{\mathrm{A}}_3(m) = N/4$, i.e., $\mathrm{A}(m) = 3N/4$ if $c_i^{(s)} c_i^{(t)*} = 1$. On the other hand, if $d_{st}(\tau_1) \neq d_{st}(\tau_2) \neq d_{st}(\tau_3) \neq d_{st}(\tau_4)$, then $\mathrm{A}(m) = N/4$. That is, the same differences of cyclic shift values increase VC even if $c_i^{(s)} c_i^{(t)*} \neq 1$. Therefore, to minimize VC, it is required that $d_{st}(\tau_i) \neq d_{st}(\tau_{i'})$ mod $N/4$ for all $\binom{U}{2}$ pairs of alternative OFDM signal sequences. ■

Note that VC only depends on $d_{st}(\tau_i)$, not on $c_i^{(s)} c_i^{(t)*}$ because the rotation by multiplying $c_i^{(u)}$ does not change the energy. Therefore, the rotation value does not affect the PAPR reduction performance in this case.

Based on the optimal condition, a simple selection of optimal cyclic shift values satisfying the optimal condition is proposed in Table II, where u is the index of alternative OFDM signal sequence. The probability that randomly selected cyclic shift values do not satisfy the optimal condition is 0.4362 when $N = 256$ and $U = 4$. Since 43.62% is not negligible, deterministic selection of the optimal cyclic shift values is important.

The next theorem derives the maximum number of optimal alternative OFDM signal sequences for Class-III SLM scheme.

Theorem 2: When the optimal cyclic shift values in Table II are used for $N = 2^m$ with $m > 2$, the maximum number of optimal alternative OFDM signal sequences is $N/8$.

Proof: Let s and t be the indices of alternative OFDM signal sequences and $0 < s < t < U$. Then, the corresponding differences of cyclic shift values are $d_{st}(\tau_1) = 0$, $d_{st}(\tau_2) = t - s$, $d_{st}(\tau_3) = 2(t-s)$, and $d_{st}(\tau_4) = 3(t-s) = 2(t-s) + (t-s)$, respectively. Now, we consider two cases.

Case 1) $\max(t-s) < N/8$;

Note that $d_{st}(\tau_{i'}) - d_{st}(\tau_i) = k(t-s) \mod N/4$, where $i, i' \in \{1,2,3,4\}, i < i'$, and $k = i' - i \in \{1,2,3\}$. When $k = 1$ or 2, $d_{st}(\tau_{i'}) - d_{st}(\tau_i) \neq 0$ because $2(t-s) < N/4$. When $k = 3$, $d_{st}(\tau_4) - d_{st}(\tau_1) = 3(t-s) < 3N/8$. We have to check whether $3(t-s)$ can be either 0 or $N/4$ which are the cases of $d_{st}(\tau_4) = d_{st}(\tau_1)$. It is clear that $3(t-s) \neq 0$ by $s < t$ and $3(t-s) \neq 2^{m-2} = N/4$. Thus, the optimal condition $d_{st}(\tau_i) \neq d_{st}(\tau_{i'})$ mod $N/4$ always hold for Case 1.

Case 2) $\max(t-s) \geq N/8$;

Suppose that $t - s = N/8$. Then, $d_{st}(\tau_2) = d_{st}(\tau_4)$ mod $N/4$. Thus, Case 2 does not satisfy the optimal condition. ■

C. Selection of Additional Alternative OFDM Signal Sequences

In Theorem 2, it is shown that the maximum number of optimal alternative OFDM signal sequences is $N/8$. However, it may be necessary to generate more alternative OFDM signal sequences by sacrificing the optimality. Thus, a simple method to generate good additional alternative OFDM signal sequences is proposed by properly adjusting the rotation values. For good PAPR reduction performance, we only consider $c_1^{(u)} \neq c_2^{(u)} \neq c_3^{(u)} \neq c_4^{(u)}$.

Let us consider a case of generating $N/4$ alternative OFDM signal sequences. In Section III-B, $N/8$ optimal alternative OFDM signal sequences without rotation values can be generated. However, by adjusting the rotation values for these $N/8$ optimal alternative OFDM signal sequences, good additional $N/8$ alternative OFDM signal sequences can be generated. Note that the same cyclic shift values in Table II are used for the first $N/8$ optimal sequences and the second additional $N/8$ sequences. For example, to generate total $N/4$ alternative OFDM signal sequences, the rotation values $c_1^{(u)} = 1, c_2^{(u)} = -1$, $c_3^{(u)} = j$, and $c_4^{(u)} = -j$ are multiplied to each of the $N/8$ optimal alternative OFDM signal sequence cases to generate additional $N/8$ sequences.

Let $c_i^{(u)} = e^{j\theta_i^{(u)}}$, and if we use $\theta_i^{(u)} = (i-1)(\pm\pi/2)$ or $(i-1)\pi$ for the second $N/8$ alternative OFDM signal sequences, the PAPR reduction performances of the first $N/8$ and the second $N/8$ sequences are the same because the second $N/8$ sequences are just cyclic-shifted version of the first $N/8$ optimal sequences in time domain. Therefore, to generate good additional alternative OFDM signal sequences, we need to use the rotation values which do not have linear relation as above. Consequently, total $4N/8$ good alternative OFDM signal sequences can be generated by multiplying the rotation values $\{c_1^{(u)}, c_2^{(u)}, c_3^{(u)}, c_4^{(u)}\} = \{1, j, -j, -1\}, \{1, -j, j, -1\}, \{1, -1, j, -j\}, \{1, -1, -j, j\}$ to each of the $N/8$ optimal sequences.

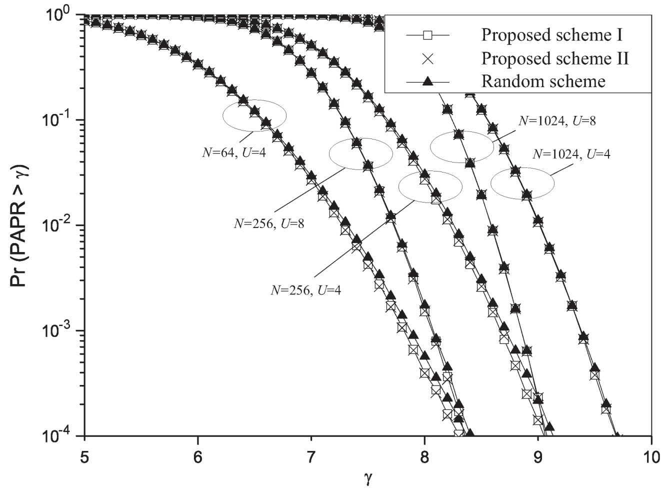

Fig. 2. Comparison of PAPR reduction performance of Class-III SLM scheme by random selection and proposed selection I and II when 16-QAM, $N = 64, 256, 1024$, and $U = 4$ and 8 are used.

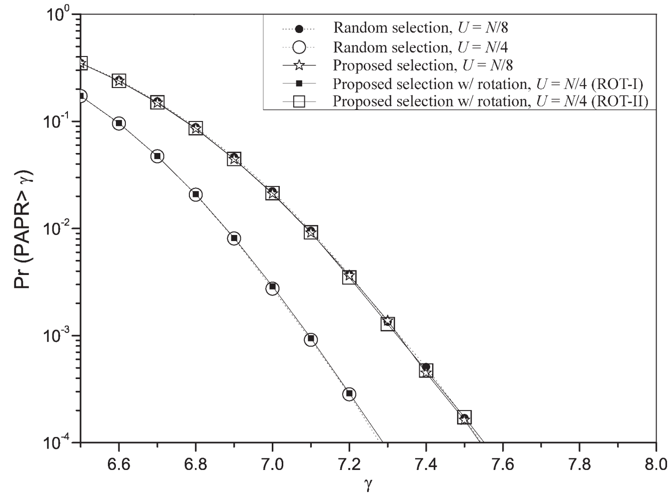

Fig. 3. Comparison of PAPR reduction performance of Class-III SLM scheme by random selection, proposed selection, ROT-I, and ROT-II when 16-QAM, $N = 256$, and $U = N/8$ or $N/4$ are used.

IV. Simulation Results and Conclusions

In this section, we compare the PAPR reduction performances of Class-III SLM scheme by random selection and the proposed selection of the cyclic shift and rotation values. For random selection, cyclic shift and rotation values are randomly selected from $0 \le \tau_i^{(u)} < N/4$ and $c_i^{(u)} \in \{\pm 1, \pm j\}$, and for the proposed scheme, rotation values are not used to generate the optimal $N/8$ alternative OFDM signal sequences.

In Fig. 2, the proposed scheme I and proposed scheme II use the cyclic shift values in Table II and $\tau_1^{(u)} = 0$, $\tau_2^{(u)} = k$, $\tau_3^{(u)} = 3k$, and $\tau_4^{(u)} = 5k$ which also satisfies the optimal condition, respectively. When $N = 64, 256$ and $U = 4$, the PAPR reduction performances of random scheme are degraded compared to the proposed schemes. When U and N are large, the performance gap becomes negligible. Proposed schemes I and II show the identical performance. Since random scheme satisfies near optimal condition for many pairs of alternative OFDM signal sequences, it is possible that the proposed schemes show little performance enhancement. However, we can find that the proposed schemes show the optimal PAPR reduction performance.

In Fig. 3, ROT-I denotes the case of using $c_1^{(u)} = 1, c_2^{(u)} = -1, c_3^{(u)} = j, c_4^{(u)} = -j$ for generating additional $N/8$ alternative OFDM signal sequences. ROT-II denotes the case of using $c_1^{(u)} = 1, c_2^{(u)} = j, c_3^{(u)} = -1, c_4^{(u)} = -j$ for generating additional $N/8$ alternative OFDM signal sequences. The proposed selection ROT-I with $U = N/4$ shows almost the same PAPR reduction performance as random selection with $U = N/4$. This means that good additional alternative OFDM signal sequences can be generated by the proposed method ROT-I. Note that the same cyclic shift values in Table II are used for the first optimal $N/8$ and the second additional $N/8$ sequences with different rotation values. Whereas, the proposed selection ROT-II with $U = N/4$ shows almost the same PAPR reduction performance as random selection with $U = N/8$ because the rotation values have linear relation, which causes no additional PAPR reduction gain for ROT-II.

In this letter, a selection method of optimal cyclic shift values for Class-III SLM scheme is proposed. Also, a selection method of good additional alternative OFDM signal sequences by using proper rotation values is proposed.

Although the analysis to derive the optimal condition is complicated, we do not need to compute the optimal condition for each OFDM symbol when we apply the proposed scheme to real systems. To use the proposed scheme, we only need to use U pre-determined optimal cyclic shift values given in Table II. Therefore, the computational complexity of the proposed scheme is basically the same as random scheme.

There are some advantages of the proposed scheme. First, random scheme requires memory for 3 complex numbers (rotation values), whereas the proposed scheme does not need the memory for rotation values. Second, random scheme requires $\lceil \log_2 (N/4)^3 \rceil$ bits of side information for cyclic shift values and $\lceil \log_2 4^3 \rceil$ bits of side information for rotation values. Whereas, the proposed scheme requires only $\lceil \log_2 U \rceil$ bits of side information if the cyclic shift values in Table II are shared by the transmitter and receiver. Third, random scheme has a risk to select the cases of bad PAPR reduction performance, whereas the proposed scheme always guarantees the optimal PAPR reduction performance in terms of minimizing VC.

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