Analysis of Oversampling Effect on Selected Mapping Scheme Using CORR Metric

Jun-Young WOO†(a), Kee-Hoon KIM†(b), Kang-Seok LEE†(c), Nonmembers, Jong-Seon NO††(d), Member, and Dong-Joon SHIN†††(e), Nonmember

Summary

It is known that in the selected mapping (SLM) scheme for orthogonal frequency division multiplexing (OFDM), correlation (CORR) metric outperforms the peak-to-average power ratio (PAPR) metric in terms of bit error rate (BER) performance. It is also well known that four times oversampling is used for estimating the PAPR performance of continuous OFDM signal. In this paper, the oversampling effect of OFDM signal is analyzed when CORR metric is used for the SLM scheme in the presence of nonlinear high power amplifier. An analysis based on the correlation coefficients of the oversampled OFDM signals shows that CORR metric of two times oversampling in the SLM scheme is good enough to achieve the same BER performance as four times and 16 times oversampling cases. Simulation results confirm that for the SLM scheme using CORR metric, the BER performance for two times oversampling case is almost the same as that for four and 16 times oversampling cases.

Key words: correlation (CORR) metric, high power amplifier (HPA), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), selected mapping (SLM)

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has been widely used in various wireless communication systems. Since OFDM signals show high peak-to-average power ratio (PAPR), many schemes have been proposed to mitigate the PAPR problem. Among them, partial transmit sequence (PTS) and selected mapping (SLM) schemes can be good answers to solve the PAPR problem [1], where many alternative OFDM signals are generated and the one with the minimum PAPR is selected for transmission. Since these schemes have high computational complexity due to multiple inverse fast Fourier transform (IFFT) operations, many low-complexity schemes have been proposed.

While above schemes focus on reducing the PAPR metric, many other metrics are proposed to improve the bit error rate (BER) performance in the presence of nonlinear devices such as high power amplifier (HPA). It is known that intermodulation distortion [2], distortion-to-signal power ratio [3], mean squared error [4], and correlation (CORR) metrics [5] outperform the PAPR metric in terms of BER performance of OFDM systems. Except for the PAPR metric, CORR metric shows the lowest computational complexity while its BER performance is almost the same as those of other metrics. Although CORR shows relatively high PAPR than PAPR metric, BER performance of CORR metric can be enhanced than that of PAPR metric.

In this paper, it is shown that two times oversampling is enough to be used instead of four times oversampling for SLM scheme using CORR metric. Four times oversampling causes high computational complexity due to $U \times 4N$-point IFFT operations and $U \times$ CORR metric calculations. Note that one CORR metric calculation consists of $LN$ complex multiplications. To reduce the computational complexity, two times oversampling instead of four times oversampling for SLM scheme using CORR metric is proposed.

Oversampled signal sequences can be obtained by linear combination of Nyquist-rate signal sequences. Based on this, the resultant sequences after CORR calculation with different oversamplings are obtained. By deriving the Pearson correlation coefficients of these resultant sequences, it is shown that the Pearson correlation coefficient of two and 16 times oversampling cases is relatively high. This implies that the BER performance of two times oversampling case is the same as that of 16 times oversampling case.

The rest of this paper is organized as follows. In Sect. 2, CORR metric is briefly reviewed and it is shown that CORR metric is near optimal metric in terms of BER performance. In Sect. 3, the oversampling effect is analyzed for the SLM scheme using CORR metric in the presence of nonlinear HPA by computing CORR metric and Pearson correlation coefficient values. Simulation results and conclusion are given in Sect. 4.

2. Overview of SLM Scheme Using CORR Metric

2.1 Overview of CORR Metric

Binary data sequences are modulated by $M$-ary quadrature amplitude modulation (QAM) to generate an input symbol sequence $[X_0, X_1, \cdots, X_{N-1}]$. Then, $(L - 1)N$ zeros are padded to the end (or middle) of an input symbol sequence, which is called $L$ times oversampling, as

$$X = [X_0, X_1, \cdots, X_{N-1}, 0, \cdots, 0]$$

$(L-1)N$ 0's
where $N$ is the number of subcarriers and $L$ is the oversampling factor. In general, it is known that four times oversampling ($L = 4$) is good enough for estimating the PAPR of continuous OFDM signal [6]. Then, $X$ in (1) performs IFFT operation and the $n$th element of the resulting OFDM signal is expressed as

$$x_n = \frac{1}{\sqrt{LN}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/L}, \quad 0 \leq n \leq LN - 1. \quad (2)$$

Figure 1 shows a block diagram of the SLM scheme using CORR metric. This scheme generates $U$ alternative symbol sequences $X^{(a)}$ by componentwisely multiplying each of $U$ different phase sequences to an input symbol sequence $[X_0, X_1, \cdots, X_{N-1}]$. Let $P^{(a)} = [P^{(a)}_0, P^{(a)}_1, \cdots, P^{(a)}_{N-1}]$ be the $u$th phase sequence with $P^{(a)}_k = e^{j\phi_k}$, where $\phi_k \in [0, 2\pi)$, $0 \leq k < N - 1$, and $0 \leq u \leq U - 1$. It is customary to use $P^{(0)}_k \in \{1, -1\}$ or $P^{(0)}_k \in \{\pm 1\}$.

Each alternative symbol sequence is zero padded and performs IFFT operation to generate $x^{(a)} = [x^{(a)}_0, x^{(a)}_1, \cdots, x^{(a)}_{LN-1}]$ and it passes through HPA such as solid state power amplifier (SSPA) which is commonly used in mobile communications. It is known that the polynomial model for estimating the real SSPA to calculate the CORR metric is sufficiently accurate and has relatively low computational complexity [3]. The polynomial model is usually expressed as a third order nonlinearity such that the $n$th element of the output of SSPA can be expressed as

$$y^{(a)}_n \approx \alpha_1 x^{(a)}_n + \alpha_3 x^{(a)}_n |x^{(a)}_n|^2 \quad (3)$$

where $x^{(a)}_n$ and $y^{(a)}_n$ are the $n$th elements of the $u$th alternative input and output OFDM signals of SSPA, respectively [3]. As the polynomial coefficients in (3), $\alpha_1 = 1$ and $\alpha_3 = -0.1769$ are used to match the practical SSPA used in WiMAX [5]. The CORR metric [5] for the $u$th alternative OFDM signal is obtained as

$$R^{(a)}_{uy} = \sum_{n=0}^{LN-1} x^{(a)}_n y^{(a)*}_n = \alpha_1 \sum_{n=0}^{LN-1} |x^{(a)}_n|^2 + \alpha_3 \sum_{n=0}^{LN-1} |x^{(a)}_n|^4 \quad (4)$$

where $\cdot$ indicates the complex conjugation. Then, this scheme calculates $U$ CORR metrics between the input signal $x^{(a)}$ and the output signal $y^{(a)}$ of SSPA. Among $U$ CORR metrics $R^{(a)}_{uy}$, the signal $x^{(a)}$ with the maximum CORR is selected for transmission. Also, the side information $\tilde{u}$ should be transmitted to the receiver.

As an HPA to amplify the selected alternative OFDM signal at the transmitter, Rapp model [5] is used because it is also widely used in mobile communications and more accurate one than the polynomial model in (3). The output of Rapp model is given as

$$y_n = A(|x_n|) e^{j[\arg(x_n) + \phi(|x_n|)]} \quad (5)$$

where $x_n$ is the input to Rapp model and $A(\cdot)$ and $\phi(\cdot)$ denote amplitude to amplitude (AM/AM) and amplitude to phase (AM/PM) conversions of the nonlinear amplifier, respectively. Since SSPA is used in this paper, AM/PM conversion is assumed flat, i.e., $\phi(|x_n|) = 0$, and the AM/AM conversion is given as

$$A(|x_n|) = |x_n| \left[1 + \left(\frac{|x_n|}{A_0}\right)^2\right]^{-\frac{1}{2}} \quad (6)$$

where $A_0$ is the maximum amplifier output and $p$ is the smoothness factor [7] with $p = 3.286$ as given in [5].

To determine the operating point of HPA, we use output back-off (OBO) defined as

$$\text{OBO} = 10 \log_{10} \frac{A_0^2}{P_{\text{out}}} \quad (7)$$

where $P_{\text{out}}$ denotes the average power of OFDM signal at the output of HPA.

2.2 BER Performance of SLM Scheme under HPA

The performance of SLM scheme was analyzed in the presence of nonlinear HPA [8]. First, we review the analysis in [8] as follows. The output of nonlinear HPA can be formulated as

$$y_n = K_0 x_n + d_n, \quad 0 \leq n \leq N - 1 \quad (8)$$

where $x_n$ is the input to the nonlinear HPA, $d_n$ is the uncorrelated distortion, and $K_0$ is the linear scaling factor defined as

$$K_0 = \frac{E[|x_n y_n|]}{E[|x_n|^2]} \quad (9)$$

Also, signal-to-distortion-plus-noise ratio (SDNR) measured at the receiver after FFT is defined as

$$\text{SDNR} = \frac{E[|K_0 x_k|^2]}{E[|D_k + W_k|^2]} \quad (10)$$

where $D_k = \text{FFT}(d_k)$ and $W_k$ is additive white Gaussian noise (AWGN) at the $k$th subcarrier. Assume that $X_k$, $D_k$, and $W_k$ are mutually uncorrelated. Then, the average BER [8] is calculated as

$$P_b = \frac{4}{\log_2 M} \left[1 - \frac{1}{\sqrt{M}}\right] Q\left(\sqrt{\frac{3}{M - 1}}\text{SDNR}\right) \quad (11)$$
where $M$ is the constellation order and $Q(\cdot)$ denotes the $Q$-function.

According to (11), to achieve good BER performance, SDNR should be large. That is, the numerator in (10) should be large because $D_x$ and $W_i$ cannot be controlled. Since $E[|K_0X_i|^2] = K_0^2E[|X_i|^2]$ and $E[|X_i|^2]$ is constant, for good BER performance, $E[\tilde{x}_i^{(0)}\tilde{y}_n^{(\alpha)}]$ in (9) should be large, which is identical to the CORR metric in (4). Therefore, the CORR metric can be regarded as near optimal in terms of BER performance of the SLM scheme for OFDM signals.

From this reason, we will focus on the CORR metric among various metrics targeting the BER performance and analyze the oversampling effect on the SLM scheme using CORR metric in the presence of nonlinear HPA.

3. Oversampling Effect on SLM Scheme Using CORR Metric

3.1 Expression of Oversampled Signal and CORR Metric

Oversampled signal can be expressed by linear combination of Nyquist-rate samples [10]. The oversampling operator is called interpolator and the impulse response of an interpolator for $L$ times oversampling is defined as

$$h_L[n] = \frac{\sin(n\pi/L)}{n\pi/L}. \quad (12)$$

Since an ideal interpolator cannot be implemented, a finite-length filter of length $I$ is used in practice. Then, the output of finite-length interpolator can be expressed as

$$\tilde{x}_L[n_L] = \sum_{k=\left[\frac{n_L}{I}\right]}^{\left[\frac{n_L+LI}{I}\right]} x[k]h_L[n_L - Lk] \quad (13)$$

where $\tilde{x}_L[n_L]$ is the estimated $n_L$th element of $L$ times oversampled signal and $x[k]$ denotes the $k$th element of Nyquist-rate signal. In this paper, we assume $I = 2$ for simplicity and for $I > 2$, a similar analysis can be applied.

To represent the continuous signal, it is known that 16 times oversampling ($L = 16$) is good enough. By substituting $L = 16$ and $I = 2$ to (13), 16 times oversampled signal $\tilde{x}_{16}[16m + s]$ can be obtained as

$$\tilde{x}_{16}[16m + s] = h_{16}[16 + s]x[m - 1] + h_{16}[s]x[m] + h_{16}[-16 + s]x[m + 1] + h_{16}[-32 + s]x[m + 2]. \quad (14)$$

where $0 \leq m \leq N - 1$ denotes the indices of Nyquist-rate samples and $0 \leq s \leq 15$. We can also obtain $\tilde{y}_{16}[16m + s]$ in the similar manner.

Note that the purpose of this analysis is investigating how frequently the same phase sequences are selected when $L$ times oversamplings are used compared to the case of 16 times oversampling. Therefore, we first investigate the result of CORR metric when 16 times oversampling is used and then compare the result from $L$ times oversampling cases. In order to do this, we only need to consider 16 samples for CORR metric computation in (4) instead of total $16N$ samples. Since the same filter coefficients $h_{16}[n]$ in (12) are used for all $m$ (see Eq. (14)), the resulting coefficients of CORR metric for $16N$ samples is just multiple of resulting coefficients of CORR metric for 16 samples.

Then, a partial CORR metric of 16 samples within two adjacent Nyquist-rate samples can be given as

$$\tilde{R}_{16}[m] = \sum_{s=0}^{15} \tilde{x}_{16}[16m + s]\tilde{y}_{16}[16m + s] = \sum_{s=0}^{15} v_m(s). \quad (15)$$

For better understanding, we can rewrite $v_m(s)$ as (16) at the top of next page by substituting (14) and $s = 8$ to (15). We can similarly rewrite all the other values $v_m(s)$. Note that in (16), there are 16 coefficients, i.e., $\{0.04, -0.13, -0.13, \cdots, -0.13, 0.04\}$ and we will call the coefficient set as a coefficient sequence.

Now, suppose that arbitrary $L \in \{1, 2, 4, 16\}$ times oversamplings are used for CORR metric computation. Then, from (4), we only need $v_m(0)$ in (15) for Nyquist-rate sampling. Also, note that $v_m(0) + v_m(8), v_m(0) + v_m(4) + v_m(8) + v_m(12)$, and $\sum_{s=0}^{15} v_m(s)$ are needed to compute $\tilde{R}_{16}[m]$ in (15) when two times, four times, and 16 times oversamplings are used, respectively.

3.2 Correlation Coefficients between Coefficient Sequences Derived from CORR Metric Computation

In this subsection, we will investigate which oversampling rate is required to show the similar performance as the continuous OFDM signal case. Note that BER performance of SLM scheme using CORR metric is determined by the selected phase sequence $P^{(0)}$. Therefore, to find the proper oversampling rate, we have to investigate which oversampling case selects the same phase sequences as the continuous OFDM signal case, i.e., 16 times oversampling case. Since the selection of phase sequences is determined by (15) for all $m$, we need the resultant coefficient sequences after computing (15) when $L \in \{1, 2, 4, 16\}$. Then, by investigating the correlation coefficients of the resultant coefficient sequences, we can find the proper oversampling rate.

Table 1 shows the resultant coefficient sequences after processing (15) when $L = 1, 2, 4$, and 16. Note that indices are arranged from left to right and top to bottom (1 to 16) in a coefficient sequence as given in (16). For example, when $L = 2$, we need $v_m(0)$ and $v_m(8)$ to compute (15). Since $v_m(0) = \tilde{x}_{16}[16m]^{[a]}\tilde{y}_{16}[16m]^{[a]} = 1 \cdot x[m]^{[a]}y[m]^{[a]}$, there is only one nonzero coefficient 1 at index 6. Therefore, the coefficient 1 should be added to 0.4 at index 6 in $v_m(8)$, i.e., $1 + 0.4 = 1.4$ (see Table 1 when $L = 2$ and Eq. (16)). The rests can also be computed in the same manner.

Now, we want to investigate the similarity between coefficient sequences $\{v_m(0), v_m(0) + v_m(8), v_m(0) + v_m(4) + v_m(8) + v_m(12), \sum_{s=0}^{15} v_m(s)\}$. In this paper, we use the Pear-
\[ v_m(8) = \hat{x}_{16}[16m+8] + y_{16}[16m+8] + 0.3 \hat{x}_m[m+1] + 0.63 \hat{x}_m[m+1] - 0.21 \hat{x}_m[m+2] + 0.91 \hat{x}_m[m+2] + 0.63 \hat{x}_m[m+1] - 0.21 \hat{x}_m[m+2] \]

Table 2: Values of Pearson correlation coefficients between coefficient sequences with different L.

<table>
<thead>
<tr>
<th>L = 1 and L = 16</th>
<th>L = 2 and L = 16</th>
<th>L = 4 and L = 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(I = 2) )</td>
<td>0.6023</td>
<td>0.9133</td>
</tr>
<tr>
<td>( r(I = 3) )</td>
<td>0.5980</td>
<td>0.9145</td>
</tr>
</tbody>
</table>

Table 3: Probability of choosing different phase sequences compared with 16 times oversampling case when \( N = 256 \).

<table>
<thead>
<tr>
<th>L = 1</th>
<th>L = 2</th>
<th>L = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U = 4 )</td>
<td>0.343</td>
<td>0.443</td>
</tr>
<tr>
<td>( U = 8 )</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( U = 16 )</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( U = 32 )</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 1: Resultant coefficient sequences after computing (15) when \( L \) times oversampling is used.

Table 2: Values of Pearson correlation coefficients between coefficient sequences with different L.

Table 3: Probability of choosing different phase sequences compared with 16 times oversampling case when \( N = 256 \).

\[ r = \frac{\sum (a_i - \bar{a})(b_i - \bar{b})}{\sqrt{\sum (a_i - \bar{a})^2 \sum (b_i - \bar{b})^2}} \]

where \( \bar{a} \) denotes the sample mean of \( a \) and \( a_i \) denotes the \( i \)th element of the sequence \( a \). Clearly, \( r \) takes a value in \([-1, 1]\), where 0 implies no correlation and 1 or -1 implies positive or negative correlation, respectively.

Table 2 shows \( r \) between two coefficient sequences obtained from Table 1 when different \( L \)'s are used. When the cases of \( L = 1 \) and \( L = 16 \) are considered, the value of \( r \) is 0.6023, which shows relatively low correlation. On the other hand, when the cases of \( L = 2 \) and \( L = 16 \) and the cases of \( L = 4 \) and \( L = 16 \) are considered, the values of \( r \) are 0.9133 and 0.9839, respectively, which show relatively high correlation. Also, \( I = 3 \) case is also derived for comparison. As seen in Table 2, the values of \( r \) look almost the same for \( I = 2 \) and 3. These results imply that when two times or four times oversamplings are used, the probability of choosing the same phase sequences as the 16 times oversampling case in the SLM scheme using CORR metric is very high.

Table 3 shows the probabilities of choosing different phase sequences from 16 times oversampling case for various values of \( L \) when \( N = 256 \). 10^4 OFDM signals are randomly generated and the unequal cases are counted. When \( L = 1 \) and \( U = 4 \), the probability is 0.343 and it increases as \( U \) increases. However, when \( L = 2 \) and 4, the probabilities are all zero. That is, they always choose the same phase sequence as the case of \( L = 16 \). This result confirms that analysis using Pearson correlation coefficient is reasonable.

4. Simulation Results and Conclusion

In this section, the BER performances of the SLM scheme using CORR metric for different oversampling factor \( L \) are compared when \( N = 256 \), \( U = 4 \), and \( OBO = 3, 3.5, 4, 4.5, \) and 5dB. \( A_0 \) in (7) is calculated by multiple simulations for a given OBO. Also, 16-QAM is used for data modulation. We only consider the AWGN channel since the BER performance of the SLM scheme using CORR in selective fading channel is shown in [5]. For simulation, nonlinear HPA is used and 16 times oversampling is used for the Rapp model to estimate the performance of continuous OFDM signal case. We assume the perfect knowledge of side information.

In Fig. 2, Original, CL, and No HPA indicate the original OFDM signal with nonlinear HPA, OFDM signal using CORR metric under nonlinear HPA where \( L \) denotes the oversampling factor for computing CORR metrics in (4), and the original OFDM signal without nonlinear HPA, respectively. Note that C2, C4, and C16 show almost the same BER performances for various OBO values, while C1 is degraded compared to them.

Figure 3 shows the BER performance of the SLM schemes using CORR and PAPR metrics. Since it is already shown that C2 shows the same BER performance as C4 and C16 and thus, C4 and C16 cases are excluded. PL
Fig. 2 BER performance of the SLM schemes using CORR metric when $N = 256$, $U = 4$, and $L = 1, 2, 4, \text{ and } 16$ for various OBOs.

denotes the OFDM signal using PAPR metric under nonlinear HPA, where $L$ denotes the oversampling factor for computing CORR metrics in (4). The BER performance of the SLM scheme using PAPR metric is degraded compared to that using CORR metric.

Figure 4 shows the PAPR reduction performances of the SLM schemes using CORR and PAPR metrics when $N = 64, 256, \text{ and } 1024$, $U = 4$, and $L = 4$. The PAPR reduction performances of SLM scheme using CORR metric are severely degraded compared to that using PAPR metric. However, Fig. 3 shows that the BER performance of the SLM scheme using CORR metric is better than that of the SLM scheme using PAPR metric.

In this paper, the oversampling effect for the SLM scheme using CORR metric is analyzed in the presence of nonlinear HPA. The oversampled signals for CORR metric computation for $L = 1, 2, 4, \text{ and } 16$ are obtained by linear combination of Nyquist-rate samples. As a result, two and four times oversampling cases show relatively high correlation with 16 times oversampling case, but Nyquist-rate sampling case shows relatively low correlation with 16 times oversampling case. These results imply that, when two times oversampling is used for CORR metric computation, the probability of choosing the same phase sequence as the 16 times oversampling case is very high.

Simulation results show that BER performance of two times oversampling for CORR metric calculation is almost the same as that of four or 16 times oversampling cases. On the other hand, the BER performance for Nyquist-rate sampling case is degraded.

Consequently, two times oversampling for CORR metric computation is good enough to achieve the same BER performance as those of the four or 16 times oversampling case in the SLM scheme. By using two times oversampling for CORR metric computation, the computational complexity can be reduced to $1/2$ as that of the four times oversampling case.

References


Jun-Young Woo received the B.S. degree in electrical and computer engineering from University of Seoul, Seoul, Korea, in 2009 and the M.S. degree in electrical engineering and computer science from Seoul National University, Seoul, Korea, in 2011. Currently, he is pursuing the Ph.D. degree in electrical and computer engineering from Seoul National University. His research interests include orthogonal frequency division multiplexing, cryptography, and error-correcting codes.

Kee-Hoon Kim received the B.S. and M.S. degrees in electrical engineering and computer engineering from Seoul National University, Seoul, Korea, in 2008 and 2010, respectively. Currently, he is pursuing the Ph.D. degree in electrical and computer engineering from Seoul National University. His research interests include orthogonal frequency division multiplexing, compressed sensing, and communication theory.

Jong-Seon No received the B.S. and M.S.E.E. degrees in electronics engineering from Seoul National University, Seoul, Korea, in 1981 and 1984, respectively and the Ph.D. degree in electrical engineering from the University of Southern California, Los Angeles, in 1988. He was a Senior MTS at Hughes Network Systems from February 1988 to July 1990. He was also an Associate Professor in the Department of Electronic Engineering, Konkuk University, Seoul, Korea, from September 1990 to July 1999. He joined the faculty of the Department of Electrical and Computer Engineering, Seoul National University, in August 1999, where he is currently a Professor. From 1996 to 2008, he served as a Founding Chair of Seoul Chapter, IEEE Information Theory Society. He was a General Chair for Sequence and Their Applications 2004 (SETA2004) in Seoul, Korea. He also served as a General Co-Chair for International Symposium on Information Theory and Its Applications 2006 (ISITA 2006) and International Symposium on Information Theory 2009 (ISIT 2009) in Seoul, Korea. He was a recipient of IEEE Information Theory Society Chapter of the Year Award in 2007. He is elevated to IEEE Fellow in Research Engineer/Scientist through IEEE Information Theory Society, November, 2011. He was Co-Editor-in-Chief of Journal of Communications and Networks, January, 2012. His area of research interests includes error-correcting codes, sequences, cryptography, LDPC codes, interference alignment and wireless communication systems.

Dong-Joon Shin received the B.S. degree in electronics engineering from Seoul National University, Seoul, Korea, the M.S. degree in electrical engineering from Northwestern University, Evanston, USA, and the Ph.D. degree in electrical engineering from University of Southern California, Los Angeles, USA. From 1999 to 2000, he was a member of technical staff in Wireless Network Division and Satellite Network Division, Hughes Network Systems, Maryland, USA. Since September 2000, he has been an Associate Professor in the Division of ECE at Hanyang University, Seoul, Korea. His current research interests include error-correcting codes, sequences, and discrete mathematics.

Kang-Seok Lee received the B.S. degree in electrical engineering from Yonsei University, Seoul, Korea, in 2012. Currently, he is pursuing the Ph.D. degree in electrical and computer engineering from Seoul National University. His research interests include orthogonal frequency division multiplexing, and error-correcting codes.