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Low-Complexity PTS Schemes Using Dominant Time-Domain Samples in OFDM Systems

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***Abstract*—Partial transmit sequence (PTS) is a probabilistic peak-to-average power ratio (PAPR) reduction scheme for orthogonal frequency division multiplexing (OFDM) signals, but it requires an exhaustive search for the one with the minimum PAPR over all alternative OFDM signal vectors. In order to reduce such computational complexity, it was proposed that the PAPR value of each alternative OFDM signal vector is approximately estimated by using dominant time-domain samples selected based on a simple metric. In this paper, two new effective metrics for selecting dominant time-domain samples are proposed for low-complexity PTS scheme. For further lowering the computational complexity, two low-complexity PTS schemes are proposed by sorting the dominant time-domain samples in decreasing order of their metric values. Simulation results confirm that compared with the conventional PTS scheme, the proposed PTS schemes show identical PAPR reduction performance with substantially reduced computational complexity.**

***Index Terms*—Low complexity, orthogonal frequency division multiplexing (OFDM), partial transmit sequence (PTS), peak-to-average power ratio (PAPR).**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is an essential technique in modern wireless communication systems due to its bandwidth efficiency and robustness to the multipath fading. Therefore, OFDM has been adopted in various wireless communication systems such as digital video broadcasting (DVB) and IEEE 802.11 wireless local area network (WLAN) [1]. However, high peak-to-average power ratio (PAPR) of OFDM signals results in signal distortion due to the nonlinear effect of high power amplifier (HPA), which causes in-band distortion, out-of-band radiation, and bit error rate (BER) degradation at the receiver.

In order to alleviate the PAPR problem of OFDM signals, various PAPR reduction schemes have been proposed such as clipping [2], selected mapping (SLM) [3], partial transmit sequence (PTS) [4], active constellation extension (ACE) [5], companding [6], and tone reservation (TR) [7].

Among them, PTS and SLM schemes effectively reduce the PAPR of OFDM signals without causing signal distortion. In general, compared with SLM schemes, PTS schemes require less inverse fast Fourier transform (IFFT) operations which increase the system implementation complexity. In the conventional PTS scheme, an input data vector is partitioned into several subblocks, which are separately IFFTed. Each of IFFTed subblocks is multiplied by phase rotating factor and then they are added together to produce an alternative OFDM signal vector. Since the conventional PTS scheme requires large computational complexity, low-complexity PTS schemes have been proposed [8]–[11].

Recently, efficient combinatorial optimization (CO) algorithms have been used to simplify the search for the optimal phase rotating vector over many phase rotating vectors in the PTS scheme such as the simulated anealing (SA) [12], the particle swarm optimization (PSO-PTS) [13], the artificial bee colony algorithm (ABC-PTS) [14], the greedy and genetic algorithm [15], and the modified chaos clonal shuffled frog leaping algorithm (MCCSFLA-PTS) [16]. Also, low-complexity PTS schemes using sequences are proposed [17], [18]. However, the low-complexity PTS schemes using CO algorithms and sequences can work well only for the case of a large number of subblocks.

The reduced-complexity PTS (RC-PTS) [19] has been proposed to reduce the computational complexity by estimating the PAPRs of alternative OFDM signal vectors based on the selected dominant time-domain samples and find the alternative OFDM signal vector with the minimum estimated PAPR. Also, the improved PTSs have been proposed to further reduce the computational complexity and enhance the PAPR reduction performance of RC-PTS by using the dominant time-domain samples but different selection method for the dominant samples with RC-PTS [20].

In this paper, two new metrics are proposed for selecting dominant time-domain samples in RC-PTS. For further lowering the computational complexity, dominant time-domain samples are sorted in decreasing order of their metric values and then each sample power is compared with the minimum PAPR obtained from the previously examined alternative OFDM signal vectors.

The rest of the paper is organized as follows. In Section II, OFDM system and the conventional PTS and RC-PTS schemes are briefly explained. Then, low-complexity PTS schemes using new metrics and sorting method are proposed in Section III. In Section IV, the computational complexity of the proposed PTS schemes is analyzed and the simulation results are provided. Finally, conclusions are given in Section V.

II. OFDM SYSTEM AND PTS SCHEMES

A. OFDM System

Let $\mathbf{X} = [X_0, X_1, \cdots, X_{N-1}]^T$ denote an input data vector of length N where X_k is the input data symbol. Then the

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OFDM signal x_n is calculated as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}}, \quad 0 \le n \le N-1 \tag{1}$$

and the PAPR of an OFDM signal vector $\mathbf{x} = [x_0, x_1, \cdots, x_{N-1}]^T$ is defined as

$$PAPR = \frac{\max\limits_{n=0}^{N-1} |x_n|^2}{E\left[|x_n|^2\right]} \tag{2}$$

where $j = \sqrt{-1}$ and $E[\cdot]$ denotes the expectation. In order to estimate the PAPR of continuous-time OFDM signals, L-times oversampled OFDM signal is used. In [21], it is shown that $L = 4$ is sufficient to approximate the real peak of continuous-time OFDM signals. Also, the ratio of the maximum magnitude to the root mean square of the signal envelope is called the crest factor (CF) which is equivalent to the square root of PAPR as

$$CF = \sqrt{PAPR}. \tag{3}$$

For large N, the OFDM signal x_n can be modeled as a zero-mean complex Gaussian random variable and thus the magnitude of x_n follows Rayleigh distribution. In order to analyze PAPR, it is convenient to check the complementary cumulative distribution function (CCDF) of PAPR, i.e., the probability that the PAPR of OFDM signals exceeds a given threshold δ, which can be calculated as

$$P(PAPR > \delta) = 1 - \left(1 - e^{-\delta}\right)^{\alpha N} \tag{4}$$

where α is in general 2.8 from numerical analysis [22]. Equivalently, by simply changing PAPR by CF, the CCDF of CF is also given as

$$P\left(CF > \sqrt{\delta}\right) = 1 - \left(1 - e^{-\delta}\right)^{\alpha N}. \tag{5}$$

In the next subsections, for simplicity, we will explain some PTS schemes for the Nyquist-rate case ($L = 1$) and the over-sampling case ($L > 1$) is quite straightforward.

B. Conventional PTS Scheme

In the conventional PTS scheme, an input data vector $\mathbf{X}$ is partitioned into M disjoint input data subvectors $\mathbf{X}_m = [X_{m,0}, X_{m,1}, \cdots, X_{m,N-1}]^T$, $0 \le m \le M-1$, such that

$$\mathbf{X} = \sum_{m=0}^{M-1} \mathbf{X}_m. \tag{6}$$

By applying IFFT to each $\mathbf{X}_m$, the OFDM signal subvectors $\mathbf{x}_m = [x_{m,0}, x_{m,1}, \cdots, x_{m,N-1}]^T$ are generated and each $\mathbf{x}_m$ is independently rotated by multiplying the phase rotating factor $b_m = e^{j\phi_m}$ where $\phi_m \in [0, 2\pi)$ for $m = 0, \cdots, M-1$. In practice, the phase rotating factor b_m is an element of the finite set given as

$$b_m \in \left\{ e^{j2\pi l/W} \mid l = 0, 1, \cdots, W-1 \right\} \tag{7}$$

where W is the number of allowed phase rotating factors. Then, the phase rotating vectors are defined as $\mathbf{b}^{(u)} = [b_0^{(u)}, b_1^{(u)}, \cdots, b_{M-1}^{(u)}]$, $u = 0, 1, \cdots, U-1$, and the u-th alternative OFDM signal vector $\mathbf{x}^{(u)}$ is generated by

$$\begin{aligned}\mathbf{x}^{(u)} &= \left[x_0^{(u)}, x_1^{(u)}, \cdots, x_{N-1}^{(u)}\right]^T \\ &= \sum_{m=0}^{M-1} b_m^{(u)} \mathbf{x}_m, \qquad u = 0, 1, \cdots, U-1\end{aligned} \tag{8}$$

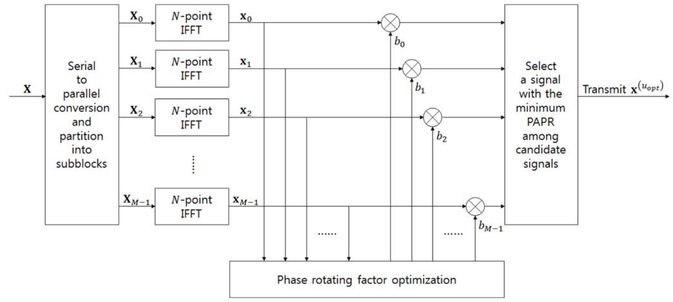

Fig. 1. A block diagram of the conventional PTS scheme.

where U is the number of alternative OFDM signal vectors. Since all the phase rotating factors $b_m^{(0)}$ for the first subblock $\mathbf{x}_0$ are fixed to 1, we have $U = W^{M-1}$ [4]. Finally, among U alternative OFDM signal vectors, the optimal OFDM signal vector $\mathbf{x}^{(u_{opt})}$ with the minimum PAPR is selected and transmitted. Fig. 1 shows a block diagram of the conventional PTS scheme.

Two disadvantages of the conventional PTS scheme are loss of data transmission rate due to the side information about u_{opt} and large computational complexity. The side information $\lceil \log_2 W^{M-1} \rceil$ bits for denoting the index of the selected phase rotating vector should be transmitted accompanying with the selected alternative OFDM signal vector. The computational complexity of the conventional PTS scheme mainly comes from M IFFTs and the generation and PAPR calculation of U alternative OFDM signal vectors.

C. Reduced-Complexity PTS Scheme

A reduced-complexity PTS (RC-PTS) scheme in [19] was proposed to reduce the computational complexity of the conventional PTS scheme, where the sum of sample powers of the OFDM signal subvectors $\mathbf{x}_m$ is used as a metric to find dominant time-domain samples which are used for calculating the PAPRs and selecting the alternative OFDM signal vector with the minimum PAPR. The RC-PTS is explained in detail as follows.

The power of the n-th sample of the u-th alternative OFDM signal vector can be written as

$$\begin{aligned}\left|x_n^{(u)}\right|^2 &= \left|\sum_{m=0}^{M-1} b_m^{(u)} x_{m,n}\right|^2 = \underbrace{\sum_{m=0}^{M-1} |x_{m,n}|^2}_{Q_n} \\ &\quad + \underbrace{\sum_{m_1=0}^{M-1} \sum_{\substack{m_2=0 \\ m_2 \ne m_1}}^{M-1} \left(b_{m_1}^{(u)} x_{m_1,n}\right)\left(b_{m_2}^{(u)} x_{m_2,n}\right)^*}_{V_n^{(u)}}\end{aligned} \tag{9}$$

where $(\cdot)^*$ denotes the complex conjugate, Q_n is a metric independent of the phase rotating vector $\mathbf{b}^{(u)}$ for a given $\mathbf{X}$, and $V_n^{(u)}$ is the cross-product term affected by $\mathbf{b}^{(u)}$. Let $\mathbf{S}_Q$ be the set of indices of samples defined as $\mathbf{S}_Q = \{n \mid Q_n \geqslant Th_Q, \quad 0 \le n \le N-1\}$ where Th_Q is a threshold determined by considering the PAPR reduction performance and the computational complexity. Then, only the samples $x_{m,n}$ of $\mathbf{x}_m$, $m = 0, 1, \cdots, M-1$, with indices $n \in \mathbf{S}_Q$ are multiplied by the phase rotating factors and used to estimate the PAPR values of U alternative OFDM signal vectors. The OFDM signal vector $\mathbf{x}^{(\tilde{u})}$ with the minimum PAPR is selected and transmitted, that is,

$$\tilde{u} = \arg\min_{u} \max_{n \in \mathbf{S}_Q} \left|\sum_{m=0}^{M-1} b_m^{(u)} x_{m,n}\right|^2. \tag{10}$$

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Therefore, the RC-PTS scheme can reduce the computational complexity because only a subset of time-domain samples is considered. Note that there exists a tradeoff between the accuracy of PAPR estimation and the reduction of computational complexity, which can be controlled by Th_Q.

III. New Low-Complexity PTS Schemes

A. Selection of Dominant Time-Domain Samples Using New Metrics

In the RC-PTS scheme, more accurate PAPR estimation is achieved by using a smaller threshold Th_Q because more time-domain samples are selected, but it gives rise to an increase in the computational complexity. In this subsection, two new metrics are proposed by analyzing the crest factor of alternative OFDM signal vectors, which enable the RC-PTS scheme to do better PAPR estimation with less dominant time-domain samples compared with the RC-PTS scheme using the metric Q_n in (9).

To select the alternative OFDM signal vector with the minimum PAPR, the amplitude of the n-th sample for all alternative OFDM signal vectors should be calculated as follows.

$$\left|x_n^{(u)}\right| = \left|\sum_{m=0}^{M-1} b_m^{(u)} x_{m,n}\right|. \tag{11}$$

Since $|x_n^{(u)}|$ clearly depends on the phase rotating vector $\mathbf{b}^{(u)}$, $|x_n^{(u)}|$ should be calculated for all $u \in \{0, 1, \cdots, U-1\}$. Therefore, to reduce the complexity for selecting dominant time-domain samples in the RC-PTS scheme, it is desirable for the metric to be independent of phase rotating vector as Q_n in (9). Now, two new metrics are proposed as follows by analysing $|x_n^{(u)}|$.

In (11), $|x_n^{(u)}|$ can be rewritten as

$$\left|x_n^{(u)}\right| = \left|\sum_{m=0}^{M-1} |x_{m,n}| e^{j\left(\theta_{m,n}+\phi_m^{(u)}\right)}\right| \tag{12}$$

where $\theta_{m,n}$ and $\phi_m^{(u)}$ are the phase of $x_{m,n}$ and $b_m^{(u)}$, respectively. The first proposed metric Y_n is obtained by removing the phase part of the summand in (12) as

$$Y_n = \sum_{m=0}^{M-1} \left|x_{m,n}\right|. \tag{13}$$

By using Q_n in (9), there always exists an error term $V_n^{(u)}$ shown in (9) in estimating the maximum power among the U n-th samples $x_n^{(u)}$, $0 \le u \le U-1$. Moreover, it is clear that even if W increases, Q_n does not approach to $\max_{u=0}^{U-1} |x_n^{(u)}|$ because of the error term $V_n^{(u)}$. On the contrary, Y_n approaches to $\max_{u=0}^{U-1} |x_n^{(u)}|$ as W increases. Therefore, for large W, it is highly probable that the dominant time-domain samples obtained by using Y_n are the real peaks. Moreover, through numerical analysis, it has been shown that the proposed low-complexity PTS scheme using Y_n outperforms the RC-PTS scheme using Q_n even for small W.

In order to derive the second metric, $|x_n^{(u)}|$ is rewritten as

$$\left|x_n^{(u)}\right| = \left|\sum_{m=0}^{M-1} b_m^{(u)} \left(\mathrm{Re}\{x_{m,n}\} + j\mathrm{Im}\{x_{m,n}\}\right)\right| \tag{14}$$

where $\mathrm{Re}\{x_{m,n}\}$ and $\mathrm{Im}\{x_{m,n}\}$ are the real and imaginary parts of $x_{m,n}$, respectively. Then, the second metric A_n is defined as

$$A_n = \left|\sum_{m=0}^{M-1} \left(\left|\mathrm{Re}\{x_{m,n}\}\right| + j\left|\mathrm{Im}\{x_{m,n}\}\right|\right)\right|. \tag{15}$$

In case of $W = 2$, it is clear that $A_n \geqslant |x_n^{(u)}|$ and A_n is equal to $\max_{u=0}^{U-1} |x_n^{(u)}|$ when the signs of real and imaginary parts of the n-th samples of the M OFDM signal subvectors $\mathbf{x}_m$ are the same, respectively. However, if $W > 2$, $A_n \geqslant |x_n^{(u)}|$ is not always true.

Now, the sets of indices of the dominant time-domain samples selected by using the metrics Y_n and A_n are defined as

$$\begin{aligned} \mathbf{S}_Y &= \{n \mid Y_n \geqslant Th_Y\} \\ \mathbf{S}_A &= \{n \mid A_n \geqslant Th_A\} \end{aligned} \tag{16}$$

where Th_Y and Th_A are the thresholds on Y_n and A_n, respectively. The cardinality of $\mathbf{S}_Y$ and $\mathbf{S}_A$ is denoted by $|\mathbf{S}_Y| = K_Y$ and $|\mathbf{S}_A| = K_A$, respectively. In general, the threshold is determined by considering the tradeoff between the computational complexity and the PAPR reduction performance. Note that only the dominant time-domain samples with the indices in $\mathbf{S}_Y$ or $\mathbf{S}_A$ are multiplied with the phase rotating factors are used to estimate the PAPRs of the U alternative OFDM signal vectors.

B. Sorting of Dominant Time-Domain Samples

For further lowering the computational complexity of the low-complexity PTS schemes in Section III-A without degrading the PAPR reduction performance, the selected dominant time-domain samples are sorted in decreasing order of their metric values as

$$\begin{aligned} \hat{\mathbf{S}}_Y &= \left\{p_0, \cdots, p_k, \cdots, p_{K_Y-1} \mid Y_{p_i} \geqslant Y_{p_{i+1}},\ p_k \in \mathbf{S}_Y\right\}, \\ \hat{\mathbf{S}}_A &= \left\{q_0, \cdots, q_k, \cdots, q_{K_A-1} \mid A_{q_i} \geqslant A_{q_{i+1}},\ q_k \in \mathbf{S}_A\right\}. \end{aligned} \tag{17}$$

Next, it will be explained how to further reduce the computational complexity only by using the sorted index sets $\hat{\mathbf{S}}_Y$ and $\hat{\mathbf{S}}_A$. For the first alternative OFDM signal vector, the power of each sample with the index in $\hat{\mathbf{S}}_Y$ (or $\hat{\mathbf{S}}_A$) is calculated in that order to estimate the PAPR. Then, denote the maximum sample power of the first alternative OFDM signal vector by γ. This γ is compared with the power of each sample of the second alternative OFDM signal vector, which has the index in $\hat{\mathbf{S}}_Y$ (or $\hat{\mathbf{S}}_A$) in that order. For instance, γ is compared with the power of $x_{p_k}^{(1)}$. If the power of $x_{p_k}^{(1)}$ is larger than γ, then stop calculating the power of the remaining samples and move to the third alternative OFDM signal vector to start the sample power calculation and comparison with γ similar to the previous case. If the power of $x_{p_k}^{(1)}$ is less than or equal to γ, then move to the next sample $x_{p_{k+1}}^{(1)}$ to calculate the power and compare it with γ. If all the sample powers are smaller than γ, then γ is updated with the maximum sample power of the second alternative OFDM signal vector and move to the third alternative OFDM signal vector to start the sample power calculation and comparison with γ similar to the previous case. This procedure is repeated until all alternative OFDM signal vectors are checked, the phase rotating vector giving the final value of γ is selected, and the corresponding alternative OFDM signal vector is transmitted.

Since the dominant time-domain samples are rearranged in decreasing order of their metric values, it is highly probable that samples with larger power are dealt earlier than those with smaller power. Let η be the average number of samples to be compared with γ until a sample with power bigger than γ is found. Using Monte Carlo simulation, Fig. 2 compares η for the sorted $\hat{\mathbf{S}}_Y$ and the unsorted $\mathbf{S}_Y$ for various set size, when $L = 4$, $M = 8$, $W = 2$, $N = 1024$, and 16-QAM are used. It is clear that η for the sorted case is much smaller than that for the unsorted case and therefore, the complexity of the proposed scheme is reduced by sorting the samples without performance degradation.

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TABLE I
COMPUTATIONAL COMPLEXITY OF THE CON-PTS, RC-PTS, AND PROPOSED PTS SCHEMES

<table>
<tr><td>Complexity</td><td colspan="4">No. of real multiplications</td><td colspan="4">No. of real additions</td></tr>
<tr><td>Step #</td><td>1)</td><td>3), 4)</td><td>5), 6), 7)</td><td>8)</td><td>2)</td><td>3), 4)</td><td>5), 6), 7)</td><td>8)</td></tr>
<tr><td>Con-PTS</td><td colspan="4">$4MLNU + 2LNU$</td><td colspan="4">$2MLNU + 2LNU(M-1) + 2LNU + U$</td></tr>
<tr><td>RC-PTS[i]</td><td>$2MLN$</td><td rowspan="3">$4KMU$ $+2KU$</td><td rowspan="3">0</td><td rowspan="3">$4LNM$ $+2LN$</td><td>$LN(M-1)+$ $MLN + LN$</td><td rowspan="3">$2KU(M-1)$ $+2KMU$ $+2KU + U$</td><td rowspan="3">0</td><td rowspan="3">$2LN(M-1)$ $+2LNM$</td></tr>
<tr><td>Proposed(Y)[i]</td><td>$3MLN$</td><td>$LN(M-1)+$ $MLN + LN$</td></tr>
<tr><td>Proposed(A)[i]</td><td>$2LN$</td><td>$2LN(M-1)+$ $2LN$</td></tr>
<tr><td>Proposed(SY)</td><td>$3MLN$</td><td rowspan="2">0</td><td rowspan="2">$4\eta MU$ $+2\eta U$</td><td rowspan="2">$4LNM$ $+2LN$</td><td>$2LN(M-1)$</td><td rowspan="2">$K \log_2 K$</td><td rowspan="2">$2\eta MU+$ $2\eta U(M-1) + \eta U$</td><td rowspan="2">$2LN(M-1)$ $+2LNM$</td></tr>
<tr><td>Proposed(SA)</td><td>$2LN$</td><td>$2LN(M-1)$</td></tr>
</table>

i) These PTS schemes require Steps 1), 2), 3), and 8).

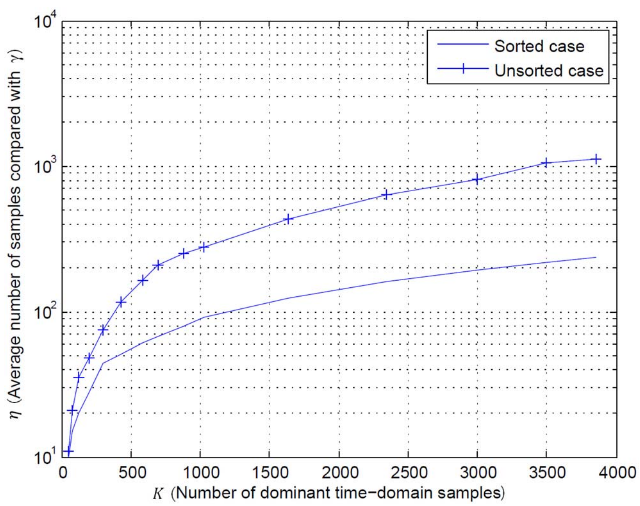

Fig. 2. Comparison of the average number of compared samples for the sorted and unsorted cases when $L = 4$, $M = 8$, $W = 2$, $N = 1024$, 16-QAM, and the metric Y are used.

The proposed low-complexity PTS schemes are summarized as follows.

1) An input data vector is divided into M disjoint subblocks and each of them is IFFTed.
2) Determine $\mathbf{S}_Y$ (or $\mathbf{S}_A$) in (16).
3) If the proposed scheme without sorting in Section III-A is used, go to Step 5). Otherwise, go to Step 4).
4) Sort the elements in $\mathbf{S}_Y$ or ($\mathbf{S}_A$) in decreasing order of their corresponding metric values and denote it again as $\mathbf{S}_Y$ (or $\mathbf{S}_A$).
5) Set γ as the maximum sample power among samples of the first alternative OFDM signal vector, which have indices in $\mathbf{S}_Y$ (or $\mathbf{S}_A$).
6) Each sample power having index in $\mathbf{S}_Y$ (or $\mathbf{S}_A$) of the second alternative OFDM signal vector is compared with γ. If a sample power is larger than γ, stop calculating sample power and go to Step 7). Otherwise, γ is updated by the maximum sample power of the second alternative OFDM signal vector and go to Step 7).
7) Repeat Step 6) for all the remaining alternative OFDM signal vectors and, the phase rotating vector giving the final γ is used to generate the optimal OFDM signal vector $\mathbf{x}^{(u_{opt})}$.
8) Transmit the optimal OFDM signal vector $\mathbf{x}^{(u_{opt})}$ with the side information on u_{opt}.

C. Computational Complexity Analysis

Table I compares the computational complexity in terms of real multiplication and real addition for the conventional PTS (Con-PTS), RC-PTS, two proposed PTS schemes using $\mathbf{S}_Y$ (Proposed(Y)) and using $\mathbf{S}_A$ (Proposed(A)) in Section III-A, and two proposed PTS schemes using sorted $\hat{\mathbf{S}}_Y$ (Proposed(SY)) and using sorted $\hat{\mathbf{S}}_A$ (Proposed(SA)) in Section III-B. When the number of subblocks is fixed, the computational complexity for Step 1) is the same for all PTS schemes and therefore, only the other steps are considered. In general, one complex multiplication requires four real multiplications and two real additions, and one complex addition requires two real additions. Also, one comparison and one square root operation are equivalent to one real addition and one real multiplication, respectively. The number of dominant time-domain samples is denoted by K and quick sorting algorithm [23] is used at Step 4), which requires $K \log_2 K$ real additions. Note that the average number of samples compared with γ is denoted by η as shown in Fig. 2 which shows the relation between η and K.

It is clear that K is an important factor for the computational complexity of Proposed(Y) and Proposed(A), whereas η is an important factor for the computational complexity of Proposed(SY) and Proposed(SA). The detailed numerical analysis is given in the following section.

IV. SIMULATION RESULTS

Fig. 3 compares the PAPR reduction performances of four proposed PTS schemes (Proposed(Y), Proposed(A), Proposed(SY), and Proposed(SA)) with those of RC-PTS and Con-PTS when $N = 1024$, $L = 4$, $M = 8$, $W = 2$, and 16-QAM are used. For fair comparison, the same K is used for the proposed low-complexity PTS schemes by properly adjusting the threshold value for each scheme and the corresponding η is determined as $\eta = 76$ for $K = 800$ and $\eta = 92$ for $K = 1100$ through intesive simulation. Fig. 3 shows that the PAPR reduction performance of Proposed(SY) and Proposed(SA) is exactly the same as that of Proposed(Y) and Proposed(A), respectively, as expected. For the same K, the proposed PTS schemes show better PAPR reduction performance than RC-PTS scheme. Moreover, to show the same PAPR reduction performance as the conventional PTS (Con-PTS), $K = 800$ for Proposed(A), $\eta = 76$ ($K = 800$) for Proposed(SA), $K = 1100$ for Proposed(Y), $\eta = 92$ ($K = 1100$) for Proposed(SY), and $K = 1400$ for RC-PTS are needed.

Fig. 4 compares the computational complexity in terms of relative computational complexity (%) compared with that of the conventional PTS when all schemes show the same PAPR reduction performance as explained in the above sentence. The relative

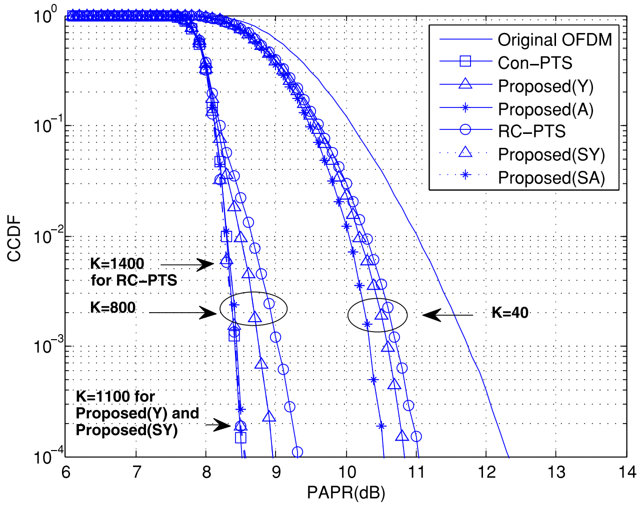

Fig. 3. Comparison of PAPR reduction performance of Con-PTS, RC-PTS, and four proposed PTS schemes when $N = 1024$, $L = 4$, $M = 8$, $W = 2$, and 16-QAM are used.

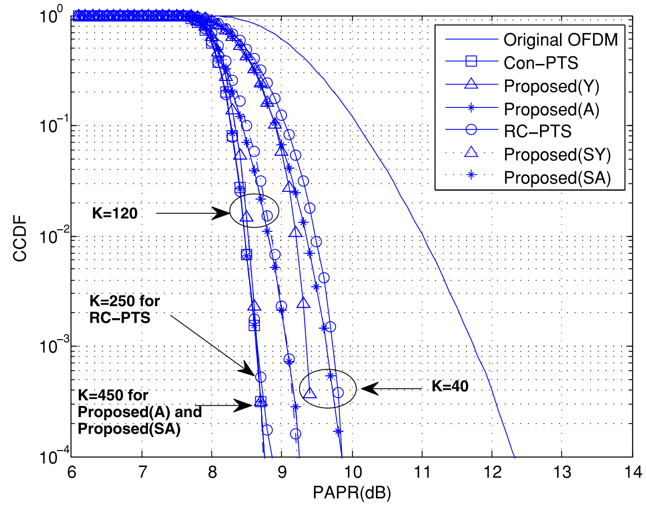

Fig. 5. Comparison of PAPR reduction performance of Con-PTS, RC-PTS, and four proposed PTS schemes when $N = 1024$, $L = 4$, $M = 4$, $W = 4$, and 16-QAM are used.

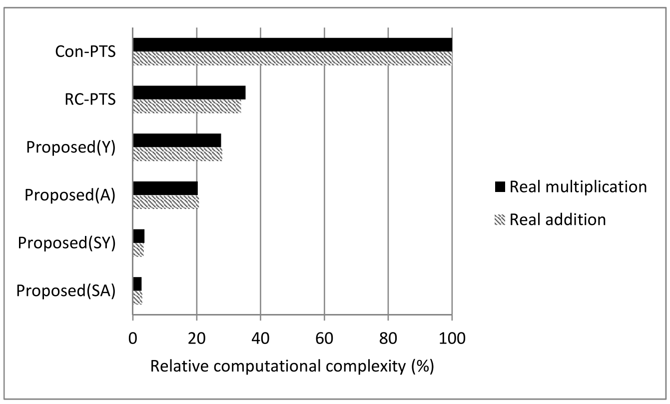

Fig. 4. Comparison of computational complexity of Con-PTS, RC-PTS, and four proposed PTS schemes when all schemes show the same PAPR reduction performance for $N = 1024$, $L = 4$, $M = 8$, $W = 2$, and 16-QAM being used.

computational complexity of RC-PTS, Proposed(Y), Proposed(A), Proposed(SY), and Proposed(SA) in terms of real multiplication is 35.3%, 27.6%, 20.3%, 3.6%, and 2.7%, respectively. Also, the relative computational complexity in terms of real addition shows a similar tendency as the relative computational complexity in terms of real multiplication, such as 33.9%, 28%, 20.7%, 3.4%, and 2.9%, respectively. Under the same PAPR reduction performance, the Proposed(SA) requires the least computational complexity, 3% of the computational complexity of the conventional PTS (Con-PTS).

Fig. 5 compares the PAPR reduction performance of Con-PTS, RC-PTS, and four proposed PTS schemes with $N = 1024$, $L = 4$, $M = 4$, $W = 4$, and 16-QAM. It shows that Proposed(Y) requires $K = 120$ to achieve the same PAPR reduction performance as Con-PTS, meanwhile RC-PTS and Proposed(A) require $K = 250$ and $K = 450$, respectively. The value of η of Proposed(SY) is 24 for $K = 120$, whereas the value of η of Proposed(SA) is 55 for $K = 450$. It can be easily shown that η is propotional to K. From this observation, we can see that in order to achieve the same PAPR reduction performance as Con-PTS, η of RC-PTS ($K = 250$) using the sorting as in Section III-A should be larger than that of Proposed(SY) ($K = 120$). By using Table I, when $N = 1024$, $L = 4$, $M = 4$, $W = 4$, compared with the conventional PTS, the relative computational complexity in terms of real multiplication for RC-PTS, Proposed(Y), Proposed(A), Proposed(SY), and Proposed(SA) is calculated as 8.4%, 5.5%, 12.7%, 3.2%, and 3.1% of that for conventional PTS, respectively, and the relative computational complexity in terms of real addition is calculated as 8.3%, 5.1%, 13.1%, 2.5%, and 3.2%, respectively. Note that the computational complexity of Proposed(A) is higher than that of RC-PTS but the computational complexity of Proposed(SA) is much lower while keeping the same PAPR reduction performance.

V. Conclusion

In this paper, two effective metrics are derived to be used for selecting dominant time-domain OFDM signal samples and two low-complexity PTS schemes based on these two metrics are proposed. Two proposed metrics A_n and Y_n show very good PAPR reduction performance for $W = 2$ and $W \geqslant 4$, respectively. Furthermore, two proposed schemes using A_n and Y_n require much lower computational complexity to achieve the same PAPR reduction performance as the conventional PTS scheme. For more complexity reduction, sorting the selected dominant time-domain samples is proposed. Numerical analysis shows that the proposed PTS schemes using sorting achieve the same PAPR reduction performance as that of the conventional PTS scheme with substantially reduced computational complexity. Also, it is shown that compared with RC-PTS scheme, the proposed PTS schemes using sorting require much lower computational complexity while achieving the same PAPR reduction performance as that of the conventional PTS scheme.

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